Date 09/09/21
Page

Unit-I

Relativistic Mechanics

☆ **Degree of freedom :-**

Degree of freedom may be defined as the no. of independent coordinates required to specifies a motion in that direction.

☆ **Frame of reference**

The system relative to which the position OR the motion of a body specified is called frame of reference.

OR

An arbetery set of axis with reference to which the position or Motion of something is describe or physical laws are formilated.

→ Most simple frame of reference is the cartessian coordinate system. In which the position of a moving particle at any instant is express in terms of coordinates (x, y, z) or by position vector

$$\vec{OP} = \vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Velocity of the particle is

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx\hat{i}}{dt} + \frac{dy\hat{j}}{dt} + \frac{dz\hat{k}}{dt}$$

Date 10/09/21
Page

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2 x}{dt^2}\hat{i} + \frac{d^2 y}{dt^2}.\hat{j} + \frac{d^2}{dt^2} z\hat{k}$$

☆ Types of frame of reference

There are two types of frame of reference.

1. Inertial frame of reference
2. Non- Inertial frame of reference

1. Inertial frame of reference

- The frame of reference in which the Newton's law followed.
- All those frame of reference which are either relationally related to each other on in uniform motion are known as Inertial frame of Reference.
- Inertial frame of Reference are necessary the unaccelerated frame of reference.

2. Non- Inertial frame of reference.

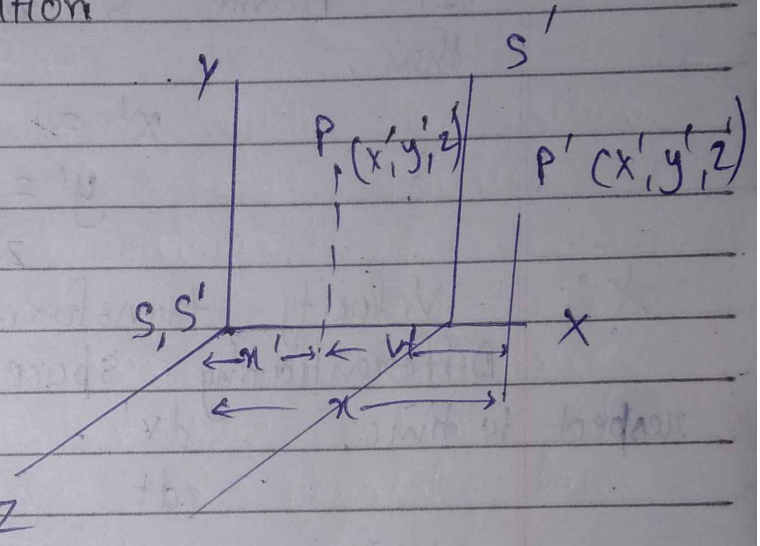
1. The frame of Reference in which Newton's law are not valid.
2. All the accelerated frame are Non-inertial frame.

Date
Page

3. Non-inertial frame of reference gives the Concept of pseudo force.

4. Pseudo force
It is an apperant force. which is used to balance action force.

☆ Gallilean transformation



A particle has different coordinate in a different frame at the same instant. The equations which relate the Coordinate of two frame of reference are called transformation equation.

The equations relative the coordinate of a particle in two different inertial frame are called The Gallian transformation.

Suppose A particle be in frame (s') which moves at Constant velocity (v) with respect to frame (s).

suppose The position of particle after moving is (x, y, z) from s and position of particle from s' is (x', y', z')

If frame s' moving along +ve x-direction then,

$$x' = x - vt$$
$$y' = y$$
$$z' = z$$

☆ Velocity transformation

Differentiating space Gallilean transformation with respect to time.

$$\frac{dx'}{dt} = \frac{dx}{dt} - v\frac{dt}{dt}$$

$$\boxed{v'_x = u_x - v} \quad \text{— ①}$$

$$\frac{dy'}{dt} = \frac{dy}{dt}$$

$$\boxed{u'_y = u_y} \quad \text{— ②}$$

$$\frac{dz'}{dt} = \frac{dz}{dt}$$

$$\boxed{u'_z = u_z} \quad \text{— ③}$$

Acceleration transformation

Again Differentiating the above equation.

$$\frac{d}{dt} u_x' = \frac{du_x}{dt} - \frac{dv}{dt}$$

$$\boxed{a_x' = a_x} \quad — \quad ①$$

$$\frac{du_y'}{dt} = \frac{du_y}{dt}$$

$$\boxed{a_y' = a_y}$$

$$\frac{du_z'}{dt} = \frac{du_z}{dt}$$

$$\boxed{a_z' = a_z}$$

Inverse Galilian transformation Equation.

$$\boxed{x = x' + vt}$$

$$y = y'$$

$$z = z'$$

Velocity :-

$$u_x = u_x' + v$$

$$u_y = u_y'$$

$$u_z = u_z'$$

Accelerated :-

$$a_x = a_x'$$

$$a_y = a_y'$$

$$a_z = a_z'$$

Date 18/09/21
Page

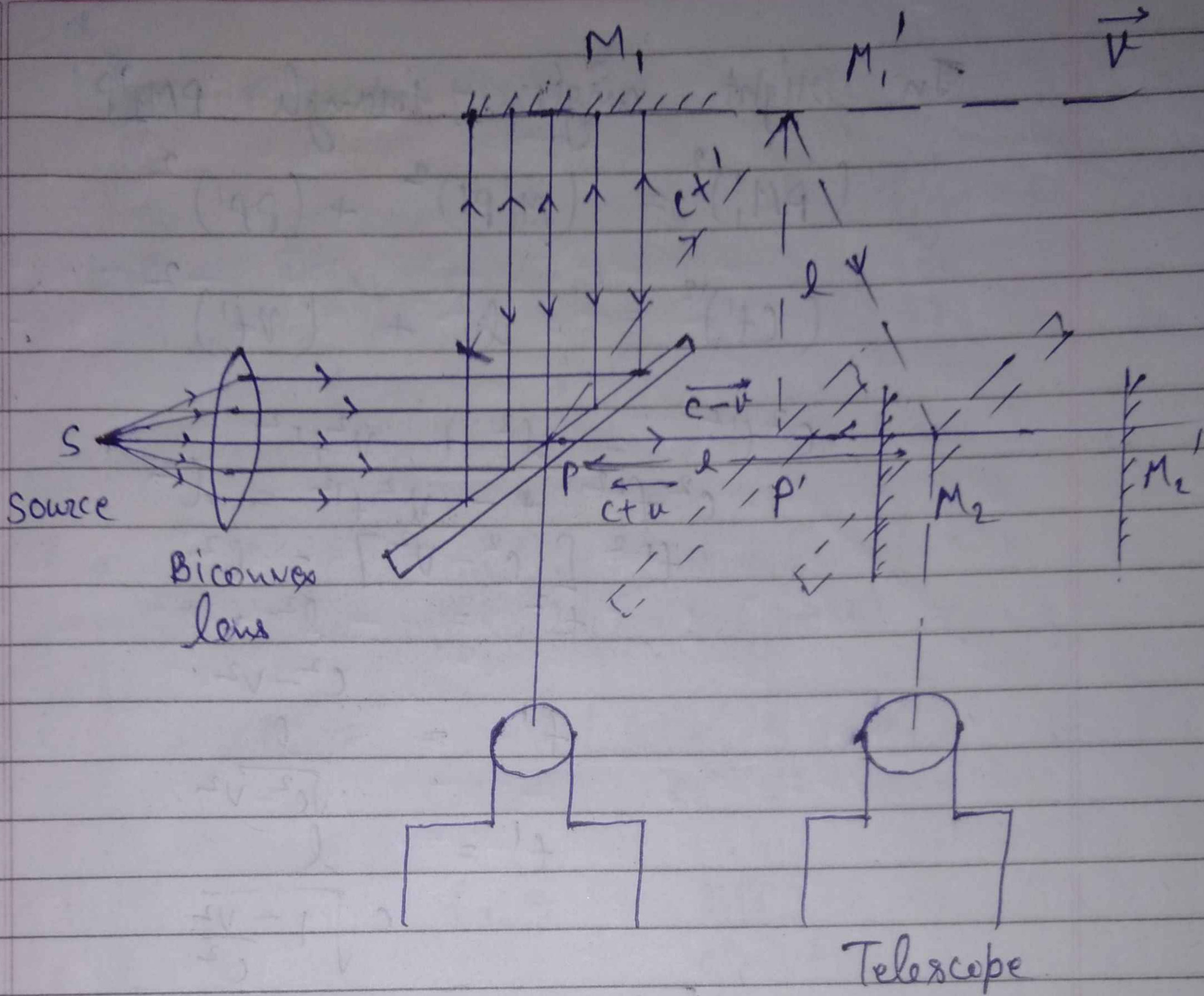
★ Michelson - Morley Experiment :-

To detect relative motion b/w the body (Earth) and a hypothetical medium (Luminus ferrus Ether), Michaelson and Morley perform a very significant experiment in 1887. Using Michaelson Interferrometer.
The main object of this experiment is to show the presence of ether in the space.

1. This experiment consist of a monochromatic source of light (S), Biconvex lens (l) plane mirror (M_1 and M_2) and A transparent glass slab, which is inclined at 45° from the principle axis of light rays.

2. So that light may be divided into two parts.
① Reflection ② Refraction

3. Mirror (M_1 and M_2) are placed equal distance (l) from the glass slab: at an angle 90° to each other.



If speed of light is 'c'
The ray travel PM_1' is covered in t' sec then distance PM_1' is equal to ct'

$$\boxed{PM_1' = ct'}$$

Path PP' have consume same time (that is) t' is taken, by the Apparatus to covered this Path

$$\boxed{PP' = vt'}$$

In right angle triangle $PM_1'P'$

$$(PM_1')^2 = (M_1'P')^2 + (PP')^2$$

$$(ct')^2 = l^2 + (vt')^2$$

$$c^2t'^2 = l^2 + v^2t'^2$$

$$c^2t'^2 - v^2t'^2 = l^2$$

$$t'^2[c^2 - v^2] = l^2$$

$$t'^2 = \frac{l^2}{c^2 - v^2}$$

$$t' = \frac{l}{\sqrt{c^2 - v^2}}$$

$$t' = \frac{l}{c\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{t' = \frac{l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}}$$

The ray of light consume same time (t') for the path $M_1'P_2$ then time taken by the light to cover path $PM_1'P_2$ is equal to

$$t_1 = PM_1'P_2 = 2t' = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}$$

total time of light $\boxed{t_1 = \frac{2l}{c}\left(1 - \frac{v^2}{c^2}\right)^{-1/2}}$

For path PM_2P

Time consume by ray of light to cover path $PM_2 \longrightarrow M_2P$

$$t_2 = \frac{l}{c-v} + \frac{l}{c+v}$$

$$t_2 = \frac{l[c+v] + l[c-v]}{(c-v)(c+v)}$$

$$t_2 = \frac{l[c+\cancel{v} + c-\cancel{v}]}{c^2-v^2}$$

$$t_2 = \frac{2lc}{c^2-v^2}$$

$$t_2 = \frac{2lc}{c^2\left(1-\frac{v^2}{c^2}\right)}$$

$$\boxed{t_2 = \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}}$$

Time difference b/w $PM_1' \rightarrow M_1'P_2$ and $PM_2 \rightarrow M_2P$

$$\Delta t = t_1 - t_2$$

$$= \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1/2} - \frac{2l}{c}\left(1-\frac{v^2}{c^2}\right)^{-1}$$

$=$

$$= \frac{2l}{c}\left[\left(1-\frac{v^2}{c^2}\right)^{-\frac{1}{2}} - \left(1-\frac{v^2}{c^2}\right)^{-1}\right]$$

$$= \frac{2l}{c}\left[1 - \frac{1}{2}\left(\frac{-v^2}{c^2}\right) + \left(-\frac{1}{2}\right)\left(\frac{v^2}{c^2}\right)^2 \times \frac{1}{2} - 1 - \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{2}\left(\frac{v^2}{c^2}\right)\right]$$

we neglect $\frac{v^2}{c^2}$ is ~~ver~~ small so $\left(\frac{v^2}{c^2}\right)^2$ is very small.

$$= \frac{2l}{c}\left[\cancel{1} + \frac{1}{2}\frac{v^2}{c^2} - \cancel{1} - \frac{v^2}{c^2}\right]$$

$$= \frac{2l}{c}\left[-\frac{1}{2}\frac{v^2}{c^2}\right]$$

$$= -\frac{\cancel{2}l}{c} \times \frac{v^2}{\cancel{2}c^2}$$

$$\boxed{\Delta t = -\frac{lv^2}{c^3}}$$

If the apprratus arrangement rotate through 90°, then the time difference b/w reflected rays from both the mirrors (similarly above derivation)

$$\Delta t_2 = \frac{lv^2}{c^3}$$

hence total time different b/w initial and final apparatus arrangement that is

Date
Page

Total time difference (Δ total) $= \Delta t_2 - \Delta t_1$

$$= \frac{lv^2}{c^3} - \left(-\frac{lv^2}{c^3}\right)$$

$$= \frac{lv^2}{c^3} + \frac{lv^2}{c^3}$$

$$= \frac{2lv^2}{c^3}$$

Total path difference will be speed of light multiply by total time difference.

Total path difference (δ) $= c \times \Delta$ total

$$= \cancel{c} \times \frac{2lv^2}{\cancel{c^3}\,c^2}$$

$$= \frac{2lv^2}{c^2}$$

If the no. of fringes made, In interfearence pattern is 'N' And wavelength of visible light is 'λ' then no. of fringes will be.

$$\delta = N \times \lambda$$

$$N = \frac{\delta}{\lambda}$$

$$\boxed{N = \frac{2lv^2}{\lambda c^2}}$$ Imp

Michael son and Morley haftane Values of different terms,

$l = 11$ metre $\quad V_e = 3 \times 10^4$ m/s

$\lambda = 6000\, A^{o} = 6 \times 10^{-7}$ m

$c = 3 \times 10^8$ m/s

$$N = \frac{\cancel{2} \times 11 \times (3 \times 10^4)^2}{\overset{3}{\cancel{6}} \times 10^{-7} \times (3 \times 10^8)^2}$$

$$= \frac{11 \times \cancel{9} \times 10^8}{3 \times 10^{-7} \times \cancel{9} \times 10^{16}}$$

$$= \frac{11 \times 10^8}{3 \times 10^9}$$

$$N = \frac{11}{30} = 0.37$$

$$\boxed{N = 0.37}$$

Michael son and Morley observe that the fringes makes b/w these two beams is approx 0.37 which is very very small that means there is no fringe shift in this experiment occur.

Hence According to Michelson & Morley experiment no relative motion b/w the earth and ether

① Date 06/10/24
Page

Q-1 What will be the astracted fringes shift on the basis of stationary ether hypothesis in Michaelson & Morley experiment. If the effective length of each path is 8m And wavelength used is 8000 A°. Take $v = 3 \times 10^4$ m/s.

Sol:- $$N = \frac{2 l V^2}{\lambda c}$$

$$= \frac{2 \times \cancel{8} \times \cancel{9} \times \cancel{10^8}}{\cancel{8} \times 10^{-7} \times \cancel{3} \times \cancel{10^8} \times \cancel{3} \times 10^8}$$

$$= \frac{2}{10} = 0.2 \text{ Ans}$$

☆ Postulate of special theory of Relativity :-

1. The frame of reference in which the particle move are consider inertial frame of reference.

2. A/c to Second postulate of special theory of Relativity speed of light is constant in all inertial frame of reference.

SHOT ON OPPO
By Ghanendra Kumar

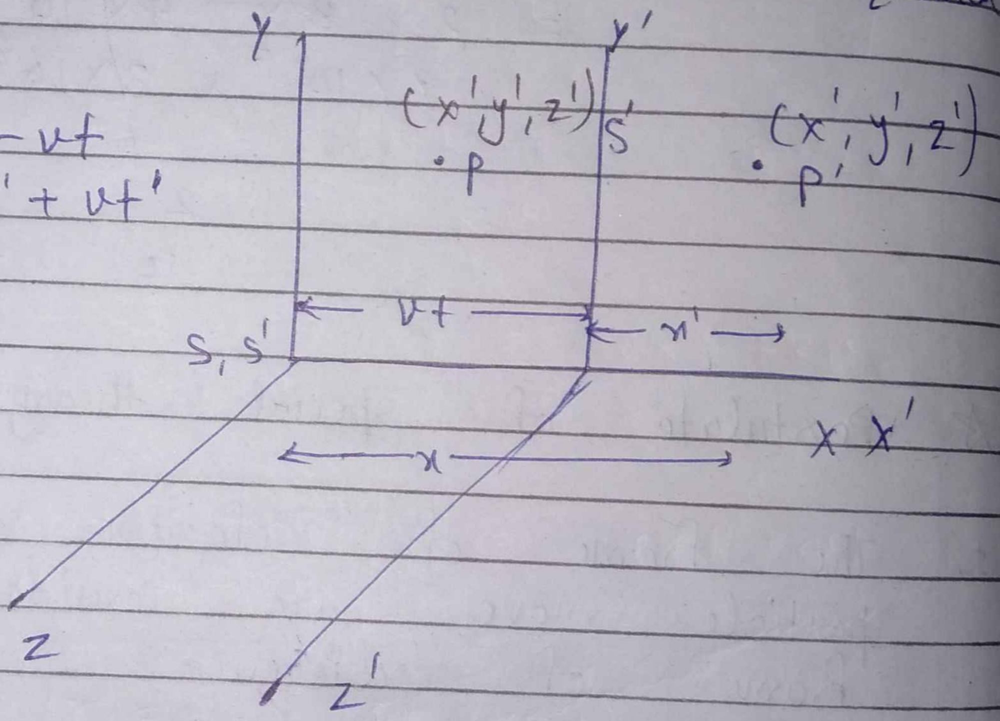
Date 07/10/21
Page

☆ Lorentz's transformation Equation :-

The equation in which special theory of relativity which relate to the space and time coordinate of an event in two inertial frame of reference moving with uniform velocity relative to one another are called Lorentz's transformation equation

$x' = x - vt$

$x = x' + vt'$



Let us consider two frame of reference S and S' in which S' is moving with velocity v' along positive x-direction. The coordinates of point or P particle from S and S' are (x, y, z) and (x', y', z') respectively.

Date
Page

According to first postulate of theory of Relativity.

$$x' \propto (x - vt)$$
$$x' = k\ (x - vt) \quad — (1)$$

where k proportionality Constant

$$x \propto (x' + vt')$$
$$x = k\ (x' + vt') \quad — (2)$$

Substituting the Value of x' in eq (2)

$$x = k\left[\ k(x - vt) + vt'\right]$$

$$\frac{x}{k} = \left[kx - kvt + vt'\right]$$

$$vt' = \frac{x}{k} - kx + kvt$$

$$vt' = \frac{x - k^2x + k^2vt}{k}$$

$$t' = \frac{x - k^2x + k^2vt}{vk}$$

$$\boxed{t' = \frac{x - k^2x}{vk} + kt} \quad — (iii)$$

Date
Page

According to second postulate of theory of relativity.

$$x = ct \quad - \text{(iv)}$$
$$x' = ct' \quad - \text{(v)}$$

on Substitution the value of x' & t' in eq (v)

$$k(x - vt) = c\left[\frac{(1-k^2)x}{kv} + kt\right]$$

$$kx - kvt = \frac{(1-k^2)xc}{kv} + ckt$$

$$\left[kx - \frac{(1-k^2)c}{kv}x\right] = ckt + kvt$$

$$\left[k - \frac{(1-k^2)c}{kv}\right]x = ckt + kvt$$

$$\boxed{x = \frac{(kc + kv)}{\left[k - \frac{(1-k^2)c}{kv}\right]}t} \quad - \text{(vi)}$$

on comparing eq (vi) and eq (iv) we get

SHOT ON OPPO

$$c = \frac{(kc + kv)}{\left[k - \frac{(1-k^2)c}{kv}\right]}$$

$$ck - \frac{(1-k^2)c^2}{kv} = kc + kv$$

$$\frac{vck^2 - c^2 + k^2c^2}{kv} = (kc + kv)$$

$$vck^2 - c^2 + k^2c^2 = k^2vc + k^2v^2$$

$$vck^2 + k^2c^2 - k^2vc - k^2v^2 = c^2$$

$$\left[\cancel{vc} + c^2 - \cancel{vc} - v^2\right]k^2 = c^2$$

$$c^2 = k^2(c^2 - v^2)$$

$$k^2 = \frac{c^2}{c^2 - v^2}$$

$$k^2 = \frac{\cancel{c^2}}{\cancel{c^2}\left[1 - \frac{v^2}{c^2}\right]}$$

$$k^2 = \frac{1}{\left[1 - \frac{v^2}{c^2}\right]}$$

$$\boxed{k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

SHOT ON OPPO

Substitutive the value of t' in eq (5)

$$x = k(x' - vt')$$

$$x' = k(x - vt)$$

$$\boxed{x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}}$$ Lorentz first transformation equation

$$\boxed{y' = y}$$ Second

$$\boxed{z' = z}$$ Third

Substitutive the value of k in eq (3)

$$t' = \frac{(1-k^2)x}{kv} + kt$$

$$= \frac{\left[1 - \frac{1}{\left(1 - \frac{v^2}{c^2}\right)}\right] x}{\frac{v}{\sqrt{1 - \frac{v^2}{c^2}}}} + \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x \, \dfrac{\left[\cancel{1} - \frac{v^{\cancel{2}}}{c^2} - \cancel{1}\right]}{\left(1 - \frac{v^2}{c^2}\right)}}{\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \cdot \cancel{v}} + \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{-xv\sqrt{1-\frac{v^2}{c^2}}}{c^2\left(1-\frac{v^2}{c^2}\right)} + \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{-vx}{c^2\sqrt{1-\frac{v^2}{c^2}}} + \frac{t}{\sqrt{1-\frac{v^2}{c^2}}}$$

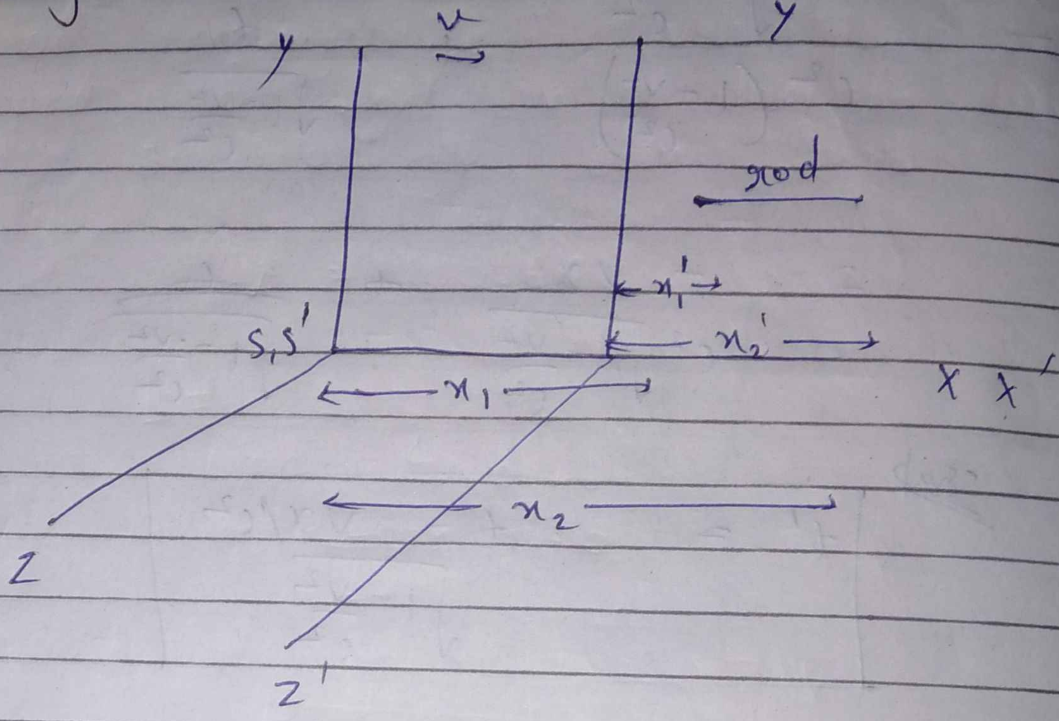
Imp

$$\boxed{t' = \frac{t - vx/c^2}{\sqrt{1-\frac{v^2}{c^2}}}}$$

SHOT ON OPPO
By Ghanendra Kumar

Date 09/10/21

☆ Length Contraction :-



When a rod of length L_0 replaced in frame of refrence S' which moves with velocity v along xx' axis. when an observer, observe this length of rod from frame of reference S, the appreared length are observed decreases along the direction of motion. This length is known as length Contraction.

Suppose L_0 is actual length or proper length and L is apperent length of a rod placed horizontally in frame S'.

$x_2' - x_1' = L_0$ (Actual or proper length) — ①
$x_2 - x_1 = L$ (Apperent length)

Using Lorentz's transformation equation →

$$x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

on Substitution the Value x_1' and x_2' in eq ①

$$L_0 = x_2' - x_1'$$

$$= \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} - \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{x_2 - \cancel{vt} - x_1 + \cancel{vt}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$L_0 = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

v. Imp

$$\boxed{L = L_0\sqrt{1 - \frac{v^2}{c^2}}}$$

where $L < L_0$

Q-1 Calculate the %age contraction of a rod moving with velocity of 0.6 of speed of light in a direction inclined at 30° to its own length.

Sol:-

$$v = 0.6c$$

$$L_y = L_0 \sin 30°$$

$$L_y = 0.5 L_0$$

$$\frac{L_y}{L_0} = 0.5$$

$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$L_x = L_0 \cos 30° \sqrt{1 - \frac{v^2}{c^2}}$$

$$= L_0 \times \frac{\sqrt{3}}{2} \times \sqrt{1 - 0.36}$$

$$= L_0 \times \frac{\sqrt{3}}{\cancel{2}} \times \cancel{0.8}$$

$$L_x = 0.4\sqrt{3} L_0$$

$$L_R = \sqrt{L_x^2 + L_y^2}$$

$$= \sqrt{(0.4\sqrt{3} L_0)^2 + (0.5 L_0)^2}$$

③

Date
Page

$$L_R = L_0\sqrt{0.48+0.29}$$

$$L_R = 0.854L_0$$

$$\%\text{age Reduction} = \frac{L_0 - 0.854L_0}{L_0} \times 100$$

$$= 0.146 \times 100 = 14.6\%$$

Q→2 How much time a metre stick moving at 0.100 of speed of light c Relative to an observer take to pass the observer? The metre stick is parallel to its direction of motion.

Sol:

$$v = 0.1c$$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1 \times \sqrt{1-\left(\frac{0.1c}{c}\right)^2}$$

$$= 1 \times \sqrt{1-0.01}$$

$$= \sqrt{0.99}$$

$$L = 0.9$$

$$v = \frac{d}{t}$$

$$t = \frac{d}{v} = \frac{0.9}{0.1 \times c}$$

$$t = \frac{9}{3} \times 10^{-8}$$

$$\boxed{t = 3 \times 10^{-8}\ \text{sec}}$$

Q-3 How fast should a rocket move relative to an observer that its length may appear to the observer to be 99% of its proper length.

Sol

$$L = 99\% \; L_0$$

$$L = \frac{99}{100} L_0$$

$$\frac{L}{L_0} = \frac{99}{100}$$

$$\frac{L}{L_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$(0.99)^2 = 1 - \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - (0.99)^2$$

$$\frac{v^2}{c^2} = 0.0199$$

$$v^2 = 0.0199 \times (3 \times 10^8)^2$$

$$v = 3 \times 10^8 \sqrt{0.0199}$$

$$v = 3 \times 10^8 \times 0.141$$

$$v = 0.423 \times 10^8$$

$$\boxed{v = 42.3 \times 10^6 \; m/s}$$

Q-4 What will be the apperent length of a meter stick measured by an observer at rest when the stick is moving along its length with a velocity equal to $\frac{\sqrt{3}}{2}c$.

Sol,

$$v = \frac{\sqrt{3}}{2}c \quad , L_0 = 1\,m$$

$$L = L_0\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1 \times \sqrt{1-\left(\frac{\sqrt{3}\cancel{c}}{2\cancel{c}}\right)^2}$$

$$= 1 \times \sqrt{1-\frac{3}{4}}$$

$$L = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

$$\boxed{L = 0.5\ m}$$

Q-5 In Michaelson Morley experiment the length of the path of the two beams is 11m each the wavelength of light is ~~the~~ each ~~the~~ ~~w~~ use 6000 A°. If the expected fringe shift in 0.4. Calculate the velocity of earth relation to eather.

Sol,

$$N = 0.4, \quad \lambda = 6 \times 10^{-7}\ m, \quad l = 11\ m$$

$$c = 3 \times 10^{8}$$

$$v = ?$$

$$N = \frac{2 l v^2}{\lambda c^2}$$

$$v^2 = \frac{N \lambda c^2}{2l}$$

$$= \frac{0.4 \times \cancel{6}^{3} \times 10^{-7} \times 9 \times 10^{16}}{\cancel{2} \times 11 \times 10}$$

$$= \frac{12 \times 9}{11} \times 10^{8}$$

$$= \frac{108}{11} \times 10^{8}$$

$$v^2 = 9.8 \times 10^{8}$$

$$v = \sqrt{9.8 \times 10^{8}}$$

$$\boxed{v = 3.1 \times 10^{4} \ m/s}$$

6. As measured by O off flash bulb these of at $x = 100km$, $y = 10 Km$ and $z = 1Km$ at $t = 5 \times 10^{-4}$ sec. what are the Co-ordinate x', y', z' and t' of this event has determined by a second observer O'. Moving with relative to O at $-0.8C$ where c is the speed of light along the common xx' axis.

Given ⇒ $x = 100$ km, $y = 10$ km, $z = 1$ Km, $t = 5 \times 10^{-4}$, $v = -0.8C$

$x = 10^{5}$ m, $y = 10^{4}$ m, $z = 10^{3}$ m

Date 11/10/21

Page

(3) Sol→

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{10^5 - (-0.8c) \times 5 \times 10^{-4}}{\sqrt{1 - \left(\frac{-0.8\cancel{c}}{\cancel{c}}\right)^2}}$$

$$= \frac{10^5 + 0.8 \times 5 \times 3 \times 10^8 \times 10^{-4}}{\sqrt{1 - 0.64}}$$

$$= \frac{10^5 + 4 \times 3 \times 10^4}{0.6}$$

$$= \frac{10^5 + 1.2 \times 10^5}{0.6}$$

$$= \frac{10^5 [1 + 1.2]}{0.6}$$

$$= \frac{2.2}{0.6} \times 10^5$$

$$= 3.67 \times 10^5$$

$$\boxed{x' = 367 \text{ km}}$$

$$y' = 10 \text{ km}$$

$$z' = 1 \text{ km}$$

$$t' = \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{5 \times 10^{-4} - \frac{-0.8\cancel{c} \times 10^5}{c^{\cancel{2}}}}{0.6}$$

$$= \frac{5 \times 10^{-4} + \frac{0.8 \times 10^5}{3 \times 10^8 \times 10}}{0.6}$$

$$= \frac{5 \times 10^{-4} + 2.67 \times 10^{-4}}{0.6} = 10^{-4} \times \frac{7.67}{0.6} \times 10$$

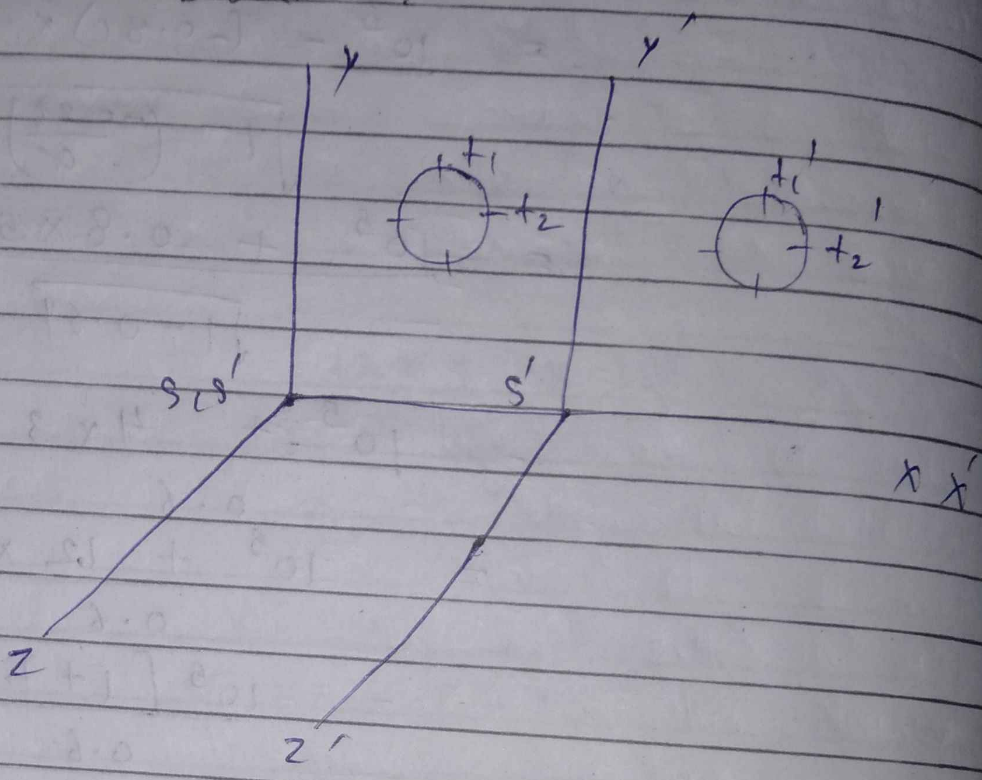
$$= 127.83 \times 10^5$$

$$t' = 1.28 \times 10^7 \text{ sec}$$

Date 12/10/24
Page

(2) ~~How much time~~

☆ Time Dilation :-



The time interval b/w two event that occurs at the same place in an observer frame of Refrence is called the proper time of the interval b/w the events.

Let clock placed at the frame S' which is moving with velocity v with r-t frame 'S'.
An observer in frame S' obserse that the clock gives two ticks at time t_1' and t_2'. And observer in frame S observe that the clock gives two ticks at the time t_1 and t_2.

$t_0 = t_2' - t_1'$ — (A) (In frame S')

$t = t_2 - t_1$ — (B) (In frame S)

A/c to lorentz Inverse transformation Equation

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{— (1)} \quad (v = -u)$$

$$t_1 = \frac{t_1' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}, \quad t_2 = \frac{t_2' - \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

on Substituting the value t_1 and t_2 in eq (B)

$$t = \frac{t_2' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} - \frac{t_1' - \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$t = \frac{t_2' - t_1'}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\boxed{t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

where $t > t_0$

Date
Page

The equation show that to the Stationary observer in S, the interval appears to be Longest by a factor $\sqrt{1-\frac{v^2}{c^2}}$

A moving clock appears to be slow down to a Stationary observer. This effect is known as Time Dilation.

Q-1 The proper mean life time of $+\mu$ meason is 2.5×10^{-8} sec. Calculate first mean life time of μ meason travelling with velocity 2.4×10^{8} m/s.
(b) The distance travelled by this Meason during one mean life time.
(c) The distance travelled without relativistic Me...

Sol: (a)

$$t_0 = 2.5\times10^{-8}\ \text{sec}$$
$$v = 2.4\times10^{8}\ \text{m/s}$$

$$t = \frac{t_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$= \frac{2.5\times10^{-8}}{\sqrt{1-\left(\frac{2.4\times\cancel{10^8}}{3\times\cancel{10^8}}\right)^2}}$$

$$= \frac{2.5\times10^{-8}}{\sqrt{1-0.64}} =$$

$$= \frac{2.5 \times 10^{-8}}{0.6} = 4.16 \times 10^{-8} \text{ sec}$$

(b) Distance (s) = $v t$

$$= 2.4 \times 10^{8} \times 4.16 \times 10^{-8}$$

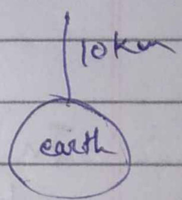
$$= 9.984 \text{ m}$$

(c) Distance (s) = $v t_0$

$$= 2.4 \times 10^{8} \times 2.5 \times 10^{-8}$$

$$= 6 \text{ m}$$

★ Time Dilation is a Real effect :-



Take an example of cosmic ray particle called meson. Meason are created at high Altitude in the earth surface (10km). And its speed 2.994×10^{8} m/s $(0.998c)$ And average mean life time, $t_0 = 2.0 \times 10^{-6}$ sec.

Hence in its life time a meason can travel a distance,

$$(s)_{\text{without relativistic}} = v t_0$$

$$= 2.994 \times 10^{8} \times 20 \times 10^{-6}$$

$$\cong 600 \text{ m}$$

But the question is that How a meason travel a distance of 10km to reach the earth surface this is possible only because of time-dilation.

In its own frame of reference μ-meason have an average life-time

$t_0 = 2.0 \times 10^{-6}$ sec.

In observers frame of Reference mean life time,

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$= \frac{2.0 \times 10^{-6}}{\sqrt{1 - \left(\frac{0.998c}{c}\right)^2}}$$

$$t = 3.17 \times 10^{-3} \text{ sec}$$

In this Dilated life time meason can travel the distance,

$$S_{\text{with relativistic}} = v \times t$$

$$= 2.994 \times 10^8 \times 3.17 \times 10^{-3}$$

$$= 10 \text{ km}$$

This explain the presence of μ- Meason on the earth surface - hence we can say the time dilation is a real effect

Date 19/10/21
Page

H.W

☆ Velocity Addition theorem !-

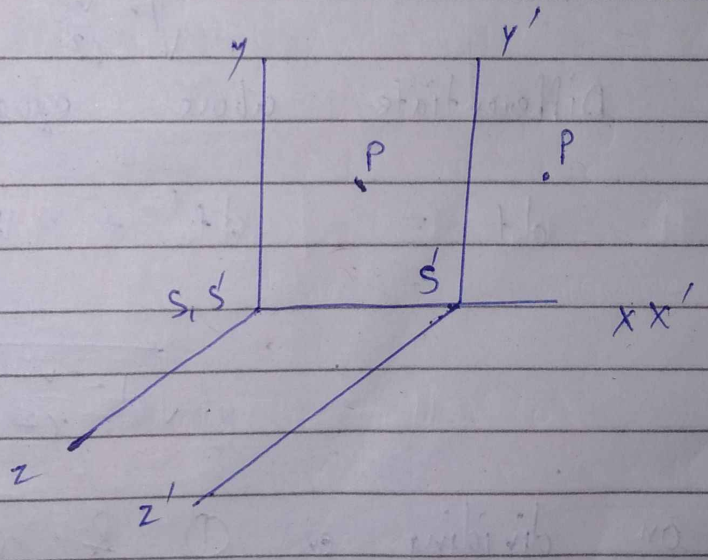
Let there are two frame of reference S and S'. frame S' is moving with velocity v relative to frame S along xx' axis

Let us Consider an observer in frame S measured its three velocity Component u_x, u_y, u_z.

$$u_x = \frac{dx}{dt} \quad , \quad u_y = \frac{dy}{dt} \quad , \quad u_z = \frac{dz}{dt}$$

And From S' frame its measure

$$u_x' = \frac{dx'}{dt'} \quad , \quad u_y' = \frac{dy'}{dt'} \quad , \quad u_z' = \frac{dz'}{dt'}$$



From Inverse Lorentz transformation Equation

$$x = \frac{x' + vt'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

differentiate the equation

$$dx = \frac{dx' + v\,dt'}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (1)}$$

on Dividing above equation by dt

$$\frac{dx}{dt} = \frac{\frac{dx'}{dt} + v\frac{dt'}{dt}}{\sqrt{1-\frac{v^2}{c^2}}}$$

From Lorentz Inverse transformation

$$t = \frac{t' + \frac{vx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

Differentiate above equation

$$dt = \frac{dt' + \frac{v\,dx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}} \quad \text{--- (2)}$$

on dividing eq (1) & (2)

$$\frac{dx}{dt} = \frac{\dfrac{dx' + v\,dt'}{\sqrt{1-\frac{v^2}{c^2}}}}{\dfrac{dt' + \frac{v\,dx'}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}}$$

Date
Page

$$= \frac{dx' + v\,dt'}{dt' + \frac{v}{c^2}dx'}$$

$$= \frac{\cancel{dt'}\left[\frac{dx'}{dt'} + v\right]}{\cancel{dt'}\left[1 + \frac{v}{c^2}\frac{dx'}{dt'}\right]}$$

$$\frac{dx}{dt} = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x}$$

$$\boxed{u_x = \frac{u'_x + v}{1 + \frac{v}{c^2}u'_x}}$$

$$dt = \frac{dt' + \frac{v}{c^2}dx'}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$y' = y$$

$$dy' = dy$$

$$u_y = \frac{dy}{dt} = \frac{dy'}{dt' + \frac{v}{c^2}dx'} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{\cancel{dt'} \times \frac{dy'}{dt'}}{\cancel{dt'}\left[1 + \frac{v}{c^2}dx'\right]} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{u_y = \frac{u'_y}{1 + \frac{v}{c^2} u'_x} \times \sqrt{1 - \frac{v^2}{c^2}}}$$

$$z' = z$$

$$dz' = dz$$

$$u_z = \frac{dz}{dt} = \frac{dz'}{dt' + \frac{v}{c^2} dx'} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{\cancel{dt'} \times \frac{dz'}{dt'}}{\cancel{dt'}\left[1 + \frac{v}{c^2} \frac{dx'}{dt'}\right]} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$\boxed{u_z = \frac{u'_z}{1 + \frac{v u'_x}{c^2}} \times \sqrt{1 - \frac{v^2}{c^2}}}$$

If $u'_x = c$ [speed of a particle in observe from S']

$$u_x = \frac{u'_x + v}{1 + \frac{v u'_x}{c^2}}$$

$$u_x = \frac{c + v}{1 + \frac{v}{c^{\cancel{2}}} \times c}$$

$$u_x = \frac{\cancel{c + v}}{(\cancel{c + v})} \times c$$

$$\boxed{u_x = c}$$

SHOT ON OPPO
By Ghanendra Kumar

Thus the two observer in different frame of reference measure the speed of light exactly same.

Thus is the ~~sp.~~ second postulate of Einstein's theory of Relativity.

SHOT ON OPPO
By Ghanendra Kumar

Date 25/10/21
Page

H.W

Physics

Q→ 1. A particle has a velocity $\vec{v}' = 3\hat{i} + 4\hat{j} + 12\hat{k}$ m/s In a coordinate system moving with velocity 0.8 of speed of light relative to laboratory along positive direction of x-axis.
Find $\vec{v}$ in laboratory frame.

Sol→

$$\vec{v}' = 3\hat{i} + 4\hat{j} + 12\hat{k} \quad m/s$$
$$\vec{v}' = u'_x\hat{i} + u'_y\hat{j} + u'_z\hat{k}$$
$$\vec{v} = u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$$

$u'_x = 3$ m/s , $u'_y = 4$ m/s , $u'_z = 12$ m/s
$v = 0.8c$

$$u_x = \frac{u'_x + v}{1 + \frac{v u'_x}{c^2}}$$

$$= \frac{3 + 0.8c}{1 + \frac{0.8\cancel{c}}{c^{\cancel{2}}} \times 3}$$

$$= \frac{3 + 0.8c}{1 + \frac{0.8 \times \cancel{3}}{\cancel{3} \times 10^8}}$$

$$= \frac{3 + 2.4 \times 10^8}{1 + 0.8 \times 10^{-8}}$$

$$u_x = 2.4 \times 10^8 \ m/s$$

SHOT ON OPPO
By Ghanendra Kumar

Date ______
Page ______

$$u_y = \frac{u'_y}{1 + \frac{v}{c^2} u'_x} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{4}{1 + \frac{0.8\cancel{c}}{c^{\cancel{2}}} \times 3} \times \sqrt{1 - \left(\frac{0.8c}{c}\right)^2}$$

$$= \frac{4 \times 0.6}{1 + 0.8 \times 10^{-8}}$$

$$= 2.4 \text{ m/s}$$

$$u_z = \frac{u'_z}{1 + \frac{v}{c^2} u'_x} \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{12}{1 + \frac{0.8\cancel{c}}{c^{\cancel{2}}} \times 3} \times \sqrt{1 - \left(\frac{0.8c}{c}\right)^2}$$

$$= \frac{12 \times 0.6}{1 + 0.8 \times 10^{-8}}$$

$$= 7.2 \text{ m/s}$$

$$\vec{V} = (2.4 \times 10^8)\hat{i} + 2.4\hat{j} + 7.2\hat{k}$$

SHOT ON OPPO
By Ghanendra Kumar

Q. 2 Observer on the earth (Assume to be an inertial frame of reference) see a spaceship 'P' receding from him at 2.0×10^8 m/s and overtaking another spaceship 'Q' receding at 1.5×10^8 m/s. Find the Relative velocity

(a) of spaceship 'Q' as observed by 'P'
(b) of spaceship 'P' as observed by 'Q'

Sol. (a)

earth   P   Q

u'_x $\rightarrow v$

$$u'_x = 2.0 \times 10^8 \text{ m/s}$$

$$v = 1.5 \times 10^8 \text{ m/s}$$

$$u_x = \frac{u'_x + v}{1 + \frac{v}{c^2} \cdot u'_x}$$

$$= \frac{2.0 \times 10^8 + 1.5 \times 10^8}{1 + \frac{1.5 \times \cancel{10^8}}{3 \times 3 \times \cancel{10^{16}}} \times 2 \times 10^8}$$

$$= \frac{10^8 \times 3.5}{1 + \frac{\cancel{3}}{3 \times \cancel{3}}}$$

$$= \frac{3.5 \times 10^8}{1 + 0.33}$$

$$= \frac{3.5}{1.33} \times 10^8$$

$$u_x = 2.63 \times 10^8 \text{ m/s}$$

Date
Page

(1) $u'_x = 2.0 \times 10^8$ m/s

$v = -1.5 \times 10^8$ m/s

$$u_x = \frac{10^8 \times 0.5}{1 + \frac{-1.5 \times \cancel{10^8} \times 2 \times \cancel{10^8}}{9 \times \cancel{10^8} \times \cancel{10^8}}}$$

$$u_x = \frac{0.5 \times 10^8}{1 - 0.33}$$

$$u_x = \frac{0.5}{0.67} \times 10^8$$

$$u_x = \frac{50}{67} \times 10^8$$

$$u_x = 7.46 \times 10^7 \text{ m/s}$$

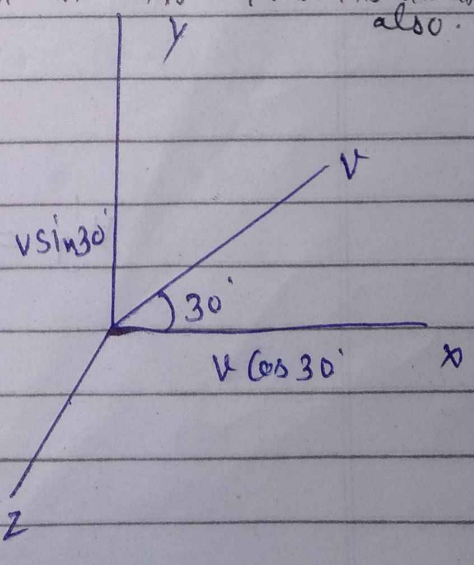
Q.3 A particle moves with a speed of 0.8C at an angle of 30° to the x-axis as determine by O. What is the velocity of the particle as determined by a second observer O' which is moving with a speed of −0.6C along the common xx' axis. Find its direction also.

Sol:

$v_x = 0.8C \cos 30$

$= 0.8 \times \frac{\sqrt{3}}{2} C$

$= 0.4\sqrt{3}C$



$v_y = 0.8C \times \sin 30$

$v_y = 0.4C$

$v = 0.6C$

Date
Page

$$v_x = \frac{v_x' + v}{1 + \frac{v}{c^2} v_x'}$$

$$v_y = \frac{v_y'}{1 + \frac{v}{c^2} v_x'} \times \sqrt{1 - \frac{v^2}{c^2}}$$

By Inverse of velocity addition theorem

$$v_x' = \frac{v_x - v}{1 - \frac{v}{c^2} v_x}$$

$$v_y' = \frac{v_y}{1 - \frac{v}{c^2} v_x} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$v_x' = \frac{0.4\sqrt{3}c - (-0.6c)}{1 + \frac{0.6\cancel{c}}{\cancel{c^2}} \times 0.4\sqrt{3} \times \cancel{c}}$$

$$= \frac{[0.4 \times 1.732 + 0.6]\, c}{1 + 0.24 \times 1.732}$$

$$= \frac{1.466}{1.4156} \times c$$

$$\boxed{v_x' = 3 \times 10^8 \text{ m/s}}$$

$$v_y' = \frac{0.4c \times \sqrt{1 - \left(\frac{0.6c}{c}\right)^2}}{1 + \frac{0.6\cancel{c}}{\cancel{c^2}} \times 0.4\sqrt{3}\cancel{c}}$$

$$= \frac{0.32c}{1.41568} = \frac{0.32}{1.42} \times c$$

$$\boxed{v_y' = 6.6 \times 10^7}$$

Date
Page

$$V_R = \sqrt{(V'_x)^2 + (V'_y)^2}$$

$$= \sqrt{(3 \times 10^8)^2 + (0.66 \times 10^8)^2}$$

$$= (10^8)^2 \sqrt{\cancel{9 \times (0.66)^2}\ (9 + (0.66)^2)}$$

$$= 10^{16} \sqrt{9 + 0.4356}$$

$$= 10^{16} \sqrt{9.4356}$$

$$= 10^{16} \times 0.66$$

$$VR = 6.6 \times 10^{15} \text{ m/s}$$

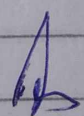
$$\tan\phi = \frac{V_y'}{V_x'}$$

$$\tan\phi = \frac{6.6 \times 10^7}{3 \times 10^8 \times 10}$$

$$\tan\phi = \frac{66}{3} \times 10^{-2}$$

$$\tan\phi = 22 \times 10^{-2}$$

$$\phi = \tan^{-1}(0.22)$$



$$\phi = 12.40$$

☆ Variation of Mass with velocity:-

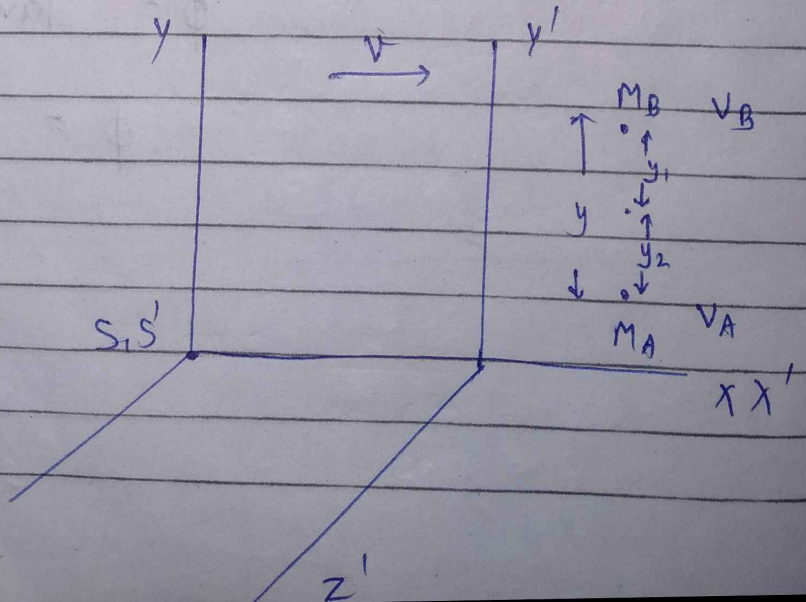
We have seen that Length and time not absolute quantities but depend upon the frame of Reference from which they are observe.
In fact mass is a function of the velocity of the body. It increases with velocity representative by the

$$\boxed{m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}}$$

where,

m = Variable mass
m_0 = Rest Mass
v = speed of particle
c = speed of light

Let us consider two frame S and S'. Frame S' is moving with a Constant Velocity 'v' along the xx' axis.



Date
Page

Let us consider an elastic collision b/w two exactly identical perfectly elastic spherical partical. A (in frame S) and B (in frame S') and their velocity V_A (in frame S) and V_B (in frame S') in positive y direction and -ve y direction respectively.

Collision occurs at the mid point of the position of two bodies.

The distance travelled by particle A.
In frame S

$$y_1 = \frac{y}{2} = y \quad — \quad (1)$$

The distance travelled by particle B
In frame S'

$$y_2 = \frac{y}{2} = y \quad — \quad (2)$$

In frame A velocity of particle A
In frame S

$$V_A = \frac{y_1}{T_0} \quad — \quad (3)$$

In frame B velocity of particle B.
In frame S'

$$V_B' = \frac{y_2}{T_0}\sqrt{1-\frac{V^2}{C^2}}$$

Linear momentum is Conserved in frame

$$M_A V_A = M_B V_B$$

Momentum is Consvered also in theory of Relativity (Relativistic Mechanism)

$$M_A V_A = M_B V_B'$$

$$M_A \times \frac{y_1}{\cancel{T_0}} = M_B \times \frac{y_2}{\cancel{T_0}} \sqrt{1 - \frac{V^2}{c^2}}$$

$$M_A \times \cancel{y} = M_B \times \cancel{y} \sqrt{1 - \frac{V^2}{c^2}}$$

$M_A = m_0$ (Rest mass)
$M_B = m$ (variable mass)

$$m_0 = m \sqrt{1 - \frac{V^2}{c^2}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1 - \frac{V^2}{c^2}}}}$$

Conclusion :- As the velocity v of the partical relative to observer increase the mass of the partical increases.

If $V = c$

$$m = \frac{m_0}{1 - \frac{\cancel{c^2}}{\cancel{c^2}}}$$

SHOT ON OPPO

Date
Page

$$m = \frac{1}{0}$$

$$m = \infty$$

(2) That is no material partical can have a velocity equal to or greater that velocity of light.

(3) If $v << c$, The value of $\frac{v^2}{c^2}$ is very very small hence

Variable mass = m_0 (approx).

☆ Relativistic Momentum and Force.

→ Momentum

According to classical physics momentum is defined as

$$P = mv \quad \text{(Classical physics)}$$

But in relativistic mechanism mass 'm' also very with speed of particle.

According to relativistic Mechanism

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where m_0 = rest mass
m = variable mass

hence momentum will be $\boxed{P = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}}$

Conservation of momentum is Valid in Special theory of relativity just as in classical physics.

★ Force

In relativistic mechanics the force acting on a particle can not be define as the product of mass and acceleration of the particle. But as the time rate of change of its momentum.

$$F = ma \quad \text{(classical physics)}$$

According to relativistic Mechanics)
m is not Constant

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv)$$

$$\boxed{F = m\frac{dv}{dt} + v\frac{dm}{dt}}$$

Date ____
Page ____

Q. 1 Show that the relativistic form of newton's Second law when $\vec{F}$ is parallel to $\vec{v}$ is $\vec{F} = m_0 \frac{d\vec{v}}{dt}\left(1 - \frac{v^2}{c^2}\right)^{-3/2}$.

Sol.
$$\vec{F} = \frac{d}{dt}(mv)$$

$$\vec{F} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

$$\vec{F} = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}\frac{dv}{dt} + v\frac{d}{dt}\frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$\vec{F} = m_0\left[1-\frac{v^2}{c^2}\right]^{-1/2}\frac{dv}{dt} + vm_0\frac{d}{dt}\left[1-\frac{v^2}{c^2}\right]^{-1/2}$$

$$\vec{F} = m_0\left[1-\frac{v^2}{c^2}\right]^{-1/2}\frac{d\vec{v}}{dt} + vm_0\left(\frac{-1}{\cancel{2}}\right)\left[1-\frac{v^2}{c^2}\right]^{-3/2} \times \frac{-\cancel{2}v}{c^2} \times \frac{dv}{dt}$$

$$\vec{F} = m_0\frac{d\vec{v}}{dt}\left[\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} + \frac{v^2}{c^2\sqrt{1-\frac{v^2}{c^2}}\left(1-\frac{v^2}{c^2}\right)}\right]$$

$$\vec{F} = m_0\frac{d\vec{v}}{dt}\left[\frac{c^2\left(1-\frac{v^2}{c^2}\right) + v^2}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right]$$

$$\vec{F} = m_0\frac{d\vec{v}}{dt}\left[\frac{\cancel{c^2} - \cancel{v^2} + \cancel{v^2}}{\cancel{c^2}\left(1-\frac{v^2}{c^2}\right)^{3/2}}\right]$$

$$\boxed{\vec{F} = m_0\frac{d\vec{v}}{dt}\left[1-\frac{v^2}{c^2}\right]^{-3/2}}$$

Hence proved

Q→1. If mass of a particle is 20% of its rest mass then. What will be the velocity of that particle.

Sol→

$$m = 20\% \, m_0$$

$$m = \frac{20}{100_5} m_0$$

$$m = 0.2 m_0$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\frac{m_0}{m_0} = \sqrt{1 - \frac{v^2}{c^2}}$$

$$1 - \frac{v^2}{c^2} = \left(\frac{m_0}{m}\right)^2$$

$$v = c\sqrt{1 - \left(\frac{m_0}{m}\right)^2}$$

$$v = 3 \times 10^8 \sqrt{1 - \left(\frac{m_0 \times 10}{0.2 m_0}\right)^2}$$

$$v = 3 \times 10^8 \sqrt{1 - 25}$$

$$v = 3 \times 10^8 \sqrt{24} \, i$$

$$\boxed{v = c\sqrt{24}\, i} \text{ m/s}$$

Date
Page

Q→2 The rest mass of an e^- is 9×10^{-31} kg. What will be its mass. If it is moving with $\frac{4}{5}c$.

Sol→

$$m_0 = 9 \times 10^{-31} \text{ kg}$$

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$m = \frac{9 \times 10^{-31}}{\sqrt{1 - \left(\frac{4c}{5c}\right)^2}}$$

$$m = \frac{9 \times 10^{-31}}{\sqrt{1 - \frac{16}{25}}}$$

$$m = \frac{\overset{3}{\cancel{9}} \times 10^{-31} \times 5}{\cancel{3}}$$

$$\boxed{m = 15 \times 10^{-31} \text{ kg}}$$ Ans

Q→3 Velocity of a particle is $v' = 5\hat{i} + 5\hat{j} + 20\hat{k}$ m/s in a frame of reference moving with uniform velocity $0.5c$ with respect to the laboratory along positive x-axis. Find out the velocity of a particle in the laboratory.

Sol→

$$v' = 5\hat{i} + 5\hat{j} + 20\hat{k}$$

$$v' = v'_x\hat{i} + v'_y\hat{j} + v'_z\hat{k}$$

$$v'_x = 5, \quad v'_y = 5, \quad v'_z = 20, \quad v = 0.5c$$

$$v_x = \frac{u'_x + v}{1 + \frac{v}{c^2} u'_x}$$

$$= \frac{5 + 0.5c}{1 + \frac{0.5\cancel{c} \times 5}{c^{\cancel{2}}}}$$

$$= \frac{5 + 0.5 \times 3 \times 10^8}{1 + \frac{2.5}{3 \times 10^8}}$$

$$= \frac{5 + 1.5 \times 10^8}{1 + \frac{2.5}{3 \times 10^8}}$$

$$= 1.5 \times 10^8 \text{ m/s}$$

$$v_y = \frac{v'_y}{1 + \frac{v}{c^2} v'_x} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{5}{1 + \frac{0.5\cancel{c} \times 5}{c^{\cancel{2}}}} \times \sqrt{1 - \left(\frac{0.5\cancel{c}}{\cancel{c}}\right)^2}$$

$$= \frac{5}{1 + \frac{2.5}{3 \times 10^8}} \times \sqrt{1 - \frac{1}{4}}$$

$$= 5 \times \frac{\sqrt{3}}{2}$$

$$= \frac{1.732}{2} \times 5$$

$$= 0.866 \times 5$$

$$v_y = 4.330 \text{ m/s}$$

Date
Page

$$v_z = \frac{v'_z}{1 + \frac{v v'_x}{c^2}} \times \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{20}{1 + \frac{0.5c \times 0.5}{c^2}} \times \sqrt{1 - \left(\frac{0.5c}{c}\right)^2}$$

$$= 20 \times \frac{\sqrt{3}}{2}$$

$$= 20 \times 0.866$$

$$= 17.320$$

$$v_z = 17.3 \text{ m/s}$$

$$v = v_x \hat{i} + v_y \hat{j} + v_z \hat{k}$$

$$v = (1.5 \times 10^8)\hat{i} + 4.3\hat{j} + 17.3\hat{k}$$ Ans

Q. 4 A certain young decides on her 25^{th} birthday that it is time to slendrize. She weights 100kg. She has heared that if she moves fast enough, she will appear thiner to her stationary friends.

(a) How fast must she move to appear slendrized by a factor of 50%.

(b) At this speed what will her mass appear to be her stationary friends?

(c) If she maintain her speed until the day she calls her 29^{th} birthday, how old will her stationary friend claimes she it according to their measurement.

Sol → (a) ~~So~~

$$L = 50\% \cdot L_0$$

$$L = 0.5 L_0$$

$$\frac{L}{L_0} = 0.5$$

$$L = L_0 \sqrt{1 - \frac{v^2}{c^2}}$$

$$\left(\frac{L}{L_0}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$(0.5)^2 = 1 - \frac{v^2}{c^2}$$

$$1 - 0.25 = \frac{v^2}{c^2}$$

$$\frac{v^2}{c^2} = 1 - \frac{\cancel{0.25}}{\cancel{100}\,4}$$

$$v^2 = c^2 \frac{3}{4}$$

$$v = c \frac{\sqrt{3}}{2} \text{ m/s}$$

$$v = c \times 0.866 \text{ m/s}$$ Ans

(b) Sol →

$$m_0 = 100 \text{kg}$$

$$m = \frac{100}{\sqrt{1 - \left(\frac{\sqrt{3}c}{2c}\right)^2}}$$

$$m = \frac{100}{\sqrt{1 - \frac{3}{4}}} = \frac{100}{\frac{1}{2}} = 200 \text{kg}$$ Ans

(c) Sol:

$t_0 = 4$ (difference of 25^{th} and 29^{th} birthday)

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t = \frac{4}{\sqrt{1 - \left(\frac{\sqrt{3}c}{2c}\right)^2}}$$

$$t = 4 \times 2$$

$t = 8$ year

$25 + 8 = 33$ years

Date 29/10/21
C.W.

☆ Einstein's Mass energy Relationship :-

The relationship can be derive directly from the defination of the kinitic energy of a moving body that is workdone in bringing the body from the rest to its present state of motion.

Workdone is equal to kinitic energy

$$W = K.E$$

In non-relativitic mechanic will be $\frac{1}{2} m_o v^2$

But kinitic energy ~~is not~~ in relativistic mechanics not equals to $= \frac{1}{2} m_o v^2$.

It is defined as
For small displacement ds
Workdone

$$d_w = F.ds$$

Change in kinitic energy for small displacement that is

$$d_{K.E} = d_w = F.ds$$

$$d_{K.E} = F.ds$$

$$d_{KE} = \frac{d}{dt}(mu)\, ds$$

$$= d(mu)\frac{ds}{dt} \qquad \left[\because \frac{ds}{dt} = v\right]$$

$$= d(mv)\, v$$

$$= v\left[m dv + v dm\right]$$

$$d_{KE} = mv dv + v^2 dm \quad \text{— (1)}$$

We know that,

$$m = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}$$

Squaring on boths sides

$$m^2\left(1-\frac{v^2}{c^2}\right) = m_o^2$$

$$m^2\left[c^2 - v^2\right] = m_o^2 c^2$$

$$m^2 c^2 = m_o^2 c^2 + m^2 v^2$$

differenciate on both side

$$c^2 \cdot 2m\, dm = 0 + m^2 \cdot 2v\, dv + v^2 \cdot 2m\, dm$$

$$2mc^2 dm = 2vm^2 dv + 2mv^2 dm$$

$$\cancel{2m} c^2 dm = \cancel{2m}\left[mv dv + v^2 dm\right]$$

$$c^2 dm = mv dv + v^2 dm \quad \text{— (2)}$$

On comparing eqs (1) & (2), we get

$$d_{KE} = c^2 dm$$

$$\text{Relativistic } K.E = \int_{m_o}^{m} c^2 dm$$

Downloaded from : uptukhabar.net

$$= c^2 \int_{m_0}^{m} 1\, dm$$

$$= c^2 [m]_{m_0}^{m}$$

$$K.E = c^2 [m - m_0]$$

$$\boxed{K.E = c^2 m - m_0 c^2}$$

This is the relativistic expression or equation for kinitic energy of a particle. From this expression it is clear that the increase in kinitic energy is due to the increase in mass of the particle.

Total kinitic energy is equal to rest kinitic energy + Relativistic K.E

Total K.E = Rest ~~K.E~~ energy + Relativivictic K.E

$$K.E = \cancel{m_0 c^2} + m c^2 - \cancel{m_0 c^2}$$

$$\boxed{K.E = mc^2}$$

This is known as Eienstein's mass energy relationship.

$$E = E_0 + K.E$$

$$E_0 = m_0 c^2$$

$$E = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} c^2$$

$$\boxed{E = \frac{E_0}{\sqrt{1 - \frac{v^2}{c^2}}}}$$

Date 30/10/21

☆ Relation between total energy and relativistic momentum. ⇒

Let the total energy of a particle is 'E', Velocity 'v' and momentum 'p'.
We know that from Einstein mass enery relationship.

Total energy (E) = mc^2

$$E = \frac{m_0 c^2}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— (1)}$$

Relativistic momentum (P) = mv

$$P = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{— (2)}$$

Squaring eq^n (1) & (2)

$$E^2 = \frac{m_0^2 c^4}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2}, \quad P^2 = \frac{m_0^2 v^2}{\left(\sqrt{1 - \frac{v^2}{c^2}}\right)^2}$$

$$E^2 = \frac{m_0^2 c^4}{\left(1 - \frac{v^2}{c^2}\right)} \quad \text{— (3)} \qquad P^2 = \frac{m_0^2 v^2}{\left(1 - \frac{v^2}{c^2}\right)} \quad \text{— (4)}$$

Multiply by c^2 in eq (4)

$$c^2 P^2 = \frac{m_0^2 v^2 c^2}{\left(1 - \frac{v^2}{c^2}\right)} \quad \text{— (5)}$$

on Subracting eqn (iii) and (v)

$$E^2 - p^2c^2 = \frac{m_0^2 c^4}{\left(1-\frac{v^2}{c^2}\right)} - \frac{m_0^2 v^2 c^2}{\left(1-\frac{v^2}{c^2}\right)}$$

$$= \frac{m_0^2 c^4 - m_0^2 v^2 c^2}{\left(1-\frac{v^2}{c^2}\right)}$$

$$E^2 - p^2c^2 = \frac{m_0^2 c^4 \left[\cancel{1-\frac{v^2}{c^2}}\right]}{\left(\cancel{1-\frac{v^2}{c^2}}\right)}$$

$$E^2 - p^2c^2 = m_0^2 c^4$$

$$\boxed{E^2 = p^2c^2 + m_0^2c^4}$$

Date 01/11/21

Q.1 Find the increase in mass of 100kg of copper of its temperature is increased 100°C. specific heat of copper that is $S = 93$ ~~kilo~~ cal/kg-°C

Sol:

$$\Delta E = \Delta m c^2$$

$m = 100kg$, $S = 93$ cal/kg °C
$= 93 \times 4.2$ J/kg °C

$$Q = MS\Delta T$$
$$Q = 100 \times 93 \times 4.2 \times 100$$
$$Q = 9.3 \times 10^5 \times 4.2$$
$$Q = 39.06 \times 10^5 J$$

$$d_E = c^2 dm$$
$$d_E = Q$$
$$Q = c^2 dm$$
$$dm = \frac{Q}{c^2}$$
$$= \frac{39.06 \times 10^5}{9 \times 10^{16}}$$
$$\boxed{dm = 4.34 \times 10^{-11} kg}$$ Ans

Q-2 A body whose specific heat is 0.2 kg cal/kg °C is heated through 100°C. Find the percentage increase in its mass.

Sol:

$s = 0.2$ kg cal/kg °C
$= 0.2 \times 10^3 \times 4.2$ J/kg °C
$= 0.84 \times 10^3$

Downloaded from : uptukhabar.net

$$Q = m s \Delta T$$
$$= m \times 0.84 \times 10^3 \times 100$$
$$\frac{Q}{m} = 84 \times 10^3 \ J/kg$$

$$dm = \frac{Q}{c^2}$$

$$dm = \frac{m \times 84 \times 10^3}{9 \times 10^{16}}$$

$$\frac{dm}{m} = 9.3 \times 10^{-13}$$

Percentage Error in Mass $= \frac{dm}{m} \times 100$
$$= 9.3 \times 10^{-13} \times 100$$
$$= 9.3 \times 10^{-11} \ kg$$

Date 08/11/21

Unit - II

Quantum mechanics

☆ **Black body :-**
Black body an ideal body to which absorb all electromagnetic radiation that strike on it so that all incident radiation is completely absorb.

☆ **Black body radiation :-**
A body (Black) that emits radiation of different wavelength is called Black body radiation.
For exp:-
1. our sun which is indeed almost a black body within a very wide bend of electromagnetic radiation wavelength as black physical object in opties.

2. Black Carbon is also a black body.

☆ **Stefan's law :-**

The energy radiated by a black body radiation per second per unit area is directly proportional to the Forth power of absolute temperature.

$$E \propto T^4$$
$$E = \sigma T^4$$
$$E = \sigma F T^4 \quad J/m^2 sec$$
$$\boxed{E_{actual} = \sigma A + F T^4} \ Joule$$