

SECTION - A

GENERAL APTITUDE

- Q.1 The _____ is too high for it to be considered _____.
- (a) fair / fare (b) faer / fair
(c) fare / fare (d) fare / fair

Ans. (d)

End of Solution

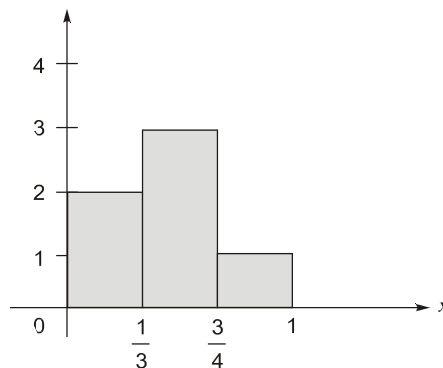
- Q.2 A function $y(x)$ is defined in the interval $[0, 1]$ on the x -axis as

$$y(x) = \begin{cases} 2 & \text{if } 0 \leq x < \frac{1}{3} \\ 3 & \text{if } \frac{1}{3} \leq x < \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x < 1 \end{cases}$$

Which one of the following is the area under the curve for the interval $[0, 1]$ on the x -axis?

- (a) $\frac{5}{6}$ (b) $\frac{6}{5}$
(c) $\frac{13}{6}$ (d) $\frac{6}{13}$

Ans. (c)



$$A = \frac{1}{3} \times 2 + \left(\frac{3}{4} - \frac{1}{3}\right) \times 3 + \left(1 - \frac{3}{4}\right) \times 1 = \frac{13}{6}$$

End of Solution

- Q.3 Let r be a root of the equation $x^2 + 2x + 6 = 0$.
Then the value of the expression $(r + 2)(r + 3)(r + 4)(r + 5)$ is
- (a) 51 (b) -51
(c) 126 (d) -126

Ans. (d)

$$\begin{aligned} &= (r + 2)(r + 3)(r + 4)(r + 5) \\ &= (r^2 + 5r + 6)(r^2 + 9r + 20) \end{aligned}$$

Now,

$$\begin{aligned} r^2 + 2r &= -6 \\ &= (3r + 6 - 6)(7r + 20 - 6) \\ &= 3r(7r + 14) \\ &= 21r(r + 2) \\ &= 21(r^2 + 2r) = 21 \times (-6) = -126 \end{aligned}$$

End of Solution

- Q.4 Given below are four statements:
- Statement 1:** All students are inquisitive.
Statement 2: Some students are inquisitive.
Statement 3: No student is inquisitive.
Statement 4: Some students are not inquisitive.

From the given four statements, find the two statements that **CANNOT BE TRUE** simultaneously, assuming that there is at least one student in the class.

- (a) Statement 1 and Statement 3 (b) Statement 1 and Statement 2
(c) Statement 2 and Statement 4 (d) Statement 3 and Statement 4
- Ans. (a)

- If all children are inquisitive is true.
Then no children are inquisitive is false.
- There is a possibility both (i) and (iii) can be false.
Assume that some children are inquisitive in that case both (i) and (iii) are false.

End of Solution

- Q.5 A palindrome is a word that reads the same forwards and backwards. In a game of words, a player has the following two plates painted with letters.

A D

From the additional plates given in the options, which one of the combinations of additional plates would allow the player to construct a five-letter palindrome. The player should use all the five plates exactly once. The plates can be rotated in their plane.

- (a) D D J (b) R V R
- (c) Z E D (d) I I Y

Ans. (b)

End of Solution

- Q.6** Some people believe that “what gets measured, improves”. Some others believe that “what gets measured, gets gamed”. One possible reason for the difference in the beliefs is the work culture in organizations. In organizations with good work culture, metrics help improve outcomes. However, the same metrics are counterproductive in organizations with poor work culture.

Which one of the following is the CORRECT logical inference based on the information in the above passage?

- (a) Metrics are useful in organizations with poor work culture.
- (b) Metrics are useful in organizations with good work culture.
- (c) Metrics are always counterproductive in organizations with good work culture.
- (d) Metrics are never useful in organizations with good work culture.

Ans. (b)

End of Solution

- Q.7** In a recently conducted national entrance test, boys constituted 65% of those who appeared for the test. Girls constituted the remaining candidates and they accounted for 60% of the qualified candidates.

Which one of the following is the correct logical inference based on the information provided in the above passage?

- (a) Equal number of boys and girls qualified
- (b) Equal number of boys and girls appeared for the test
- (c) The number of boys who appeared for the test is less than the number of girls who appeared
- (d) The number of boys who qualified the test is less than the number of girls who qualified

Ans. (d)

$$\text{Appeared} = x$$

$$\text{Boys} = \frac{65x}{100}$$

$$\text{Girls} = \frac{35x}{100}$$

$$\text{Qualified} = y$$

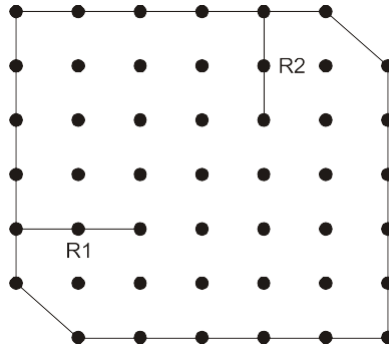
$$\text{Boys} = \frac{40y}{100}$$

$$\text{Girls} = \frac{60y}{100}$$

End of Solution

Q.10 A plot of land must be divided between four families. They want their individual plots to be similar in shape, not necessarily equal in area. The land has equally spaced poles, marked as dots in the below figure. Two ropes, R1 and R2, are already present and cannot be moved.

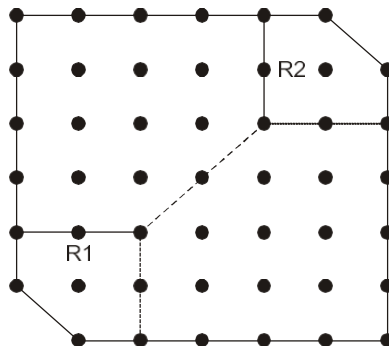
What is the least number of additional straight ropes needed to create the desired plots?
A single rope can pass through three poles that are aligned in a straight line.



- (a) 2
- (c) 5

- (b) 4
- (d) 3

Ans. (d)



End of Solution



SECTION - B

TECHNICAL

Q.11 Which one of the following statements is TRUE for all positive functions $f(n)$?

- (a) $f(n^2) = \theta(f(n)^2)$, when $f(n)$ is a polynomial
- (b) $f(n^2) = o(f(n)^2)$
- (c) $f(n^2) = O(f(n)^2)$, when $f(n)$ is an exponential function
- (d) $f(n^2) = \Omega(f(n)^2)$

Ans. (a)

True (a) $f(n^2) = \theta(f(n)^2)$ $f(n)$ polynomial
 $f(n) = n^3 \Rightarrow f(n)^2 = (n^2)^3 = n^6$
 $f(n^2) = (n^3)^2 = n^6$ } asy equal

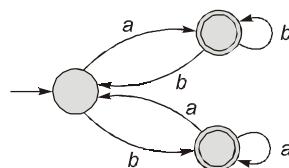
False (b) $f(n^2) = o(f(n)^2)$
 $f(n) = n^3 \Rightarrow f(n)^2 = (n^2)^3 = n^6$
 $f(n^2) = (n^3)^2 = n^6$

False (c) $f(n^2) = \Omega(f(n)^2)$ $f(n)$ exponential
 $f(n) = 2^n \Rightarrow f(n)^2 = 2^{(n^2)}$ $2^{(n^2)} > 2^{2n}$
 $f(n^2) = (2^n)^2 = 2^{2n}$
 $f(n) = n^2 \Rightarrow f(n^2) = f(n^2)$
 $f(n) = \log n \Rightarrow f(n)^2 = \log n^2 = 2 \log n$
 $f(n^2) = (\log n)^2$ } $2 \log n < (\log n)^2$

So $f(n^2) = \Omega(f(n)^2)$ is false

End of Solution

Q.12 Which one of the following regular expressions correctly represents the language of the finite automaton given below?



- (a) $ab^*bab^* + ba^*aba^*$
- (b) $(ab^*b)^* ab^* + (ba^*a)^* ba^*$
- (c) $(ab^*b + ba^*a)^* (a^* + b^*)$
- (d) $(ba^*a + ab^*b)^* (ab^* + ba^*)$

Ans. (d)

Resolving the loops on A and solving A completely,

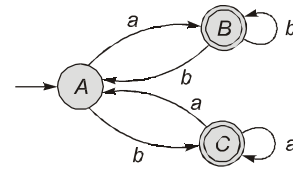
We get,

$$r_A = (ba^*a + ab^*b)^*$$

Now,

$$r = r_B + r_C = r_A(ab^* + ba^*)$$

$$= (ba^*a + ab^*b)^* (ab^* + ba^*)$$



Which is choice (d).

End of Solution

Q.13 Which one of the following statements is TRUE?

- (a) The LALR(1) parser for a grammar G cannot have reduce-reduce conflict if the LR(1) parser for G does not have reduce-reduce conflict.
- (b) Symbol table is accessed only during the lexical analysis phase.
- (c) Data flow analysis is necessary for run-time memory management.
- (d) LR(1) parsing is sufficient for deterministic context-free languages.

Ans. (d)

- (a) False: Even though LR(1) parser does not have R/R conflict for a grammar G, the LALR(1) parser may have R/R conflict for G.
- (b) False: Symbol table can be accessed by all the phases of a compiler
- (c) False: Data flow analysis is used in control flow graph for code optimization
- (d) True: LR(1) parser is sufficient for DCLF's.

End of Solution

Q.14 In a relational data model, which one of the following statements is TRUE?

- (a) A relation with only two attributes is always in BCNF.
- (b) If all attributes of a relation are prime attributes, then the relation is in BCNF.
- (c) Every relation has at least one non-prime attribute.
- (d) BCNF decompositions preserve functional dependencies.

Ans. (a)

Relation with 2 attributes always in BCNF

$$R(AB) \{A \rightarrow B\} \Rightarrow \text{BCNF}$$

$$\{B \rightarrow A\} \Rightarrow \text{BCNF}$$

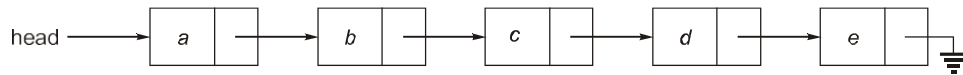
$$\{A \rightarrow B, B \rightarrow A\} \Rightarrow \text{BCNF}$$

$$\{\text{No non-trivial FD's}\} \Rightarrow \text{BCNF}$$

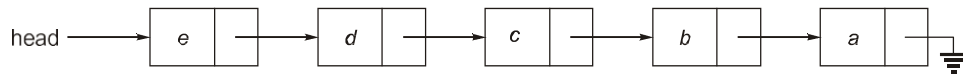
- If all prime attributes than relation always in 3NF but may not BCNF.
- Not mandatory of atleast one non-prime attribute in RDBMS table.
- Not every relation can decompose into BCNF with dependency preserving.

End of Solution

- Q.15** Consider the problem of reversing a singly linked list. To take an example, given the linked list below:



the reversed linked list should look like



Which one of the following statements is TRUE about the time complexity of algorithms that solve the above problem in $O(1)$ space?

- (a) The best algorithm for the problem takes $\theta(n)$ time in the worst case.
- (b) The best algorithm for the problem takes $\theta(n \log n)$ time in the worst case.
- (c) The best algorithm for the problem takes $\theta(n^2)$ time in the worst case.
- (d) It is not possible to reverse a singly linked list in $O(1)$ space.

Ans. (a)

End of Solution

- Q.16** Suppose we are given n keys, m hash table slots, and two simple uniform hash functions h_1 and h_2 . Further suppose our hashing scheme uses h_1 for the odd keys and h_2 for the even keys. What is the expected number of keys in a slot?

- (a) $\frac{m}{n}$
- (b) $\frac{n}{m}$
- (c) $\frac{2n}{m}$
- (d) $\frac{n}{2m}$

Ans. (b)

End of Solution

- Q.17** Which one of the following facilitates transfer of bulk data from hard disk to main memory with the highest throughput?

- (a) DMA based I/O transfer
- (b) Interrupt driven I/O transfer
- (c) Polling based I/O transfer
- (d) Programmed I/O transfer

Ans. (a)

In DMA, bulk amount of data will be transferred from the secondary memory to main memory without involvement of a CPU.

End of Solution

Q.18 Let R1 and R2 be two 4-bit registers that store numbers in 2's complement form. For the operation $R1 + R2$, which one of the following values of R1 and R2 gives an arithmetic overflow?

- (a) R1 = 1011 and R2 = 1110
- (b) R1 = 1100 and R2 = 1010
- (c) R1 = 0011 and R2 = 0100
- (d) R1 = 1001 and R2 = 1111

Ans. (b)

Stored numbers in register R1 and R2 are in 2's complement form. Register size is 4 bits. The range of numbers in 2's complement form is -8 to $+7$. If $R1 + R2$, result is out of out of the above range, then it is overflow.

- (a)

R1 = 1011 = $-(0101)$ = -5	
+R2 = 1110 = $-(0010)$ = -2	
	-7
	No over flow
- (b)

R1 = 1100 = $-(0100)$ = -4	
+R2 = 1010 = $-(0110)$ = -6	
	-10
	Over flow
- (c)

R1 = 0011 = $+(0011)$ = +3	
+R2 = 0100 = $+(0100)$ = +4	
	+7
	No over flow
- (d)

R1 = 1001 = $-(0111)$ = -7	
+R2 = 1111 = $-(0001)$ = -1	
	-8
	No over flow

End of Solution

Q.19 Consider the following threads, T_1 , T_2 and T_3 executing on a single processor, synchronized using three binary semaphore variables, S_1 , S_2 and S_3 , operated upon using standard `wait()` and `signal()`. The threads can be context switched in any order and at any time.

T_1	T_2	T_3
while (true) { wait (S_3); print ("C"); signal (S_2); }	while (true) { wait (S_1); print ("B"); signal (S_3); }	while (true) { wait (S_2); print ("A"); signal (S_1); }

Which initialization of the semaphores would print the sequence BCABCABCA.....?

- (a) $S_1 = 1; S_2 = 1; S_3 = 1$
- (b) $S_1 = 1; S_2 = 1; S_3 = 0$
- (c) $S_1 = 1; S_2 = 0; S_3 = 0$
- (d) $S_1 = 0; S_2 = 1; S_3 = 1$

Ans. (c)

Inorder to get the required output only semaphore S_1 should be initialized to 1, other semaphore should be initialized to 0.

So, $S_1 = 1; S_2 = 0; S_3 = 0$

Answer is option (c).

End of Solution

Q.20 Consider the following two statements with respect to the matrices $A_{m \times n}$, $B_{n \times m}$, $C_{n \times n}$ and $D_{n \times n}$.

Statement 1: $\text{tr}(AB) = \text{tr}(BA)$

Statement 2: $\text{tr}(CD) = \text{tr}(DC)$

where $\text{tr}(\)$ represents the trace of a matrix. Which one of the following holds?

- (a) Statement 1 is correct and Statement 2 is wrong.
- (b) Statement 1 is wrong and Statement 2 is correct.
- (c) Both Statement 1 and Statement 2 are correct.
- (d) Both Statement 1 and Statement 2 are wrong.

Ans. (c)

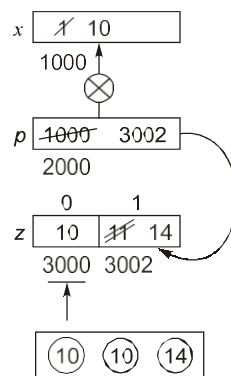
End of Solution

Q.21 What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[ ])
{
    int x = 1, z[2] = {10, 11};
    int *p = NULL;
    p = &x;
    *p = 10;
    p = &z[1];
    *(&z[0] + 1) += 3;
    printf("%d, %d, %d\n", x, z[0], z[1]);
    return 0;
}
```

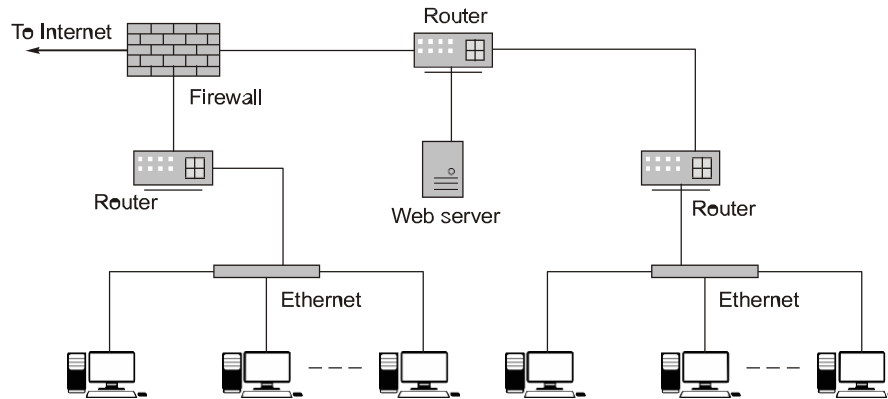
- (a) 1, 10, 11
- (b) 1, 10, 14
- (c) 10, 14, 11
- (d) 10, 10, 14

Ans. (d)



End of Solution

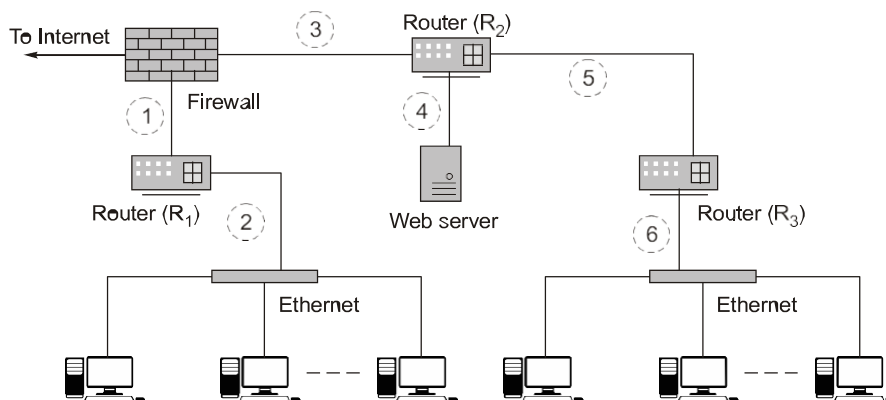
Q.22 Consider an enterprise network with two Ethernet segments, a web server and a firewall, connected via three routers as shown below.



What is the number of subnets inside the enterprise network?

- (a) 3
- (b) 12
- (c) 6
- (d) 8

Ans. (c)



Each interface of a router is a subnet. In diagram given in question, we have 3 routers R_1 , R_2 , R_3 . R_1 have 2 interfaces, R_2 have 3 interfaces and R_3 have 2 interfaces. So, total $2 + 3 + 2 = 7$ interfaces, but R_2 and R_3 sharing a common interface. So, finally there a 6 different interfaces in given diagram. So, there are 6 subnets in given enterprises.

End of Solution

Q.23 Which of the following statements is/are TRUE?

- (a) Every subset of a recursively enumerable language is recursive.
- (b) If a language L and its complement \bar{L} are both recursively enumerable, then L must be recursive.
- (c) Complement of a context-free language must be recursive.
- (d) If L_1 and L_2 are regular, then $L_1 \cap L_2$ must be deterministic context-free.

Ans. (b, c, d)

(a) No language is closed under subset operation. So subset of an RE language may or may not be REC. So option (a) is false.

(b) If L and \bar{L} are both RE $\Rightarrow L$ is REC is a theorem and is true.

(c) $(\text{CFL})^c = (\text{CSL})^c = \text{CSL}$

Every CSL is recursive.

So complement of CFL is recursive is true.

(d) $L_1 \rightarrow \text{regular}, L_2 \rightarrow \text{regular}$

$$\Rightarrow L_1 \cap L_2 = \text{Regular} \cap \text{Regular} \\ = \text{Regular}$$

Now every regular language is a DCFL.

So option (d) is true.

End of Solution

Q.24 Let WB and WT be two set associative cache organizations that use LRU algorithm for cache block replacement. WB is a write back cache and WT is a write through cache. Which of the following statements is/are FALSE?

(a) Each cache block in WB and WT has a dirty bit.

(b) Every write hit in WB leads to a data transfer from cache to main memory.

(c) Eviction of a block from WT will not lead to data transfer from cache to main memory.

(d) A read miss in WB will never lead to eviction of a dirty block from WB.

Ans. (a, b, d)

End of Solution

Q.25 Consider the following three relations in a relational database.

Employee (eld, Name), Brand (bid, bName), Own(eld, bid)

Which of the following relational algebra expressions return the set of elds who own all the brands?

(a) $\pi_{\text{eld}} (\pi_{\text{eld, bid}} (\text{Own}) / \pi_{\text{bid}} (\text{Brand}))$

(b) $\pi_{\text{eld}} (\text{Own}) - \pi_{\text{eld}} ((\pi_{\text{eld}} (\text{Own}) \times \pi_{\text{bid}} (\text{Brand})) - \pi_{\text{eld, bid}} (\text{Own}))$

(c) $\pi_{\text{eld}} (\pi_{\text{eld, bid}} (\text{Own}) / \pi_{\text{bid}} (\text{Own}))$

(d) $\pi_{\text{eld}} ((\pi_{\text{eld}} (\text{Own}) \times \pi_{\text{bid}} (\text{Own}) / \pi_{\text{bid}} (\text{Brand}))$

Ans. (a, b)

- $\pi_{\text{eld bid}} (\text{Own}) / \pi_{\text{bid}} (\text{Brand}) \Rightarrow$ Results eid's which owns every brand of brand relation.
- Option (b) expansion and division using basic operators.

End of Solution

Q.26 Which of the following statements is/are TRUE with respect to deadlocks?

(a) Circular wait is a necessary condition for the formation of deadlock.

(b) In a system where each resource has more than one instance, a cycle in its wait-for graph indicates the presence of a deadlock.

- (c) If the current allocation of resources to processes leads the system to unsafe state, then deadlock will necessarily occur.
- (d) In the resource-allocation graph of a system, if every edge is an assignment edge, then the system is not in deadlock state.

Ans. (a, d)

- (a) Circular wait is one of the necessary condition for the formation of deadlock hence it is true.
- (b) If the resource has more than one instance then cycle is just a necessary condition but not the sufficient condition, hence it is false.
- (c) Unsafe state will not always leads to deadlock, because it depends on behaviour of the processes, hence it is false.
- (d) If every edge is only allocation, it means there is no requirement of the resources, so deadlock not possible, hence it is true.

End of Solution

Q.27 Which of the following statements is/are TRUE for a group G ?

- (a) If for all $x, y \in G$, $(xy)^2 = x^2y^2$, then G is commutative.
- (b) If for all $x \in G$, $x^2 = 1$, then G is commutative. Here, 1 is the identity element of G .
- (c) If the order of G is 2, then G is commutative.
- (d) If G is commutative, then a subgroup of G need not be commutative.

Ans. (a, b, c)

- (a) Given that,

$$(xy)^2 = x^2y^2$$

$$xyxy = xxyy$$

$$yx = xy \quad (\because \text{By applying cancelation laws in group})$$

$$\therefore \forall x, y \in G, yx = xy$$

$$\therefore G \text{ is commutative.}$$
- (b) $\forall x \in G, x^2 = 1$

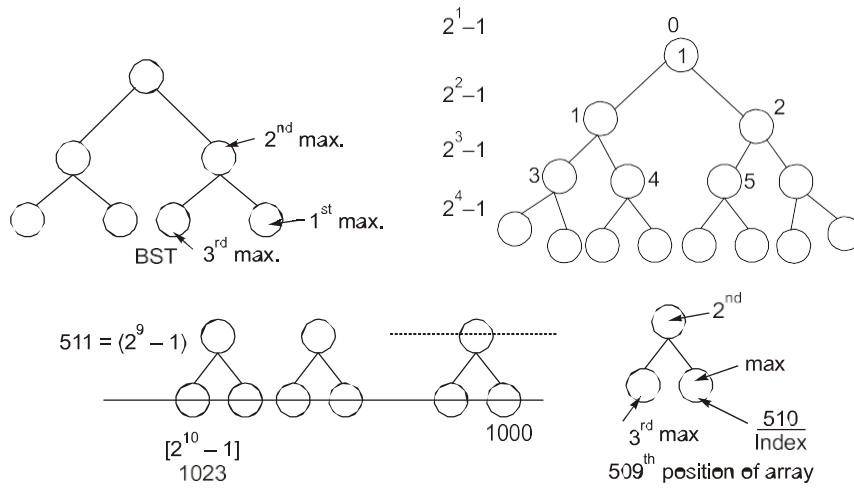
$$\Rightarrow x = x^{-1} \quad (\because x^2 = 1, xx = 1, x^{-1}xx = x^{-1}, ex = x^{-1}, x = x^{-1})$$

In a group if every element has its own inverse then group is commutative.
- (c) Every group of prime order is commutative so of $O(G) = 2$, the group ' G ' is commutative.
- (d) If G is commutative then a subgroup of ' G ' is also commutative.
Let H is subgroup of group commutative group ' G '
 $\forall a, b, \in H$, we have $a, b, \in G$ and $ab = ba$ (\because ' G ' is commutative)
 $\therefore H$ is commutative.

End of Solution

Q.28 Suppose a binary search tree with 1000 distinct elements is also a complete binary tree. The tree is stored using the array representation of binary heap trees. Assuming that the array indices start with 0, the 3rd largest element of the tree is stored at index _____.

Ans. (509)



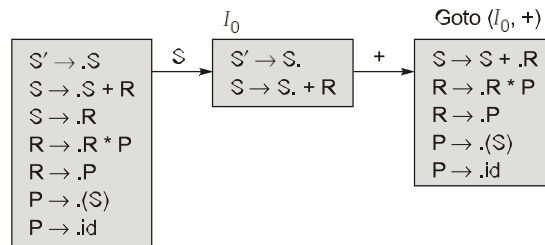
End of Solution

Q.29 Consider the augmented grammar with $\{+, *, (,), id\}$ as the set of terminals.

- $S' \rightarrow S$
- $S \rightarrow S + R \mid R$
- $R \rightarrow R * P \mid P$
- $P \rightarrow (S) \mid id$

If I_0 is the set of two LR(0) items $\{[S' \rightarrow S.], [S \rightarrow S. + R]\}$, then $\text{goto}(\text{closure}(I_0), +)$ contains exactly _____ items.

Ans. (5)



$\text{Goto}(I_0, +)$ will have 5 elements.

End of Solution

Q.30 Consider a simple undirected graph of 10 vertices. If the graph is disconnected, then the maximum number of edges it can have is _____.

Ans. (36)

If 'G' is a group with 'n' vertices and having 'k' connected components then maximum number of edges in 'G'.

$$= \frac{(n-k)(n-k+1)}{2}$$

In the given problem, $n = 10, k = 2$

$$= \frac{(10-2)(10-2+1)}{2} = 36$$

End of Solution

Q.31 Consider a relation $R(A, B, C, D, E)$ with the following three functional dependencies.

$$AB \rightarrow C; BC \rightarrow D; C \rightarrow E;$$

The number of superkeys in the relation R is _____.

Ans. (8)

AB : Candidate key
Number of superkeys 8.

End of Solution

Q.32 The number of arrangements of six identical balls in three identical bins is _____.

Ans. (7)

Given that object are 6 identical balls this means that it does not matter. Which objects are grouped together, it only matters how many objects go into each bins. In addition to this the bins are identical, this means that ordering of bins does not matter. This problem can be modeled as partition of '6' into exactly '3' parts as follows.

(6, 0, 0) (5, 1, 0) (4, 2, 0) (4, 1, 1) (3, 3, 0) (3, 2, 1) (2, 2, 2)

End of Solution

Q.33 A cache memory that has a hit rate of 0.8 has an access latency 10 ns and miss penalty 100 ns. An optimization is done on the cache to reduce the miss rate. However, the optimization results in an increase of cache access latency to 15 ns, whereas the miss penalty is not affected. The minimum hit rate (rounded off to two decimal places) needed after the optimization such that it should not increase the average memory access time is _____.

Ans. (0.85) (0.85 to 0.85)

Memory w/o optimization

$$H_C = 0.8$$

$$T_C = 10 \text{ ns}$$

$$\text{Miss penalty} = 100 \text{ ns}$$

$$T_{\text{avg}} = T_C + (1 - H_C) \text{ miss penalty}$$

$$= (10) + (1 - 0.8) 100 = 30 \text{ ns}$$

Memory with optimization

$$T_C = 15 \text{ ns}$$

$$\begin{aligned}
 \text{Miss penalty} &= 100 \text{ ns} \\
 H_C &= ? \\
 T_{\text{avg}} &= 28 \text{ ns} \\
 T_{\text{avg}} &= T_C + (1 - H_C) \text{ miss penalty} \\
 30 &= (15) + (1 - H_C) 100 \text{ ns} \\
 H_C &= 0.85
 \end{aligned}$$

End of Solution

Q.34 The value of the following limit is _____.

$$\lim_{x \rightarrow 0^+} \frac{\sqrt{x}}{1 - e^{2\sqrt{x}}}$$

Ans. (-0.5)

Apply L Hospital rule,

$$\lim_{x \rightarrow 0^+} \frac{\frac{1}{2\sqrt{x}}}{-e^{2\sqrt{x}} \times \frac{1}{\sqrt{x}}} = \lim_{x \rightarrow 0^+} \frac{-1}{2e^{2\sqrt{x}}} = \frac{-1}{2} = -0.5$$

End of Solution

Q.35 Consider the resolution of the domain name www.gate.org.in by a DNS resolver. Assume that no resource records are cached anywhere across the DNS servers and that iterative query mechanism is used in the resolution. The number of DNS query-response pairs involved in completely resolving the domain name is _____.

Ans. (4)

In the iterative query, the DNS resolver goes to root server and then forwards to top level domain and forwards to secondary to level domain and gets IP address from authoritative DNS server.

End of Solution

Q.36 Which one of the following is the closed form for the generating function of the sequence $\{a_n\}_{n \geq 0}$ defined below?

$$a_n = \begin{cases} n+1, & n \text{ is odd} \\ 1, & \text{otherwise} \end{cases}$$

(a) $\frac{x(1+x^2)}{(1-x^2)^2} + \frac{1}{1-x}$

(b) $\frac{x(3-x^2)}{(1-x^2)^2} + \frac{1}{1-x}$

(c) $\frac{2x}{(1-x^2)^2} + \frac{1}{1-x}$

(d) $\frac{x}{(1-x^2)^2} + \frac{1}{1-x}$

Ans. (a)

$$\begin{aligned}
 a_n &= n + 1 \text{ if 'n' is odd} \\
 &= 1 \quad \text{otherwise} \\
 \frac{(n+k)(n-k+1)}{2} &= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + a_7x^7 + \dots \\
 &= 1 + 2x + x^2 + 4x^3 + x^4 + 6x^5 + x^6 + 8x^7 + \dots \\
 &= (1 + x + x^2 + x^3 + x^4 + \dots) + x(1 + 3x^2 + 5x^4 + 7x^6 + \dots) \\
 &= \frac{1}{(1-x)} + x \frac{d}{dx}(x + x^3 + x^5 + x^7 + \dots) \\
 &= \frac{1}{(1-x)} + x \frac{d}{dx} \left[\frac{x}{(1-x^2)} \right] \\
 &= \frac{1}{(1-x)} + x \left[\frac{(1-x^2) - x(-2x)}{(1-x^2)^2} \right] \\
 &= \frac{1}{(1-x)} + \frac{x(1+x^2)}{(1-x^2)^2}
 \end{aligned}$$

End of Solution

Q.37 Consider a simple undirected unweighted graph with at least three vertices. If A is the adjacency matrix of the graph, then the number of 3-cycles in the graph is given by the trace of

- (a) A^3 (b) A^3 divided by 2
 (c) A^3 divided by 3 (d) A^3 divided by 6

Ans. (d)

Represent graph in adjacency matrix format.

If adjacency matrix multiply by itself 3 times matrix.

Adjacency matrix \times Adjacency matrix \times Adjacency matrix

A_{ii}^3 [diagonal element] represents cycle of length 3 with beginning and ending with vertex i .

Trace of $A^3 = A_{11}^3 + A_{22}^3 + A_{33}^3 + \dots + A_{nn}^3$ (sum of diagonals)

Since cycle has 3 vertices it counted for every vertex we need to divide by 3 for trace of A^3 and because and undirected graph $A - B - C - A$ same as $A - C - B - A$ cycle. To eliminate repeated possibility divide by 2.

Number of cycles of 3 vertices = $\frac{A^3}{6}$

End of Solution

- Q.38** Which one of the following statements is FALSE?
- The TLB performs an associative search in parallel on all its valid entries using page number of incoming virtual address.
 - If the virtual address of a word given by CPU has a TLB hit, but the subsequent search for the word results in a cache miss, then the word will always be present in the main memory.
 - The memory access time using a given inverted page table is always same for all incoming virtual addresses.
 - In a system that uses hashed page tables, if two distinct virtual addresses V1 and V2 map to the same value while hashing, then the memory access time of these addresses will not be the same.

- Ans. (c)**
- TLB performs parallel search. Hence true
 - If the TLB hit means, word will surely present in main memory. Hence true
 - Memory access time using inverted page table is always not same, because there is no indexing applied, we follows no equal linear search. Hence false
 - If two distinct virtual address map to same value while hashing, they will be resolved using linked list, so memory access time will not be same. Hence true

End of Solution

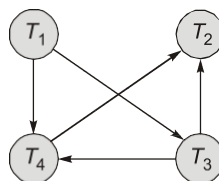
- Q.39** Let $R_i(z)$ and $W_i(z)$ denote read and write operations on a data element z by a transaction T_i , respectively. Consider the schedule S with four transactions.

$$S : R_4(x), R_2(x), R_3(x), R_1(y), W_1(y), W_2(x), W_3(y), R_4(x)$$

Which one of the following serial schedules is conflict equivalent to S ?

- | | |
|---|---|
| (a) $T_1 \rightarrow T_3 \rightarrow T_4 \rightarrow T_2$ | (b) $T_1 \rightarrow T_4 \rightarrow T_3 \rightarrow T_2$ |
| (c) $T_4 \rightarrow T_1 \rightarrow T_3 \rightarrow T_2$ | (d) $T_3 \rightarrow T_1 \rightarrow T_4 \rightarrow T_2$ |

- Ans. (a)**



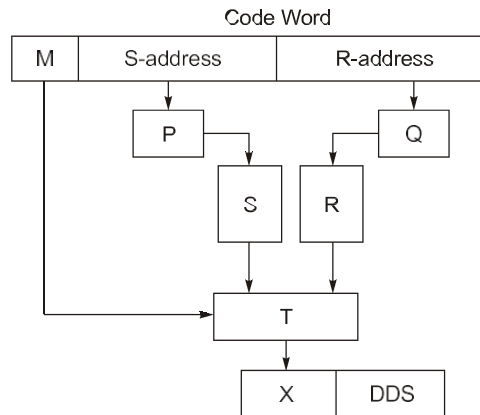
Precedence graph of schedule S .

Topological order = Conflict equal serial order

$$\Rightarrow T_1 T_3 T_4 T_2$$

End of Solution

- Q.40** Consider a digital display system (DDS) shown in the figure that displays the contents of register X. A 16-bit code word is used to load a word in X, either from S or from R. S is a 1024-word memory segment and R is a 32-word register file. Based on the value of mode bit M, T selects an input word to load in X. P and Q interface with the corresponding bits in the code word to choose the addressed word. Which one of the following represents the functionality of P, Q, and T?



- (a) P is 10 : 1 multiplexer; Q is 5 : 1 multiplexer; T is 2 : 1 multiplexer
 (b) P is 10 : 2^{10} decoder; Q is 5 : 2^5 decoder; T is 2 : 1 encoder
 (c) P is 10 : 2^{10} decoder; Q is 5 : 2^5 decoder; T is 2 : 1 multiplexer
 (d) P is 1 : 10 de-multiplexer; Q is 1 : 5 de-multiplexer; T is 2 : 1 multiplexer

Ans. (c)

S is 1024 word memory segment, it needs 10 address lines.

∴ P must be a decoder, with 10 input lines and 1024 output lines. (10 : 2^{10} decoder)

R is 32 word register file, it needs 5 address lines.

∴ Q must be a decoder, with 5 input lines and 32 output lines. (5 : 2^5 decoder)

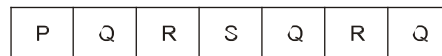
Based on mode bit M, T selects an input word to load X.

∴ T must be 2 : 1 multiplexer, M is select input to the multiplexer.

End of Solution

- Q.42** Consider four processes P, Q, R, and S scheduled on a CPU as per round robin algorithm with a time quantum of 4 units. The processes arrive in the order P, Q, R, S, all at time $t = 0$. There is exactly one context switch from S to Q, exactly one context switch from R to Q, and exactly two context switches from Q to R. There is no context switch from S to P. Switching to a ready process after the termination of another process is also considered a context switch. Which one of the following is NOT possible as CPU burst time (in time units) of these processes?
- (a) P = 4, Q = 10, R = 6, S = 2
 - (b) P = 2, Q = 9, R = 5, S = 1
 - (c) P = 4, Q = 12, R = 5, S = 4
 - (d) P = 3, Q = 7, R = 7, S = 3

Ans. (d)



The above gantt chart satisfies all conditions given in the question the option (a), (b), (c) but not option (d).

So, answer is option (d)

End of Solution

- Q.43** What is printed by the following ANSI C program?

```
#include<stdio.h>
int main(int argc, char *argv[ ])
{
    int a[3][3][3] = {{1, 2, 3, 4, 5, 6, 7, 8, 9},
        {10, 11, 12, 13, 14, 15, 16, 17, 18},
        {19, 20, 21, 22, 23, 24, 25, 26, 27}};
    int i = 0, j = 0, k = 0;
    for(i = 0; i < 3; i++) {
        for(k = 0; k < 3; k++)
            printf("%d", a[i][j][k]);
        printf("\n");
    }
    return 0;
}
```

- | | |
|--|--|
| <p>(a) 1 2 3
10 11 12
19 20 21</p> | <p>(b) 1 4 7
10 13 16
19 22 25</p> |
| <p>(c) 1 2 3
4 5 6
7 8 9</p> | <p>(d) 1 2 3
13 14 15
25 26 27</p> |

Ans. (a)

```
int a[3][3][3] = {{1, 2, 3, 4, 5, 6, 7, 8, 9}, {10, 11, 12, 13, 14, 15, 16, 17, 18},
                 {19, 20, 21, 22, 23, 24, 25, 26, 27}}
```

```
int i = 0, j = 0; k = 0;
```

```
for (i = 0; i < 3; i ++)
```

```
for(k = 0; k < 3, k ++)
```

```
printf(a[i][j][k]);
```

```
0 0 0 | 0 0 1 | 0 0 2 | 1 0 0 | 1 0 1 and so on.
i j k | i j k | i j k | i j k | i j k
```

$$= \begin{array}{ccc} 1 & 2 & 3 \\ 10 & 11 & 12 \\ 19 & 20 & 21 \end{array}$$

This is the answer.

End of Solution

Q.44 What is printed by the following ANSI C program?

```
#include <stdio.h>
```

```
int main(int argc, char *argv[ ]) {
```

```
char a = 'P';
```

```
char b = 'x';
```

```
char c = (a & b) + '*';
```

```
char d = (a | b) - '-';
```

```
char e = (a ^ b) + '+';
```

```
printf("%c %c %c\n", c, d, e);
```

```
return 0;
```

```
}
```

ASCII encoding for relevant characters is given below:

A	B	C	...	Z
65	66	67	...	90

a	b	c	...	Z
97	98	99	...	122

*	+	-
42	43	46

(a) z K S

(c) * - +

(b) 122 75 83

(d) P x +

Ans. (a)

$$\begin{aligned}
 a \rightarrow 80 &= 1010000 \\
 b \rightarrow 120 &= \underline{1111000} \\
 &\quad \& 1010000 = 80 \\
 &\quad \text{or } \underline{1111000} = 120 \\
 &\quad \wedge \underline{0101000} = 83
 \end{aligned}$$

End of Solution

Q.45 Consider solving the following system of simultaneous equations using LU decomposition.

$$\begin{aligned}
 x_1 + x_2 - 2x_3 &= 4 \\
 x_1 + 3x_2 - x_3 &= 7 \\
 2x_1 + x_2 - 5x_3 &= 7
 \end{aligned}$$

where L and U are denoted as

$$L = \begin{pmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{pmatrix}, \quad U = \begin{pmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{pmatrix}$$

Which one of the following is the correct combination of values for L_{32} , U_{32} , and x_1 ?

- (a) $L_{32} = 2, U_{33} = -\frac{1}{2}, x_1 = -1$ (b) $L_{32} = 2, U_{32} = 2, x_1 = -1$
 (c) $L_{32} = -\frac{1}{2}, U_{33} = 2, x_1 = 0$ (d) $L_{32} = -\frac{1}{2}, U_{33} = -\frac{1}{2}, x_1 = 0$

Ans. (d)

End of Solution

Q.46 Which of the following is/are undecidable?

- (a) Given two Turing machines M_1 and M_2 , decide if $L(M_1) = L(M_2)$.
 (b) Given a Turing machine M , decide if $L(M)$ is regular.
 (c) Given a Turing machine M , decide if M accepts all strings.
 (d) Given a Turing machine M , decide if M takes more than 1073 steps on every string.

Ans. (a, b, c)

A, B, C choices are all non-trivial properties of RE language (language of TM's) and hence by Rice's theorem are all UNDECIDABLE.

Choice (d) is DECIDABLE. Why?

A Turing Machine sees only at most the first 1073 symbols of input in its first 1073 steps. Hence whether it stops on first 1073 steps depends only on the first 1073 symbols of input.

Since the number of strings of length 1073 is finite, it gives a way to decide this.

Run the input machine M on all inputs of length 1073 and check whether any of them stops within 1073 steps. If so, reject, otherwise accept.

End of Solution

Q.47 Consider the following languages:

$$L_1 = \{a^n wa^n \mid w \in \{a, b\}^*\}$$

$$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$$

Note that w^R is the reversal of the string w . Which of the following is/are TRUE?

- (a) L_1 and L_2 are regular.
- (b) L_1 and L_2 are context-free.
- (c) L_1 is regular and L_2 is context-free.
- (d) L_1 and L_2 are context-free but not regular.

Ans. (a, b, c)

$$L_1 = \{a^n wa^n \mid w \in \{a, b\}^*\}$$

$$L_2 = \{wxw^R \mid w, x \in \{a, b\}^*, |w|, |x| > 0\}$$

- L_1 is regular because by putting $n = 0$, we create a subset $\{w \mid w \in \{a, b\}^*\}$ which contains all possible strings. So if subset of L_1 is $(a + b)^*$, then $L_1 = (a + b)^*$.
- L_2 is regular because by putting w as "a" and "b" we get a regular expression $a(a + b)^+a + b(a + b)^+b$, which covers all other string which can be obtained by putting w as "aa", "ab", "ba", "bb", etc.

So, $L_2 = a(a + b)^+a + b(a + b)^+b$, which is regular.

So L_1 and L_2 both are regular.

Now every regular is also context-free.

So, option (a), (b), (c) are all true and option (d) is false.

End of Solution

Q.48 Consider the following languages:

$$L_1 = \{ww \mid w \in \{a, b\}^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

Which of the following statements is/are FALSE?

- (a) L_1 is not context-free but L_2 and L_3 are deterministic context-free.
- (b) Neither L_1 nor L_2 is context-free.
- (c) L_2 , L_3 and $L_2 \cap L_3$ all are context-free.
- (d) Neither L_1 nor its complement is context-free.

Ans. (b, c, d)

$$L_1 = \{ww \mid w \in \{a, b\}^*\}$$

$$L_2 = \{a^n b^n c^m \mid m, n \geq 0\}$$

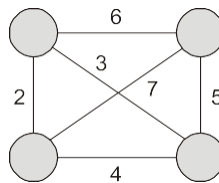
$$L_3 = \{a^m b^n c^n \mid m, n \geq 0\}$$

- L_1 is not context free because it has string matching in straight order, which PDA cannot do.
 - L_2 and L_3 are clearly DCFL's, since they have only one comparison and DPDA can accept both.
- (a) is therefore true.
 (b) is false since L_2 is DCFL and every DCFL is a CFL.
 (c) is false because $L_2 \cap L_3 = \{a^n b^n c^n \mid n \geq 0\}$ is not a CFL.
 (d) is false because complement of "ww" has CFG and is CFL.

End of Solution

- Q.49** Consider a simple undirected weighted graph G , all of whose edge weights are distinct. Which of the following statements about the minimum spanning trees of G is/are TRUE?
- The edge with the second smallest weight is always part of any minimum spanning tree of G .
 - One or both of the edges with the third smallest and the fourth smallest weights are part of any minimum spanning tree of G .
 - Suppose $S \subseteq V$ be such that $S \neq \emptyset$ and $S \neq V$. Consider the edge with the minimum weight such that one of its vertices is in S and the other in $V \setminus S$. Such an edge will always be part of any minimum spanning tree of G .
 - G can have multiple minimum spanning trees.

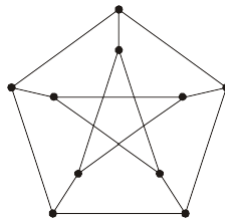
Ans. (a, b, c)



- 2nd minimum cost edge always in MST because 2nd minimum not forms cycle in MST.
- One of 3rd or 4th minimum in MST. If 3rd minimum forms cycle in MST the 4th minimum must be in MST.

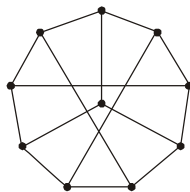
End of Solution

- Q.50** The following simple undirected graph is referred to as the Peterson graph.



Which of the following statements is/are TRUE?

- The chromatic number of the graph is 3.
- The graph has a Hamiltonian path.
- The following graph is isomorphic to the Peterson graph.



- The size of the largest independent set of the given graph is 3. (A subset of vertices of a graph form an independent set if no two vertices of the subset are adjacent.)

Ans. (a, b, c)

- (a) Chromatic number of Peterson graph is 3.
- (b) Peterson graph has Hamiltonian path.
- (c) The given graph is isomorphic to Peterson graph.
- (d) The size of the largest independent set of Peterson graph is 4.

End of Solution

Q.51 Consider the following recurrence:

$$\begin{aligned}
 f(1) &= 1; \\
 f(2n) &= 2f(n) - 1, \text{ for } n \geq 1; \\
 f(2n + 1) &= 2f(n) + 1, \text{ for } n \geq 1;
 \end{aligned}$$

Then, which of the following statements is/are TRUE?

- (a) $f(2^n - 1) = 2^n - 1$
- (b) $f(2^n) = 1$
- (c) $f(5 \cdot 2^n) = 2^{n+1} + 1$
- (d) $f(2^n + 1) = 2^n + 1$

Ans. (a, b, c)

Given functions:

$$\begin{aligned}
 f(2n) &= 2f(n) - 1 & f(2n + 1) &= 2f(n) + 1 \\
 // 2n \text{ is always even} & & // 2n + 1 \text{ is always odd} &
 \end{aligned}$$

<p>(b) $f(2^n) = f(2 \cdot 2^{n-1})$</p> $ \begin{aligned} &= 2f(2 \cdot 2^{n-2}) - 1 \\ &= 2[2f(2^{n-2}) - 1] - 1 \\ &= 2^2 f(2^{n-2}) - 2 - 1 \\ &\vdots \\ &= 2^k f(2^{n-k}) - [2^k - 1] \\ &[\because 2^{n-k} = 1 \Rightarrow 2^k = 2^n] \\ &= 2^n - [2^n - 1] = 1 \end{aligned} $	<p>(a) $f(2^n - 1)$</p> $ \begin{aligned} &f(2(2^{n-1} - 1) + 1) = 2f(2^{n-1} - 1) + 1 \\ &= 2[2f(2^{n-2} - 1) + 1] + 1 \\ &= 2^2 f(2^{n-2} - 1) + 2 + 1 \\ &\vdots \\ &= 2^k f(2^{n-k} - 1) + 2^{k-1} + \dots + 2 + 1 \\ &[\because 2^{n-k} - 1 = 1 \Rightarrow k = n - 1] \\ &= 2^{n-1} + 2^{n-1} - 1 = 2^n - 1 \end{aligned} $
---	--

True

<p>(c) $f(5 \cdot 2^n) = 2^{n+1} + 1$</p>	<p>(d) $f(2^n + 1)$</p> $ \begin{aligned} &= f(2 \cdot 2^{n-1} + 1) \\ &= 2f(2^{n-1}) + 1 \\ &= 2f(2 \cdot 2^{n-2}) + 1 \\ &= 2[2f(2^{n-2}) + 1] + 1 \\ &= 2^2 f(2^{n-2}) + 2 + 1 \\ &\vdots \\ &= 2^k f(2^{n+k}) + (2^k - 1) \\ &[\because 2^n = 2^k] \\ &= 2^n + 2^n - 1 = 2^{n+1} - 1 \end{aligned} $
--	---

End of Solution

Q.52 Which of the properties hold for the adjacency matrix A of a simple undirected unweighted graph having n vertices?

- (a) The diagonal entries of A^2 are the degrees of the vertices of the graph.
- (b) If the graph is connected, then none of the entries of $A^{n-1} + I_n$ can be zero.
- (c) If the sum of all the elements of A is at most $2(n-1)$, then the graph must be acyclic.
- (d) If there is at least a 1 in each of A 's rows and columns, then the graph must be connected.

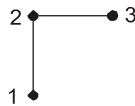
Ans. (a)

(a) Assume adjacency matrix representation of undirected graph is " A ".

$$A^2 = A \times A$$

Diagonal elements of A^2 are degree of vertices or $(A^2)_{ii}$ represents degree of node i . So (a) is true.

(b) Take following connected graph with $n = 3$,



Adjacency matrix, $(A) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

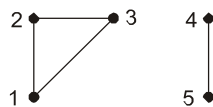
$$A^{n-1} = A^{3-1} = A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$A^{n-1} + I_n = A^2 + I_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 3 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

We can see that $A^{n-1} + I_n$ have some entries as zero. So (b) false.

(c)

Consider following acyclic graph with $n = 5$



Here, $A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$

Sum of all elements in $A = 8$

Here $n = 5$, So $2(n - 1) = 2(5 - 1) = 8$

Here above graph satisfying 'if' condition of graph, but above graph is not acyclic. So, (c) is false.

(d) Consider above graph in option (c), here, in A all rows and columns have atleast a 1, but it is disconnected, So, (d) is false.

End of Solution

Q.53 Which of the following is/are the eigen vector(s) for the matrix given below?

$$A = \begin{pmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{pmatrix}$$

(a) $\begin{pmatrix} -1 \\ 1 \\ 0 \\ 1 \end{pmatrix}$

(b) $\begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix}$

(c) $\begin{pmatrix} -1 \\ 0 \\ 2 \\ 2 \end{pmatrix}$

(d) $\begin{pmatrix} 0 \\ 1 \\ -3 \\ 0 \end{pmatrix}$

Ans. (a, c, d)

$$|A - \lambda I| = \begin{vmatrix} -9-\lambda & -6 & -2 & -4 \\ -8 & -6-\lambda & -3 & -1 \\ 20 & 15 & 8-\lambda & 5 \\ 32 & 21 & 7 & 12-\lambda \end{vmatrix}$$

$$Ax = \lambda x$$

(a) $\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

$$= \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} = \lambda \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \Rightarrow \lambda = 1$$

(b) $\begin{bmatrix} -9 & -6 & -2 & -4 \\ -8 & -6 & -3 & -1 \\ 20 & 15 & 8 & 5 \\ 32 & 21 & 7 & 12 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

156, 160, [164], [168], 172,

Multiples of 8 $\Rightarrow 8 \times 256 \Rightarrow 2^3 \times 2^8 \Rightarrow 2^{11} \Rightarrow 2^{32-21}$

152, 160, 168, 176, 184,

Multiples of 16 $\Rightarrow 16 \times 256 \Rightarrow 2^4 \times 2^8 \Rightarrow 2^{12} \Rightarrow 2^{32-21}$

154, 160, 176, 192,

End of Solution

Q.56 Consider the relational database with the following four schemas and their respective instances.

Student(sNo, sName, dNo) Dept(dNo, dName)

Course(cNo, cName, dNo) Register(sNo, cNo)

Student		
sNo	sName	dNo
S01	James	D01
S02	Rocky	D02
S03	Jackson	D02
S04	Jane	D01
S05	Milli	D02

Dept	
dNo	dName
D01	CSE
D02	EEE

Course		
cNo	sName	dNo
C11	DS	D01
C12	OS	D01
C21	DE	D02
C22	PT	D02
C23	CV	D03

Register	
sNo	cNo
S01	C11
S01	C12
S02	C11
S03	C21
S03	C22
S03	C23
S04	C11
S04	C12
S05	C11
S05	C21

SQL Query:

```
SELECT * FROM Student AS S WHERE NOT EXIST
  (SELECT cNo FROM Course WHERE dNo = "D01"
   EXCEPT
   SELECT cNo FROM Register WHERE sNo = S.sNo)
```

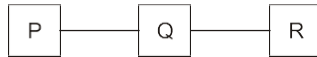
The number of rows returned by the above SQL query is _____.

Ans. (2)

- Given SQL query retrieves student records who register for all courses of dno = D01.
- Course id's and dept D01 are $\{C_{11}, C_{12}\}$.
- Result students who register and all C_{11}, C_{12} courses which are S01, S04 students records.

End of Solution

- Q.57** Consider a network with three routers P, Q, R shown in the figure below. All the links have cost of unity.



The routers exchange distance vector routing information and have converged on the routing tables, after which the link Q-R fails. Assume that P and Q send out routing updates at random times, each at the same average rate. The probability of a routing loop formation (rounded off to one decimal place) between P and Q, leading to count-to-infinity problem, is _____.

Ans. (0.5)

It is given that P and Q are sending routing update at same average rate, so in a particular time period of sending update, there are two possibilities, one is P sends before Q and other is Q sends before P.

Now, We have 2 conditions:

1. Q sends update to P first, then P finds the value of R as ∞ and count-to-infinity will not occur.
2. P sends update to Q first, then Q found that P has a way to reach R, so it is count-to-infinity problem.

As Q - R is broken, So R cannot send update to anyone.

From above two conditions (total possibilities) only one condition leads to count-to-infinity problem and each possibility have equal probability of occurrence, so probability is $1/2 = 0.5$ for count-to-infinity.

End of Solution

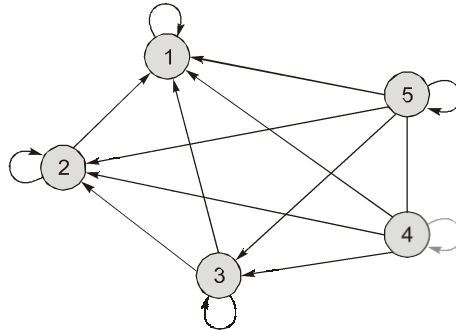
- Q.58** Let $G(V, E)$ be a directed graph, where $V = \{1, 2, 3, 4, 5\}$ is the set of vertices and E is the set of directed edges, as defined by the following adjacency matrix A .

$$A[i][j] = \begin{cases} 1, & 1 \leq j \leq i \leq 5 \\ 0, & \text{otherwise} \end{cases}$$

$A[i][j] = 1$ indicates a directed edge from node i to node j . A directed spanning tree of G , rooted at $r \in V$, is defined as a subgraph T of G such that the undirected version of T is a tree, and T contains a directed path from r to every other vertex in V . The number of such directed spanning trees rooted at vertex 5 is _____.

Ans. (24)

According to given adjacency matrix, the graph will be as following



Now, to find directed spanning tree of the above graph as per definition given in question, $r = 5$ is given,

- (i) To reach node 4, from 5 : $(5, 4) = 1$ way
- (ii) To reach node 3, from $\{5, 4\}$: $(5, 3), (4, 3) = 2$ ways
- (iii) To reach node 2, from $\{5, 4, 3\}$: $(5, 2), (4, 2), (3, 2) = 3$ ways
- (iv) To reach node 1, from $\{5, 4, 3, 2\}$: $(5, 1), (4, 1), (3, 1), (2, 1) = 4$ ways

From (i), (ii), (iii), (iv),

Total no. of ways to reach node 4, 5, 2, 1

From node 5 = $1 \times 2 \times 3 \times 4 = 24$ ways

So, there are 24 directed spanning trees for given graph.

End of Solution

Q.59 Consider a 100 Mbps link between an earth station (sender) and a satellite (receiver) at an altitude of 2100 km. The signal propagates at a speed of 3×10^8 m/s. The time taken (in milliseconds, rounded off to two decimal places) for the receiver to completely receive a packet of 1000 bytes transmitted by the sender is _____.

Ans. (7.08)

$$T.T = \frac{\text{Data size}}{\text{Bandwidth}} = \frac{1000 \times 8 \text{ bits}}{10^8 \text{ bits/sec}} = 8 \times 10^{-5} \text{ sec}$$

$$T.T = 0.08 \text{ millisecc}$$

$$P.T = \frac{\text{Length}}{\text{Velocity}} = \frac{2100 \times 10^3 \text{ m}}{3 \times 10^8 \text{ m/sec}} = 7 \times 10^{-3} \text{ sec}$$

$$= 7 \text{ millisecc}$$

$$\text{Total time} = 7 + 0.08 = 7.08 \text{ millisecc}$$

End of Solution

Q.60 Consider the data transfer using TCP over a 1 Gbps link. Assuming that the maximum segment lifetime (MSL) is set to 60 seconds, the minimum number of bits required for the sequence number field of the TCP header, to prevent the sequence number space from wrapping around during the MSL is _____.

Ans. (33)

$$\text{B.W} = 1 \text{ Gbps}$$

$$= \frac{2^{30}}{8} \text{ bytes/sec}$$

$$1 \text{ sec} = \frac{2^{30}}{8} \text{ bytes}$$

$$\Rightarrow 1 \text{ sec} = \frac{2^{30}}{8} \text{ sequence number}$$

$$\Rightarrow 60 \text{ sec} = \frac{2^{30} \times 60}{8} \text{ sequence number}$$

$$\text{Number of sequence bits required} = \log_2 \left[\frac{2^{30} \times 60}{8} \right]$$

$$= \log_2 2^{30} + \log_2 60 - \log_2 8$$

$$= 30 + 5.9 - 3$$

$$= 32.9 \approx 33$$

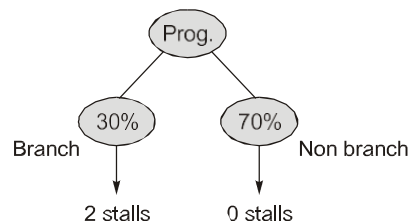
End of Solution

Q.61 A processor X_1 operating at 2 GHz has a standard 5-stage RISC instruction pipeline having a base CPI (cycles per instruction) of one without any pipeline hazards. For a given program P that has 30% branch instructions, control hazards incur 2 cycles stall for every branch. A new version of the processor X_2 operating at same clock frequency has an additional branch predictor unit (BPU) that completely eliminates stalls for correctly predicted branches. There is neither any savings nor any additional stalls for wrong predictions. There are no structural hazards and data hazards for X_1 and X_2 . If the BPU has a prediction accuracy of 80%, the speed up (rounded off to two decimal places) obtained by X_2 over X_1 in executing P is _____.

Ans. (1.42) (1.42 to 1.43)

$$\text{Cycle time} = 0.5 \text{ ns}$$

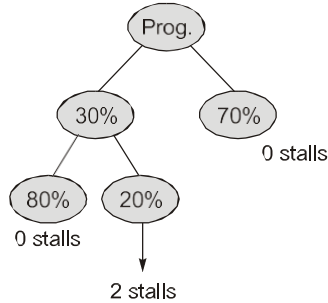
X_1 : Without branch prediction



$$E_T = (1 + \text{Number of stalls/instruction}) \text{ cycle time}$$

$$= [1 + (0.3) \times 2] 0.5 \text{ ns} = 0.8 \text{ ns}$$

X_2 : With branch prediction

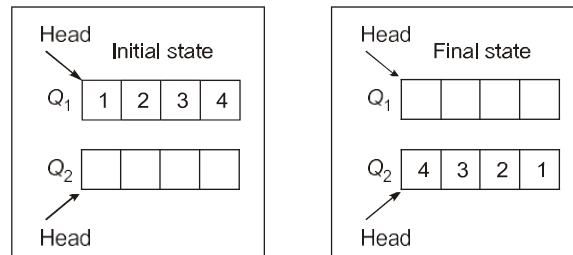


$$E_T = [1 + (0.3 \times 0.2 \times 2)] 0.5 \text{ ns} = 0.56 \text{ ns}$$

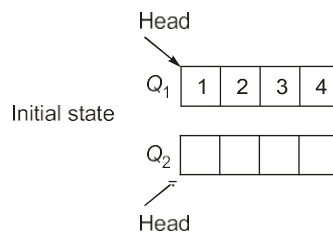
$$S = \frac{0.8}{0.56} = 1.42$$

End of Solution

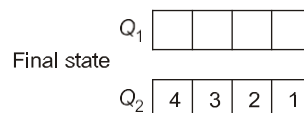
- Q.62** Consider the queues Q_1 containing four elements and Q_2 containing none (shown as the Initial State in the figure). The only operations allowed on these two queues are Enqueue(Q , element) and Dequeue(Q). The minimum number of Enqueue operations on Q_1 required to place the elements of Q_1 in Q_2 in reverse order (shown as the Final State in the figure) without using any additional storage is _____.



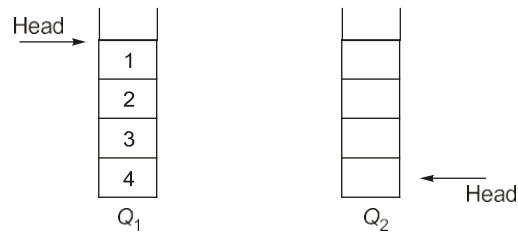
Ans. (0)



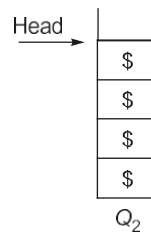
No additional storage.
Enqueues on Q_1 = ?



Consider Q_1 and Q_2 as the following



Now, Enqueue (Q_2 , \$) 4 times,



Now, one by one perform

1. $x = \text{Dequeue}(Q_1)$
2. Enqueue (Q_2, x) until Q_1 is empty.

End of Solution

Q.63 Consider two files systems A and B, that use contiguous allocation and linked allocation, respectively. A file of size 100 blocks is already stored in A and also in B. Now, consider inserting a new block in the middle of the file (between 50th and 51st block), whose data is already available in the memory. Assume that there are enough free blocks at the end of the file and that the file control blocks are already in memory. Let the number of disk accesses required to insert a block in the middle of the file in A and B are n_A and n_B , respectively, then the value of $n_A + n_B$ is _____.

Ans. (153)

Contiguous Allocation :

50 read operations + 50 write operations + 1 operation to write the newly inserted block = 101 operations (n_A)

Linked Allocation:

50 read operations + 1 operation delete next pointer of 50th element + 1 operation to connect it to the 51st element = 52 operations (n_B)

Total operations:

$$\text{Required} = n_A + n_B = 101 + 52 = 153$$

End of Solution

- Q.64** Consider a demand paging system with four page frames (initially empty) and LRU page replacement policy. For the following page reference string
7, 2, 7, 3, 2, 5, 3, 4, 6, 7, 7, 1, 5, 6, 1
the page fault rate, defined as the ratio of number of page faults to the number of memory accesses (rounded off to one decimal place) is _____.

Ans. (0.6)

9 page faults out of total 15 references $\frac{9}{15} = 0.6$

End of Solution

- Q.65** Consider the following grammar along with translation rules.

$$\begin{aligned} S &\rightarrow S_1 \# T && \{S.val = S_1.val * T.val\} \\ S &\rightarrow T && \{S.val = T.val\} \\ T &\rightarrow T_1 \% R && \{T.val = T_1.val \div R.val\} \\ T &\rightarrow R && \{T.val = R.val\} \\ R &\rightarrow id && \{R.val = id.val\} \end{aligned}$$

Here # and % are operators and id is a token that represents an integer and id.val represents the corresponding integer value. The set of non-terminals is $\{S, T, R, P\}$ and a subscripted non-terminal indicates an instance of the non-terminal.

Using this translation scheme, the computed value of S.val for root of the parse tree for the expression $20\#10\%5\#8\%2\%2$ is _____.

Ans. (80)

In the given SDT % has more precedence than # and % is left associative.

\therefore The given expression $20\#10\%5\#8\%2\%2$ will be $20 \# (10 \% 5) \# [(8 \% 2) \% 2]$.

Now replacing the operators # and % with * and \div respectively the expression will be

$$\begin{aligned} &= 20 * (10 \div 5) * [(8 \div 2) \div 2] \\ &= 20 * 2 * [4 \div 2] \\ &= 20 * 2 * 2 = 80 \end{aligned}$$

End of Solution

