1 Concept of Measurement Systems

1.1 INTRODUCTION

Measurement is the act, or the result, of a quantitative comparison between a given quantity and a quantity of the same kind chosen as a unit. The result of the measurement is expressed by a pointer deflection over a predefined scale or a number representing the ratio between the unknown quantity and the standard. A standard is defined as the physical personification of the unit of measurement or its submultiple or multiple values. The device or instrument used for comparing the unknown quantity with the unit of measurement or a standard quantity is called a *measuring instrument.* The value of the unknown quantity can be measured by direct or indirect methods. In direct measurement methods, the unknown quantity is measured directly instead of comparing it with a standard. Examples of direct measurement are current by ammeter, voltage by voltmeter, resistance by ohmmeter, power by wattmeter, etc. In indirect measurement methods, the value of the unknown quantity is determined by measuring the functionally related quantity and calculating the desired quantity rather than measuring it directly. Suppose the resistance as (R) of a conductor can be measured by measuring the voltage drop across the conductor and dividing the voltage (V) by the current (I) through the conductors, by Ohm's $R = \frac{V}{I}$

1.2 FUNDAMENTAL AND DERIVED UNITS

At the time of measuring a physical quantity, we must express the magnitude of that quantity in terms of a unit and a numerical multiplier, i.e.,

Magnitude of a physical quantity = (Numerical ratio) × (Unit)

The numerical ratio is the number of times the unit occurs in any given amount of the same quantity and, therefore, is called the *number of measures.* The numerical ratio may be called *numerical multiplier.* However, in measurements, we are concerned with a large number of quantities which are related to each other, through established physical equations, and therefore the choice of size of units of these quantities cannot be done arbitrarily and independently. In this way, we can avoid the use of awkward numerical constants when we express a quantity of one kind which has been derived from measurement of another quantity.

In science and engineering, two kinds of units are used:

- Fundamental units

- Derived units

The *fundamental units* in mechanics are measures of length, mass and time. The sizes of the fundamental units, whether foot or metre, pound or kilogram, second or hour are arbitrary and can be selected to fit a certain set of circumstances. Since length, mass and time are fundamental to most other physical quantities besides those in mechanics, they are called the *primary fundamental units.* Measures of certain physical quantities in the thermal, electrical and illumination disciplines are also represented by fundamental units. These units are used only when these particular classes are involved, and they may therefore be defined as *auxiliary fundamental units.*

All other units which can be expressed in terms of the fundamental units are called *derived units.* Every derived unit originates from some physical law defining that unit. For example, the area (A) of a rectangle is proportional to its length (l) and breadth (b), or $A = lb$. if the metre has been chosen as the unit of length then the area of a rectangle of 5 metres by 7 metres is 35 m^2. Note that the numbers of measure are multiplied as well as the units. The derived unit for area (A) is then the metre square (m^2).

A derived unit is recognized by its dimensions, which can be defined as the complete algebraic formula for the derived unit. The dimensional symbols for the fundamental units of length, mass and time are L, M and T respectively. The dimensional symbol for the derived unit of area is L^2 and that for volume is L^3 . The dimensional symbol for the unit of force is MLT , which follows from the defining equation for force. The dimensional formulas of the derived units are particularly useful for converting units from one system to another. For convenience, some derived units have been given new names. For example, the derived unit of force in the SI system is called the newton (N), instead of the dimensionally correct kg-m/s^2.

1.3 STANDARDS AND THEIR CLASSIFICATIONS

A standard of measurement is a physical representation of a unit of measurement. A unit is realised by reference to an arbitrary material standard or to natural phenomena including physical and atomic constants. The term ‘standard’ is applied to a piece of equipment having a known measure of physical quantity. For example, the fundamental unit of mass in the SI system is the kilogram, defined as the mass of the cubic decimetre of water at its temperature of maximum of 4°C. This unit of mass is represented by a material standard; the mass of the international prototype kilogram consisting of a platinum–iridium hollow cylinder. This unit is preserved at the International Bureau of Weights and Measures at Sevres, near Paris, and is the material representation of the kilogram. Similar standards have been developed for other units of measurement, including fundamental units as well as for some of the derived mechanical and electrical units.

The classifications of standards are

1. International standards
2. Primary standards

3. Secondary standards
4. Working standards
5. Current standards
6. Voltage standards
7. Resistance standards
8. Capacitance standards
9. Time and frequency standards

1.3.1 International Standards

The international standards are defined by international agreement. They represent certain units of measurement to the closest possible accuracy that production and measurement technology allow. International standards are periodically checked and evaluated by absolute measurements in terms of the fundamental units. These standards are maintained at the International Bureau of Weights and Measures and are not available to the ordinary user of measuring instruments for purposes of comparison or calibration. Table 1.1 shows basic SI Units, Quantities and Symbols.

Table 1.1 *Basic Quantities, SI Units and Symbols*

Quantity	Unit	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Luminous Intensity	Candela	cd
Thermodynamic temperature	Kelvin	K
Electric current	Ampere	A

1.3.2 Primary Standards

The primary standards are maintained by national standards laboratories in different places of the world. The National Bureau of Standards (NBS) in Washington is responsible for maintenance of the primary standards in North America. Other national laboratories include the National Physical Laboratory (NPL) in Great Britain and the oldest in the world, the Physikalisch Technische Reichsanstalt in Germany. The primary standards, again representing the fundamental units and some of the derived mechanical and electrical units, are independently calibrated by absolute measurements at each of the national laboratories. The results of these measurements are compared with each other, leading to a world average figure for the primary standard. Primary standards are not available for use outside the national laboratories. One of the main functions of primary standards is the verification and calibration of secondary standards.

1.3.3 Secondary Standards

Secondary standards are the basic reference standards used in the industrial measurement laboratories. These standards are maintained by the particular involved industry and are checked locally against other reference standards in the area. The responsibility for maintenance and calibration rests entirely with the industrial laboratory itself. Secondary standards are generally sent to the national standards laboratory on a periodic basis for calibration and comparison against the primary standards. They are then returned to the industrial user with a certification of their measured value in terms of the primary standard.

1.3.4 Working Standards

Working standards are the principle tools of a measurement laboratory. They are used to check and calibrate general laboratory instruments for accuracy and performance or to perform comparison measurements in industrial applications. A manufacturer of precision resistances, for example, may use a standard resistor in the quality control department of his plant to check his testing equipment. In this case, the manufacturer verifies that his measurement setup performs within the required limits of accuracy.

1.3.5 Current Standard

The fundamental unit of electric current (Ampere) is defined by the International System of Units (SI) as the constant current which, if maintained in two straight parallel conductors of infinite length and negligible circular cross section placed 1 meter apart in vacuum, will produce between these conductors a force equal to 2×10^{-7} newton per meter length. Early measurements of the absolute value of the ampere were made with a current balance which measured the force between two parallel conductors. These measurements were rather crude and the need was felt to produce a more practical and reproducible standard for the national laboratories. By international agreement, the value of the international ampere was based on the electrolytic deposition of silver from a silver nitrate solution. The *international ampere* was then defined as that current which deposits silver at the rate of 1.118 mg/s from a standard silver nitrate solution. Difficulties were encountered in the exact measurement of the deposited silver and slight discrepancies existed between measurements made independently by the various National Standard Laboratories. Later, the international ampere was superseded by the *absolute ampere* and it is now the fundamental unit of electric current in the SI and is universally accepted by international agreement.

1.3.6 Voltage Standard

In early times, the standard volt was based on an electrochemical cell called the *saturated standard cell* or simply *standard cell.* The saturated cell has temperature dependence, and the output voltage changes about -40 µV/°C from the nominal of 1.01858 volt. The standard cell suffers from this temperature dependence and also from the fact that the voltage is a function of a chemical reaction and not related directly to any other physical constants. In 1962, based on the work of Brian Josephson, a new standard for the volt was introduced. A thin-film junction is cooled to nearly absolute zero and irradiated with microwave energy. A voltage is developed across the junction, which is related to the irradiating frequency by the following relationship:

$$v = \frac{hf}{2e}$$

where, h =Planck's constant = 6.63×10^{-34} J-s

e =charge of an electron = 1.602×10^{-19} C

f =frequency of the microwave irradiation

In Eq. (1.1), the irradiation frequency is the only variable, thus the standard volt is related to the standard of time/frequency. When the microwave irradiating frequency is locked to an atomic clock or a broadcast frequency standard such as WWVB, the accuracy of the standard volt, including all of the system inaccuracies, is one part in 10^8.

The major method of transferring the volt from the standard based on the Josephson junction to secondary standards used for calibration of the standard cell. This device is called the normal or saturated Weston cell. The Weston cell has a positive electrode of mercury and a negative electrode of cadmium amalgam (10% cadmium). The electrolyte is a solution of cadmium sulfate. These components are placed in an H-shaped glass container as shown in Figure 1.1.

Figure 1.1 *Standard cell of emf of 1.0183 volt at 20°C (Courtesy, physics.kenyon.edu)*

1.3.7 Resistance Standard

In the SI system, the absolute value of ohm is defined in terms of the fundamental units of length, mass and time. The absolute measurement of the ohm is carried out by the International Bureau of Weights and Measures in Sevres and also by the national standard laboratories, which preserve a group of primary resistance standards. The NBS maintains a group of those primary standards (1 ohm standard resistors) which are periodically checked against each other and are occasionally verified by absolute measurements. The standard resistor is a coil of wire of some alloy like manganin which has a high electrical resistivity and a low temperature coefficient of resistance. The resistance coil is mounted in a double walled sealed container as shown in Figure 1.2 to prevent changes in resistance due to moisture conditions in the atmosphere. With a set of four or five 1-ohm resistors in this type, the unit resistance can be represented with a precision of a few parts in 10^7 over several years.

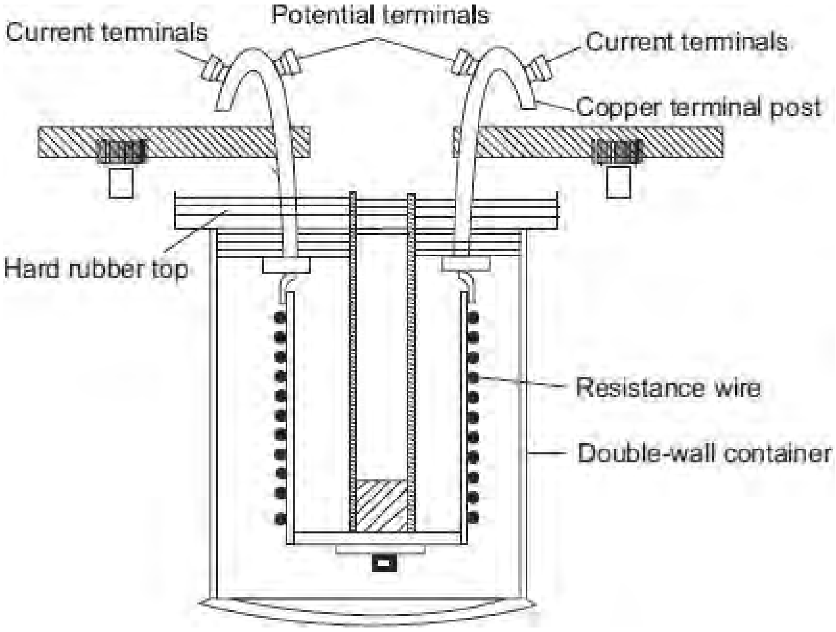


Figure 1.2 *Resistance standard*

Secondary standards and working standards are available from some instrument manufactures in a wide range of values, usually in multiples of 10 ohms. These standard resistors are sometimes called *transfer resistor* and are made of alloy resistance wire, such as manganin or Evanohm. The resistance coil of the transfer resistor is supported between polyester films to reduce stresses on the wire and to improve the stability of the resistor. The coil is immersed in moisture free oil and placed in a sealed container. The connections to the coil are silver soldered, and the terminal hooks are made of nickel-plated oxygen free copper. The transfer resistor is checked for stability and temperature characteristics at its rated power and a specified operating temperature (usually 25°C). A calibration report accompanying the resistor specifies its traceability to NBS standards and includes the *a* and β temperature coefficients. Although the selected resistance wire provides almost constant resistance over a fairly wide temperature range, the exact value of the resistance at any temperature can be calculated from the formula

$$R_t = R_{25°C} + \alpha(t - 25) + \beta(t - 25)^2$$

where R_t = resistance at the ambient temperature *t*

$R_{25°C}$ = reisitance at 25°c

α,β = temperature coefficients

Temperature coefficient *a* is usually less than $10 \times 10^{-}$, and coefficient β lies between $-3 \times 10^{-}$ to -6×10^{-7}. This means that a change in temperature of 10°C from the specified reference temperature of 25°C may cause a change in resistance of 30 to 60 ppm from the nominal value.

1.3.8 Capacitance Standard

Many electrical and magnetic units may be expressed in terms of these voltage and resistance standards since the unit of resistance is represented by the standard resistor and the unit of voltage by standard Weston cell. The unit of capacitance (the farad) can be measured with a Maxwell dc commutated bridge, where the capacitance is computed from the resistive bridge arms and the frequency of the dc commutation. The bridge is shown in Figure 1.3. Capacitor C is alternately charged and discharged through the commutating contact and resistor R. Bridge balance is obtained by adjusting the resistance R_3, allowing exact determination of the capacitance value in terms of the bridge arm constants and frequency of commutation. Although the exact derivation of the expression for capacitance in terms of the resistances and the frequency is rather involved, it may be seen that the capacitor could be measured accurately by this method. Since both resistance and frequency can be determined very accurately, the value of the capacitance can be measured with great accuracy. Standard capacitors are usually constructed from interleaved metal plates with air as the dielectric material. The area of the plates and the distance between them must be known very accurately, and the capacitance of the air capacitor can be determined from these basic dimensions. The NBS maintains a bank of air capacitors as standards and uses them to calibrate the secondary and working standards of measurement laboratories and industrial users.

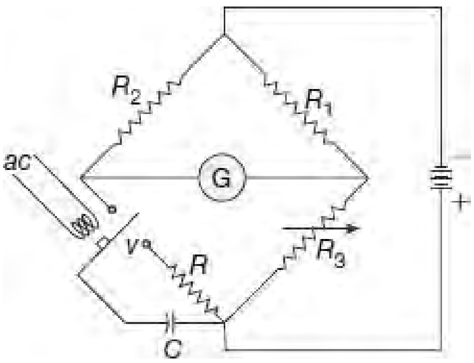


Figure 1.3 *Commutated dc method for measuring capacitance*

1.3.9 Time Standard and Frequency Standard

In early centuries the time reference used was the rotation of the earth around the sun about its axis. Later, precise astronomical observations have shown that the rotation of the earth around the sun is very irregular, owing to secular and irregular variations in the rotational speed of the earth. So the time scale based on this apparent solar time had to be changed. *Mean solar time* was thought to give a more accurate time scale. A *mean solar day* is the average of all the apparent days in the year. A *mean solar second* is then equal to 1/86400 of the mean solar day. The mean solar second is still inappropriate since it is based on the rotation of the earth which is non-uniform.

In the year 1956, the *ephemeris second* has been defined by the International Bureau of Weights and Measures as the fraction 1/31556925.99747 of the tropical year for 1900 January 01 at 12 h ET (Ephemeris Time), and adopted as the fundamental invariable unit of time. A disadvantage of the use of the *ephemeris* second is that it can be determined only several years in arrears and then only indirectly, by observations of the positions of the sun and the moon. For physical measurements, the unit of time interval has now been defined in terms of an atomic standard. The universal second and the *ephemeris* second, however, will continue to be used for navigation, geodetic surveys and celestial mechanics. The atomic units of the time was first related to UT (Universal Time) but was later expressed in terms of ET. The International Committee of Weights and Measures has now defined the second in terms of frequency of the cesium transition, assigning a value of 9192631770 Hz to the hyperfine transition of the cesium atom unperturbed by external fields.

The atomic definition of second realises an accuracy much greater than that achieved by astronomical observations, resulting in a more uniform and much more convenient time base. Determinations of time intervals can now be made in a few minutes to greater accuracy than was possible before in astronomical measurements that took many years to complete. An atomic clock with a precision exceeding 1 μs per day is in operation as a primary frequency standard at the NBS. An atomic time scale, designated NBS-A, is maintained with this clock.

Time and frequency standards are unique in that they may be transmitted from the primary standard at NBS to other locations via radio or television transmission. Early standard time and frequency transmission were in the High Frequency (HF) portion of the radio spectrum, but these transmissions suffered from Doppler shifts due to the fact that radio propagation was primarily ionospheric. Transmission of time and frequency standards via low frequency and very low frequency radio reduces this Doppler shift because the propagation is strictly ground wave. Two NBS operated stations, WWVL and WWVB, operate 20 and 60 kHz, respectively, providing precision time and frequency transmissions.

1.4 METHODS OF MEASUREMENT

As discussed above, the measurement methods can be classified as

- Direct comparison methods
- Indirect comparison methods

1.4.1 Direct Comparison Methods

In direct measurement methods, the unknown quantity is measured directly. Direct methods of measurement are of two types, namely, *deflection methods* and *comparison methods.*

In deflection methods, the value of the unknown quantity is measured by the help of a measuring instrument having a calibrated scale indicating the quantity under measurement directly, such as measurement of current by an ammeter.

In comparison methods, the value of the unknown quantity is determined by direct comparison with a standard of the given quantity, such as measurement of emf by comparison with the emf of a standard cell. Comparison methods can be classified as null methods, differential methods, etc. In null methods of measurement, the action of the unknown quantity upon the instrument is reduced to zero by the counter action of a known quantity of the same kind, such as measurement of weight by a balance, measurement of resistance, capacitance, and inductance by bridge circuits.

1.4.2 Indirect Comparison Methods

In indirect measurement methods, the comparison is done with a standard through the use of a calibrated system. These methods for measurement are used in those cases where the desired parameter to be measured is difficult to be measured directly, but the parameter has got some relation with some other related parameter which can be easily measured.

For instance, the elimination of bacteria from some fluid is directly dependent upon its temperature. Thus, the bacteria elimination can be measured indirectly by measuring the temperature of the fluid.

In indirect methods of measurement, it is general practice to establish an empirical relation between the actual measured quantity and the desired parameter.

The different methods of measurement are summarised with the help of a tree diagram in Figure 1.4.

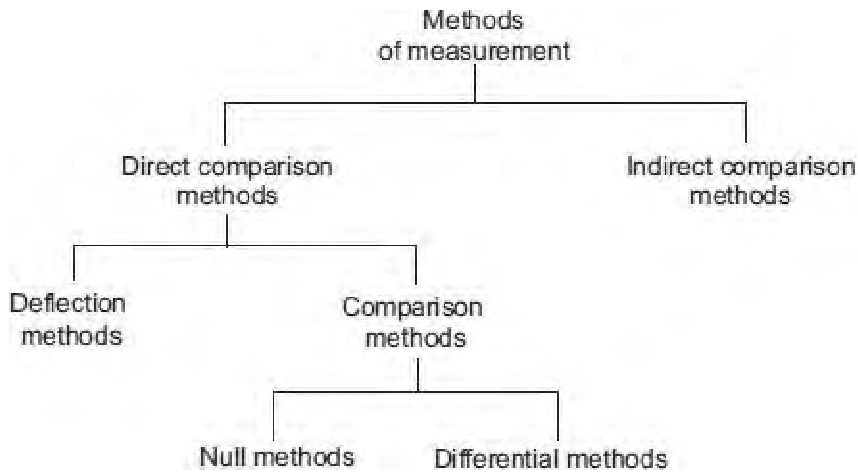


Figure 1.4 *Different methods of measurement*

1.5 MEASUREMENT SYSTEM AND ITS ELEMENTS

A measurement system may be defined as a systematic arrangement for the measurement or determination of an unknown quantity and analysis of instrumentation. The generalised measurement system and its different components/elements are shown in Figure 1.5.

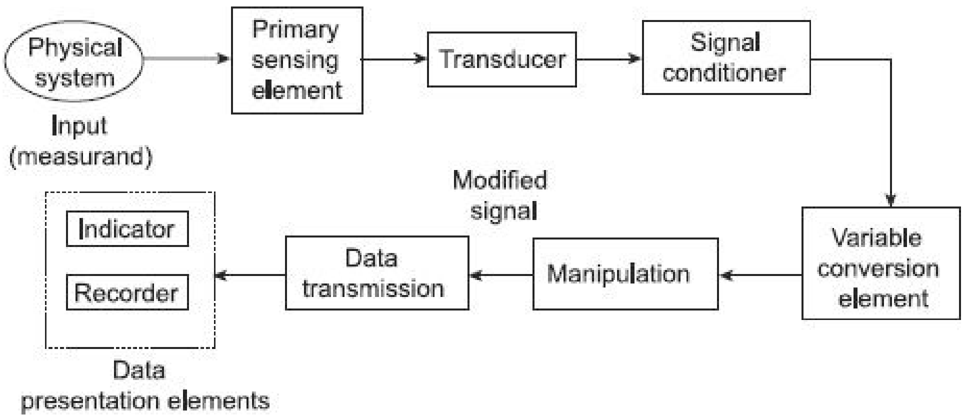


Figure 1.5 *Generalised measurement system*

The operation of a measurement system can be explained in terms of functional elements of the system. Every instrument and measurement system is composed of one or more of these functional elements and each functional element is made of distinct components or groups of components which performs required and definite steps in measurement. The various elements are the following:

1.5.1 Primary Sensing Elements

It is an element that is sensitive to the measured variable. The physical quantity under measurement, called the *measurand*, makes its first contact with the primary sensing element of a measurement system. The measurand is always disturbed by the act of the measurement, but good instruments are designed to minimise this effect. Primary sensing elements may have a non-electrical input and output such as a spring, manometer or may have an electrical input and output such as a rectifier. In case the primary sensing element has a non-electrical input and output, then it is converted into an electrical signal by means of a transducer. The transducer is defined as a device, which when actuated by one form of energy, is capable of converting it into another form of energy.

Many a times, certain operations are to be performed on the signal before its further transmission so that interfering sources are removed in order that the signal may not get distorted. The process may be linear such as amplification, attenuation, integration, differentiation, addition and subtraction or nonlinear such as modulation, detection, sampling, filtering, chopping and clipping, etc. The process is called signal conditioning. So a signal conditioner follows the primary sensing element or transducer, as the case may be. The sensing element senses the condition, state or value of the process variable by extracting a small part of energy from the measurand, and then produces an output which reflects this condition, state or value of the measurand.

1.5.2 Variable Conversion Elements

After passing through the primary sensing element, the output is in the form of an electrical signal, may be voltage, current, frequency, which may or may not be accepted to the system. For performing the desired operation, it may be necessary to convert this output to some other suitable form while retaining the information content of the original signal. For example, if the output is in analog form and the next step of the system accepts only in digital form then an analog-to-digital converter will be employed. Many instruments do not require any variable conversion unit, while some others require more than one element.

1.5.3 Manipulation Elements

Sometimes it is necessary to change the signal level without changing the information contained in it for the acceptance of the instrument. The function of the variable manipulation unit is to manipulate the signal presented to it while preserving the original nature of the signal. For example, an electronic amplifier converts a small low voltage input signal into a high voltage output signal. Thus, the voltage amplifier acts as a variable manipulation unit. Some of the instruments may require this function or some of the instruments may not.

1.5.4 Data Transmission Elements

The data transmission elements are required to transmit the data containing the information of the signal from one system to another. For example, satellites are physically separated from the earth where the control stations guiding their movement are located.

1.5.5 Data Presentation Elements

The function of the data presentation elements is to provide an indication or recording in a form that can be evaluated by an unaided human sense or by a controller. The information regarding measurand (quantity to be measured) is to be conveyed to the personnel handling the instrument or the system for monitoring, controlling or analysis purpose. Such a device may be in the form of analog or digital format. The simplest form of a display device is the common panel meter with some kind of calibrated scale and pointer. In case the data is to be recorded, recorders like magnetic tapes or magnetic discs may be used. For control and analysis purpose, computers may be used.

The stages of a typical measurement system are summarised below with the help of a flow diagram in Figure 1.6.

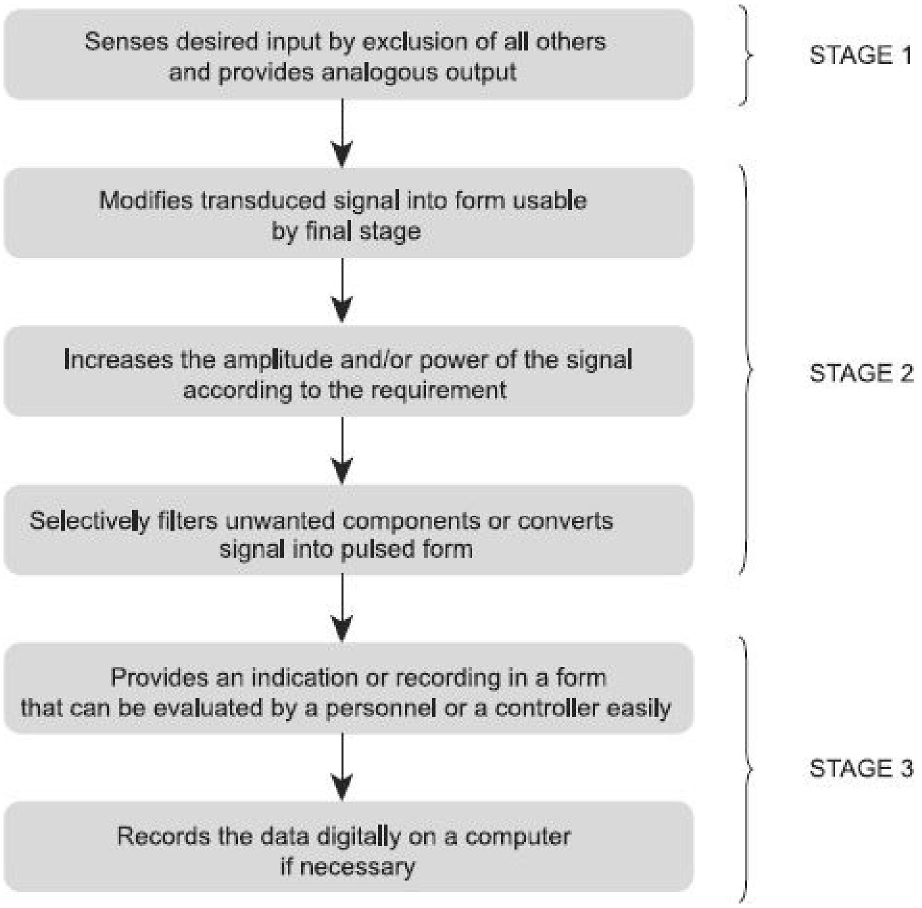


Figure 1.6 *Steps of a typical measurement system*

1.6 CLASSIFICATION OF INSTRUMENTS

The measuring instruments may be classified as follows:

1.6.1 Absolute and Secondary Instruments

1. Absolute Instruments

The instruments of this type give the value of the measurand in terms of instrument constant and its deflection. Such instruments do not require comparison with any other standard. The example of this type of instrument is tangent galvanometer, which gives the value of the current to be measured in terms of tangent of the angle of deflection produced, the horizontal component of the earth's magnetic field, the radius and the number of turns of the wire used. Rayleigh current balance and absolute electrometer are other examples of absolute instruments. Absolute instruments are mostly used in standard laboratories and in similar institutions as standardising. The classification of measuring instruments is shown in Figure 1.7.

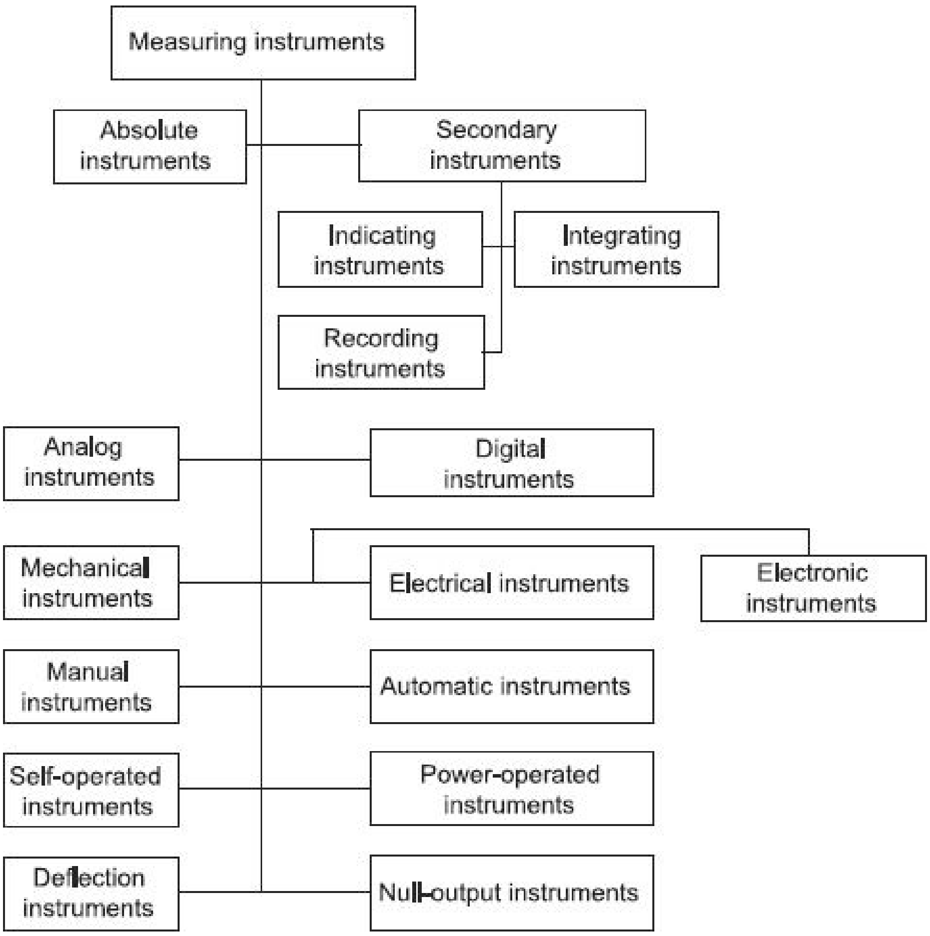


Figure 1.7 *Classification of measuring instruments*

2. Secondary Instruments

These instruments are so constructed that the deflection of such instruments gives the magnitude of the electrical quantity to be measured directly. These instruments are required to be calibrated by comparison with either an absolute instrument or with another secondary instrument, which has already been calibrated before the use. These instruments are generally used in practice.

Secondary instruments are further classified as

- Indicating instruments
- Integrating instruments

- Recording instruments

(i) Indicating Instruments

Indicating instruments are those which indicate the magnitude of an electrical quantity at the time when it is being measured. The indications are given by a pointer moving over a calibrated (pregraduated) scale. Ordinary ammeters, voltmeters, wattmeters, frequency meters, power factor meters, etc., fall into this category.

(ii) Integrating Instruments

Integrating instruments are those which measure the total amount of either quantity of electricity (ampere-hours) or electrical energy supplied over a period of time. The summation, given by such an instrument, is the product of time and an electrical quantity under measurement. The ampere-hour meters and energy meters fall in this class.

(iii) Recording Instruments

Recording instruments are those which keep a continuous record of the variation of the magnitude of an electrical quantity to be observed over a definite period of time. In such instruments, the moving system carries an inked pen which touches lightly a sheet of paper wrapped over a drum moving with uniform slow motion in a direction perpendicular to that of the direction of the pointer. Thus, a curve is traced which shows the variations in the magnitude of the electrical quantity under observation over a definite period of time. Such instruments are generally used in powerhouses where the current, voltage, power, etc., are to be maintained within certain acceptable limit.

1.6.2 Analog and Digital Instruments

1. Analog Instruments

The signals of an analog unit vary in a continuous fashion and can take on infinite number of values in a given range. Fuel gauge, ammeter and voltmeters, wrist watch, speedometer fall in this category.

2. Digital Instruments

Signals varying in discrete steps and taking on a finite number of different values in a given range are digital signals and the corresponding instruments are of digital type. Digital instruments have some advantages over analog meters, in that they have high accuracy and high speed of operation. It eliminates the human operational errors. Digital instruments can store the result for future purposes. A digital multimeter is the example of a digital instrument.

1.6.3 Mechanical, Electrical and Electronics Instruments

1. Mechanical Instruments

Mechanical instruments are very reliable for static and stable conditions. They are unable to respond rapidly to the measurement of dynamic and transient conditions due to the fact that they have moving parts that are rigid, heavy and bulky and consequently have a large mass. Mass presents inertia problems and hence these instruments cannot faithfully follow the rapid changes which are involved in dynamic instruments. Also, most of the mechanical instruments causes noise pollution.

Advantages of Mechanical Instruments

- Relatively cheaper in cost
- More durable due to rugged construction
- Simple in design and easy to use
- No external power supply required for operation
- Reliable and accurate for measurement of stable and time invariant quantity

Disadvantages of Mechanical Instruments

- Poor frequency response to transient and dynamic measurements
- Large force required to overcome mechanical friction
- Incompatible when remote indication and control needed
- Cause noise pollution

2. Electrical Instruments

When the instrument pointer deflection is caused by the action of some electrical methods then it is called an electrical instrument. The time of operation of an electrical instrument is more rapid than that of a mechanical instrument. Unfortunately, an electrical system normally depends upon a mechanical measurement as an indicating device. This mechanical movement has some inertia due to which the frequency response of these instruments is poor.

3. Electronic Instruments

Electronic instruments use semiconductor devices. Most of the scientific and industrial instrumentations require very fast responses. Such requirements cannot be met with by mechanical and electrical instruments. In electronic devices, since the only movement involved is that of electrons, the response time is extremely small owing to very small inertia of the electrons. With the use of electronic devices, a very weak signal can be detected by using pre-amplifiers and amplifiers.

Advantages of Electrical/Electronic Instruments

- Non-contact measurements are possible
- These instruments consume less power
- Compact in size and more reliable in operation
- Greater flexibility
- Good frequency and transient response
- Remote indication and recording possible

- Amplification produced greater than that produced in mechanical instruments

1.6.4 Manual and Automatic Instruments

In case of manual instruments, the service of an operator is required. For example, measurement of temperature by a resistance thermometer incorporating a Wheatstone bridge in its circuit, an operator is required to indicate the temperature being measured.

In an automatic type of instrument, no operator is required all the time. For example, measurement of temperature by mercury-in-glass thermometer.

1.6.5 Self-operated and Power-operated Instruments

Self-operated instruments are those in which no outside power is required for operation. The output energy is supplied wholly or almost wholly by the input measurand. Dial-indicating type instruments belong to this category.

The power-operated instruments are those in which some external power such as electricity, compressed air, hydraulic supply is required for operation. In such cases, the input signal supplies only an insignificant portion of the output power. Electromechanical instruments shown in Figure 1.8 fall in this category.

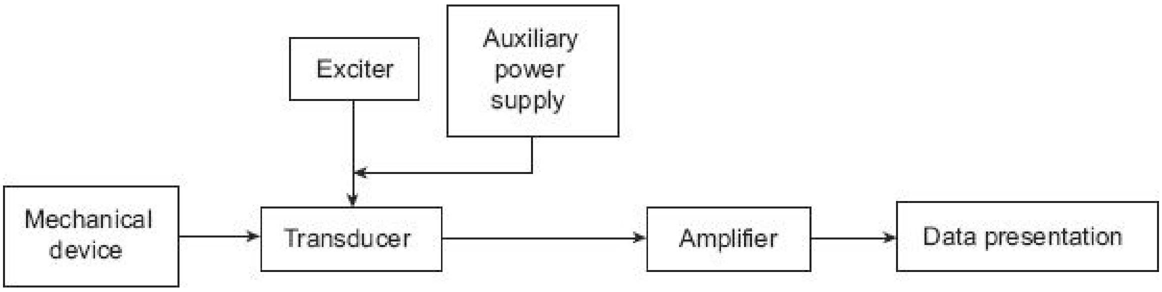


Figure 1.8 *Electromechanical measurement system*

1.6.6 Deflection and Null Output Instruments

In a deflection-type instrument, the deflection of the instrument indicates the measurement of the unknown quantity. The measurand quantity produces some physical effect which deflects or produces a mechanical displacement in the moving system of the instrument. An opposite effect is built in the instrument which opposes the deflection or the mechanical displacement of the moving system. The balance is achieved when opposing effect equals the actuating cause producing the deflection or the mechanical displacement. The deflection or the mechanical displacement at this point gives the value of the unknown input quantity. These type of instruments are suited for measurement under dynamic condition. Permanent Magnet Moving Coil (PMMC), Moving Iron (MI), etc., type instruments are examples of this category.

In null-type instruments, a zero or null indication leads to determination of the magnitude of the measurand quantity. The null condition depends upon some other known conditions. These are more accurate and highly sensitive as compared to deflection-type instruments. A dc potentiometer is a null- type instrument.

1.7 DEFINITIONS OF SOME STATIC CHARACTERISTICS

1. Accuracy

Accuracy is the closeness with which the instrument reading approaches the true value of the variable under measurement. Accuracy is determined as the maximum amount by which the result differs from the true value. It is almost impossible to determine experimentally the true value. The true value is not indicated by any measurement system due to the loading effect, lags and mechanical problems (e.g., wear, hysteresis, noise, etc.).

Accuracy of the measured signal depends upon the following factors:

- Intrinsic accuracy of the instrument itself;
- Accuracy of the observer;
- Variation of the signal to be measured; and
- Whether or not the quantity is being truly impressed upon the instrument.

2. Precision

Precision is a measure of the reproducibility of the measurements, i.e., precision is a measure of the degree to which successive measurements differ from one another. Precision is indicated from the number of significant figures in which it is expressed. Significant figures actually convey the information regarding the magnitude and the measurement precision of a quantity. More significant figures imply greater precision of the measurement.

3. Resolution

If the input is slowly increased from some arbitrary value it will be noticed that the output does not change at all until the increment exceeds a certain value called the resolution or discrimination of the instrument. Thus, the resolution or discrimination of any instrument is the smallest change in the input signal (quantity under measurement) which can be detected by the instrument. It may be expressed as an accrual value or as a fraction or percentage of the full scale value. Resolution is sometimes referred as *sensitivity*. The largest change of input quantity for which there is no output of the instrument is called the *dead zone* of that instrument.

The sensitivity gives the relation between the input signal to an instrument or a part of the instrument system and the output. Thus, the sensitivity is defined as the ratio of output signal or response of the instrument to a change of input signal or the quantity under measurement.

Example 1.1

A moving coil ammeter has a uniform scale with 50 divisions and gives a full-scale reading of 5 A. The instrument can read up to V th of a scale division with a fair degree of certainty. Determine the resolution of the instrument in mA.

Solution Full-scale reading = 5 A

Number of divisions on scale = 50

$$1 \text{ scale division} = \frac{5}{50} \times 1000 = 100 \text{ mA}$$

$$\text{Resolution} = \tfrac{1}{4}\text{th of a scale division} = \frac{100}{4} = 25 \text{ mA}$$

4. Speed of Response

The quickness of an instrument to read the measurand variable is called the speed of response. Alternately, speed of response is defined as the time elapsed between the start of the measurement to the reading taken. This time depends upon the mechanical moving system, friction, etc.

1.8 MEASUREMENT OF ERRORS

In practice, it is impossible to measure the exact value of the measurand. There is always some difference between the measured value and the absolute or true value of the unknown quantity (measurand), which may be very small or may be large. The difference between the true or exact value and the measured value of the unknown quantity is known as the absolute error of the measurement.

If δA be the absolute error of the measurement, A_m and A be the measured and absolute value of the unknown equantity then δA may be expressed as

$$\delta A = A_m - A \qquad (1.2)$$

Sometimes, δA is denoted by ε_0.

The relative error is the ratio of absolute error to the true value of the unknown quantity to be measured,

$$\text{i.e., relative error, } \varepsilon_r = \frac{\delta A}{A} = \frac{\varepsilon_0}{A} = \frac{\text{Absolute error}}{\text{True value}} \qquad (1.3)$$

When the absolute error ε_0 (=δA) is negligible, i.e., when the difference between the true value A and the measured value A_m of the unknown quantity is very small or negligible then the relative error may be expressed as,

$$\varepsilon_r = \frac{\delta A}{A_m} = \frac{\varepsilon_0}{A_m} \qquad (1.4)$$

The relative error is generally expressed as a fraction, i.e., 5 parts in 1000 or in percentage value,

$$\text{i.e., percentage error} = \varepsilon_r \times 100 = \frac{\varepsilon_0}{A_m} \times 100 \qquad (1.5)$$

The measured value of the unknown quantity may be more than or less than the true value of the measurand. So the manufacturers have to specify the deviations from the specified value of a particular quantity in order to enable the purchaser to make proper selection according to his requirements. The limits of these deviations from specified values are defined as limiting or guarantee errors. The magnitude of a given quantity having a specified magnitude A_m and a maximum or a limiting error ±δA must have a magnitude between the limits

$$A_m - \delta A \text{ and } A_m + \delta A$$

or,

$$A = A_m \pm \delta A \qquad (1.6)$$

For example, the measured value of a resistance of 100 Ω has a limiting error of ±0.5 Ω. Then the true value of the resistance is between the limits 100 ± 0.5, i.e., 100.5 and 99.5 Ω.

Example 1.2

A 0-25 A ammeter has a guaranteed accuracy of 1 percent of full scale reading. The current measured by this instrument is 10 A. Determine the limiting error in percentage.

Solution The magnitude of limiting error of the instrument from Eq. (1.1),

$$\delta A = \varepsilon_r \times A = 0.01 \times 25 = 0.25 \text{ A}$$

The magnitude of the current being measured is 10 A. The relative error at this current is

$$\varepsilon_r = \frac{\delta A}{A} = \frac{0.25}{10} = 0.025$$

Therefore, the current being measured is between the limit of

$$A = A_m(1 \pm \varepsilon_r) = 10(1 \pm 0.025) = 10 \pm 0.25 \text{ A}$$

$$\text{The limiting error} = \frac{0.25}{10} \times 100 = 2.5\%$$

Example 1.3

The inductance of an inductor is specified as 20 H ± 5 percent by a manufacturer. Determine the limits of inductance between which it is guaranteed.

Solution

$$\text{Relative error, } \varepsilon_r = \frac{\text{Percentage error}}{100} = \frac{5}{100} = 0.05$$

Limiting value of inductance, $A = A_m \pm \delta A$

$= A_m \pm \varepsilon_r A_m = A_m\,(1 \pm \varepsilon_r)$

$= 20(1 \pm 0.05) = 20 \pm 1$ H

Example 1.4

A 0-250 V voltmeter has a guaranteed accuracy of 2% of full-scale reading. The voltage measured by the voltmeter is 150 volts. Determine the limiting error in percentage.

Solution The magnitude of the limiting error of the instrument,

$\delta A = \varepsilon_r V = 0.02 \times 250 = 5.0$ V

The magnitude of the voltage being measured is 150 V.

The percentage limiting error at this voltage

$$= \frac{5.0}{150} \times 100\% = 3.33\%$$

Example 1.5

The measurand value of a resistance is 10.25 Ω, whereas its value is 10.22 Ω. Determine the absolute error of the measurement.

Solution

Measurand value $A_m = 10.25$ Ω

True value $A_m = 10.22$ Ω

Absolute error, $\delta A = A_m - A = 10.25 - 10.22 = 0.03$ Ω

Example 1.6

The measured value of a capacitor is 205.3 fiF, whereas its true value is 201.4 fiF. Determine the relative error.

Solution

Measured value $A_m = 205.3 \times 10^{-6}$ F

True value, A = 201.4×10^{-6} F

Absolute error, $\varepsilon_0 = A_m - A$

$= 205.3 \times 10^{-6} — 201.4 \times 10^{-6}$

$= 3.9 \times 10^{-6}$ F

$= 3.9 \times 10^{-6}$ F

Relative error, $\varepsilon_r = \frac{\varepsilon_0}{A} = \frac{3.9 \times 10^{-6}}{201.4 \times 10^{-6}} = 0.0194$ or 1.94%

Example 1.7 *A wattmeter reads 25.34 watts. The absolute error in the measurement is –0.11 watt. Determine the true value of power.*

Solution

Measured value $A_m = 25.34$ W

Absolute error $\delta A = -0.11$ W

True value A = Measured value - Absolute error

$= 25.34 - (-0.11)$,

$= 25.45$ W

1.8.1 Types of Errors

The origination of error may be in a variety of ways. They are categorised in three main types.

- Gross error
- Systematic error
- Random error

1. Gross Error

The errors occur because of mistakes in observed readings, or using instruments and in recording and calculating measurement results. These errors usually occur because of human mistakes and these may be of any magnitude and cannot be subjected to mathematical treatment. One common gross error is frequently committed during improper use of the measuring instrument. Any indicating instrument changes conditions to some extent when connected in a complete circuit so that the reading of measurand quantity is altered by the method used. For example, in Figure (1.9)(a) and (b), two possible connections of voltage and current coil of a wattmeter are shown.

In Figure 1.9(a), the connection shown is used when the applied voltage is high and current flowing in the circuit is low, while the connection shown in Figure 1.9(b) is used when the applied voltage is low and current flowing in the circuit is high. If these connections of wattmeter are used in opposite order then an error is liable to enter in wattmeter reading. Another example of this type of error is in the use of a well-calibrated voltmeter for measurement of voltage across a resistance of very high value. The same voltmeter, when connected in a low resistance circuit, may give a more dependable reading because of very high resistance of the voltmeter itself. This shows that the voltmeter has a loading effect on the circuit, which alters the original situation during the measurement.

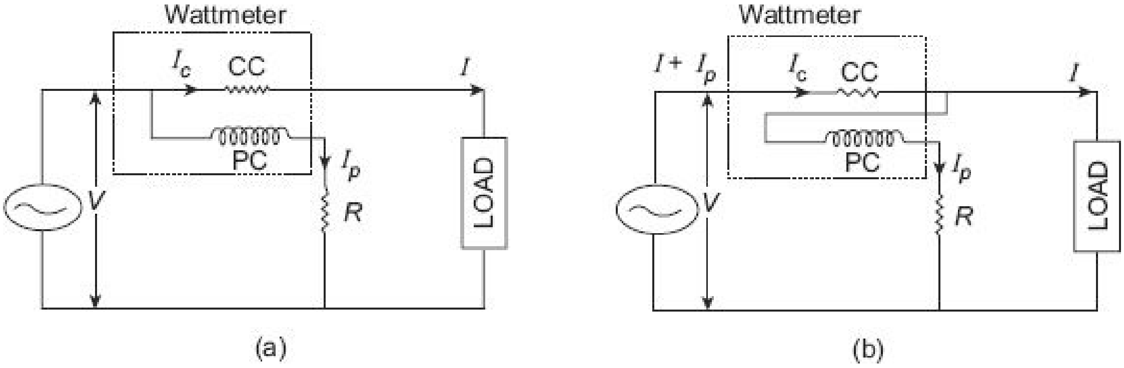
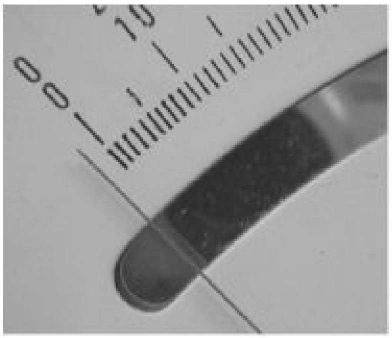
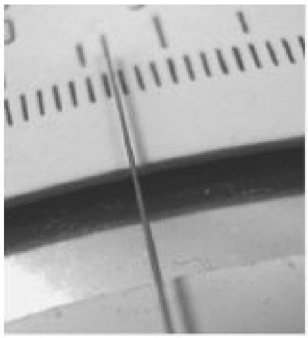


Figure 1.9 *Different connections of wattmeter*

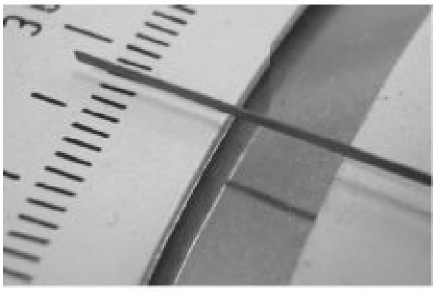
For another example, a multirange instrument has a different scale for each range. During measurements, the operator may use a scale which does not correspond to the setting of the range selector of the instrument. Gross error may also be there because of improper setting of zero before the measurement and this will affect all the readings taken during measurements. The gross error cannot be treated mathematically, so great care should be taken during measurement to avoid this error. Pictorial illustration of different types of gross error is shown in Figure 1.10.



(a) Error will occur in the measurement result if proper zero setting are not there.



(b) Gross error will occur in the measurement result if the pointer deflects between the two scaling points.



(c) Parallax error will occur in the measurement result if the pointer is not set in the vertical position.

Figure 1.10 *Different types of gross errors*

In general, to avoid gross error, at least two, three or more readings of the measurand quantity should be taken by different observers. Then if the readings differ by an unacceptably large amount, the situation can be investigated and the more erroneous readings eliminated.

2. Systematic Error

These are the errors that remain constant or change according to a definite law on repeated measurement of the given quantity. These errors can be evaluated and their influence on the results of measurement can be eliminated by the introduction of proper correction. There are two types of systematic errors:

- Instrumental error
- Environmental error

Instrumental errors are inherent in the measuring instruments because of their mechanical structure and calibration or operation of the apparatus used. For example, in D'Arsonval movement, friction in bearings of various components may cause incorrect readings. Improper zero adjustment has a similar effect. Poor construction, irregular spring tensions, variations in the air gap may also cause instrumental errors. Calibration error may also result in the instrument reading either being too low or too high.

Such instrumental errors may be avoided by

- Selecting a proper measuring device for the particular application
- Calibrating the measuring device or instrument against a standard

- Applying correction factors after determining the magnitude of instrumental errors

Environmental errors are much more troublesome as the errors change with time in an unpredictable manner. These errors are introduced due to using an instrument in different conditions than in which it was assembled and calibrated. Change in temperature is the major cause of such errors as temperature affects the properties of materials in different ways, including dimensions, resistivity, spring effect and many more. Other environmental changes also effect the results given by the instruments such as humidity, altitude, earth's magnetic field, gravity, stray electric and magnetic field, etc. These errors can be eliminated or reduced by taking the following precautions:

- Use the measuring instrument in the same atmospheric conditions in which it was assembled and calibrated.
- If the above precaution is not possible then deviation in local conditions must be determined and suitable compensations are applied in the instrumental reading.
- Automatic compensation, employing sophisticated devices for such deviations, is also possible.

3. Random Errors

These errors are of variable magnitude and sign and do not maintain any known law. The presence of random errors become evident when different results are obtained on repeated measurements of one and the same quantity. The effect of random errors is minimised by measuring the given quantity many times under the same conditions and calculating the arithmetical mean of the results obtained. The mean value can justly be considered as the most probable value of the measured quantity since random errors of equal magnitude but opposite sign are of approximately equal occurrence when making a great number of measurements.

1.9 LOADING EFFECTS

Under ideal conditions, an element used for signal sensing, conditioning, transmission and detection should not change/distort the original signal. The sensing element should not use any energy or take least energy from the process so as not to change the parameter being measured. However, under practical conditions, it has been observed that the introduction of any element in a system results invariably in extraction of the energy from the system, thereby distorting the original signal. This distortion may take the form of attenuation, waveform distortion, phase shift, etc., and consequently, the ideal measurements become impossible.

The incapability of the system to faithfully measure the input signal in undistorted form is called *loading effect.* This results in loading error.

The loading effects, in a measurement system, not only occur in the detector–transducer stage but also occur in signal conditioning and signal presentation stages as well. The loading problem is carried right down to the basic elements themselves. The loading effect may occur on account of both electrical and mechanical elements. These are due to impedances of the various elements connected in a system. The mechanical impedances may be treated similar to electrical impedances.

Sometimes loading effect occurs due to the connection of measuring instruments in an improper way. Suppose a voltmeter is connected with parallel of a very high resistance. Due to the high resistance of the voltmeter itself, the circuit current changes. This is the loading effect of a voltmeter when they are connected in parallel with a very high resistance. Similarly, an ammeter has a very low resistance. So if an ammeter is connected in series with a very low resistance, the total resistance of the circuit changes, and in succession, the circuit current also changes. This is the loading effect of ammeters when they are connected in series with very low resistance.

EXERCISE

Objective-type Questions

1. A null-type instrument as compared to a deflection-type instrument has
   (a) a lower sensitivity
   (b) a faster response
   (c) a higher accuracy
   (d) all of the above
2. In a measurement system, the function of the signal manipulating element is to
   (a) change the magnitude of the input signal while retaining its identity
   (b) change the quantity under measurement to an analogous signal
   (c) to perform non-linear operation like filtering, chopping and clipping and clamping
   (d) to perform linear operation like addition and multiplication
3. The measurement of a quantity
   (a) is an act of comparison of an unknown quantity with a predefined acceptable standard which is accurately known
   (b) is an act of comparison of an unknown quantity with another quantity
   (c) is an act of comparison of an unknown quantity with a known quantity whose accuracy may be known or may not be known
   (d) none of these
4. Purely mechanical instruments cannot be used for dynamic measurements because they have
   (a) large time constant
   (b) higher response time
   (c) high inertia
   (d) all of the above
5. A modifying input to a measurement system can be defined as an input
   (a) which changes the input–output relationship for desired as well as interfering inputs
   (b) which changes the input–output relationship for desired inputs only
   (c) which changes the input–output relationship for interfering inputs only
   (d) none of the above
6. In measurement systems, which of the following static characteristics are desirable?

(a) Sensitivity

(b) Accuracy

(c) Reproducibility

(d) All of the above

7. In measurement systems, which of the following are undesirable static characteristics?

(a) Reproducibility and nonlinearity

(b) Drift, static error and dead zone

(c) Sensitivity and accuracy

(d) Drift, static error, dead zone and nonlinearity

8. In some temperature measurement, the reading is recorded as 25.70°C. The reading has

(a) five significant figures

(b) four significant figures

(c) three significant figures

(d) none of the above

9. In the centre of a zero analog ammeter having a range of -10 A to +10 A, there is a detectable change of the pointer from its zero position on either side of the scale only as the current reaches a value of 1 A (on either side). The ammeter has a

(a) dead zone of 1 A

(b) dead zone of 2 A

(c) resolution of 1 A

(d) sensitivity of 1 A

10. A dc circuit can be represented by an internal voltage source of 50 V with an output resistance of 100 kW. In order to achieve 99% accuracy for voltage measurement across its terminals, the voltage measuring device should have

(a) a resistance of at least 10 W

(b) a resistance of 100 kW

(c) a resistance of at least 10 MW

(d) none of the above

11. In ac circuits, the connection of measuring instruments cause loading errors which may affect

(a) only the phase of the quantity being measured

(b) only the magnitude of the quantity being measured

(c) both the phase and the magnitude of the quantity

(d) magnitude, phase and also the waveform of the quantity being measured

12. A pressure gauge is calibrated form 0–50 kN/m^2. It has a uniform scale with 100 scale divisions. One fifth of the scale division can be read with certainty. The gauge has a

(a) dead zone of 0.2 kN/m^2

(b) resolution of 0.1 kN/m^2

(c) resolution of 0.5 kN/m^2

(d) threshold of 0.1 kN/m^2

13. A pressure measurement instrument is calibrated between 10 bar and 260 bar. The scale span of the instrument is

(a) 10 bar

(b) 260 bar

(c) 250 bar

(d) 270 bar

14. A Wheatstone bridge is balanced with all the four resistances equal to 1 kW each. The bridge supply voltage is 100 V. The value of one of the resistance is changed to 1010 W. The output voltage is measured with a voltage measuring device of infinite resistance. The bridge sensitivity is

(a) 2.5 mV/W

(b) 10 V/W

(c) 25 mV/W

(d) none of the above

15. The main advantage of the null balance technique of measurement is that

(a) it gives a quick measurement

(b) it does not load the medium

(c) it gives a centre zero value at its input

(d) it is not affected by temperature variation

16. The smallest change in a measured variable to which an instrument will respond is

(a) resolution

(b) precision

(c) sensitivity

(d) accuracy

17. The desirable static characteristics of a measurement are

(a) precision

(b) accuracy

(c) sensitivity

(d) all of these

18. The errors mainly caused by human mistakes are

(a) systematic error

(b) instrumental error

(c) observational error

(d) gross error

19. Systematic errors are

(a) environmental error

(b) observational error

(c) instrumental error

(d) all of the above

20. An analog ammeter is

(a) an absolute instrument

(b) an indicating instrument

(c) a controlling instrument

(d) a recording instrument

Answers

1. (c)	2. (a)	3. (a)	4. (d)	5. (a)	6. (d)	7. (d)
8. (b)	9. (b)	10. (c)	11 (d)	12. (b)	13. (c)	14. (c)
15. (b)	16. (a)	17. (d)	18. (d)	19. (d)	20. (b)	

Short-answer Questions

1. Compare the advantages and disadvantages of electrical and mechanical measurement systems.
2. Explain the various classes of measuring instruments with examples.
3. Differentiate clearly between absolute and secondary instruments.
4. Explain analog and digital modes of operation. Why are the digital instruments becoming popular now? What is meant by ADC and DAC?
5. Briefly define and explain all the static characteristics of measuring instruments.
6. Explain *loading effect* in measurement systems.
7. Explain the terms *accuracy, sensitivity* and *resolution* as used for indicating instruments.
8. What are the different types of errors in a measuring instrument? Describe their source briefly.
9. What are fundamental and derived units? Briefly explain them.
10. What are the differences between primary and secondary standards?

Long-answer Questions

1. (a) What is measurement? What is meant by the term measurand? What is a measuring instrument?
   (b) Write down the important precautions that should be taken while carrying out electrical measurements.
   (c) With an example, explain the term *loading effect* in a measurement system.
2. (a) Explain the terms:
   (i) Measurement
   (ii) Accuracy
   (iii) Precision
   (iv) Sensitivity
   (v) Reproducibility
   (b) Define *random errors* and explain how they are analysed statistically.
3. (a) What are environmental, instrumental and observational errors? Briefly explain each of them.
   (b) Three resistors have the following ratings:
   R_1 = 47 W ± 4%, R_2 = 65 W ± 4%, R_3 = 55 W ± 4%
   Determine the magnitude and limiting errors in ohms and in percentage of the resistance of these resistors connected in series.
4. (a) What is the necessity of units in measurements? What are various SI units?
   (b) Define the terms *units, absolute units, fundamental units* and *derived units* with suitable examples.
   (c) What is systematic error and how can we reduce it?
5. (a) Distinguish between international, primary, secondary and working standards.
   (b) What are the primary standards for time and frequency? Briefly discuss each of them.
   (c) Describe the working principle, operation and constructional detail of a primary standard of emf.

2 Analog Meters

2.1 INTRODUCTION

An analog device is one in which the output or display is a continuous function of time and bears a constant relation to its input. Measuring instruments are classified according to both the quantity measured by the instrument and the principle of operation. Three general principles of operation are available: (i) electromagnetic, which utilises the magnetic effects of electric currents; (ii) electrostatic, which utilises the forces between electrically charged conductors; (iii) electro-thermal, which utilises the heating effect.

Electric measuring instruments and meters are used to indicate directly the value of current, voltage, power or energy. In this chapter, we will consider an electromechanical meter (input is as an electrical signal which results in mechanical force or torque as an output) that can be connected with additional suitable components in order to act as an ammeter and a voltmeter. The most common analog instrument or meter is the permanent magnet moving coil instrument and it is used for measuring a dc current or voltage of an electric circuit. On the other hand, the indications of alternating current ammeters and voltmeters must represent the rms values of the current, or voltage, respectively, applied to the instrument.

2.2 CLASSIFICATION OF ANALOG INSTRUMENTS

In a broad sense, analog instruments may be classified into two ways:

1. Absolute instruments
2. Secondary instruments

Absolute instruments give the value of the electrical quantity to be measured in terms of the constants of the instruments and to its deflection, no comparison with another instrument being required. For example, the tangent galvanometer gives the value of the current to be measured in terms of the tangent of the angle of deflection produced by the current, the radius and the number of turns of galvanometer coil, and the horizontal component of the earth's magnetic field. No calibration of the instrument is thus necessary.

Secondary instruments are so constructed that the value of current, voltage or other quantity to be measured can be determined from the deflection of the instruments, only if the latter has been calibrated by comparison with either an absolute instrument or one which has already been calibrated. The deflection obtained is meaningless until such a calibration has been made.

This class of instruments is in most general use, absolute instrument being seldom used except in standard laboratories and similar institutions.

The secondary instruments may be classifies as

1. Indicating instruments
2. Recording instruments
3. Integrating instruments

Indicating instruments are instruments which indicate the magnitude of a quantity being measured. They generally make use of a dial and a pointer for this purpose.

Recording instruments give a continuous record of the quantity being measured over a specified period. The variation of the quantity being measured are recorded by a pen (attached to the moving system of the instrument; the moving system is operated by the quantity being measured) on a sheet of paper that moves perpendicular to the movement of the pen.

Integrating instruments record totalised events over a specified period of time. The summation, which they give, is the product of time and an electrical quantity. Ampere hour and watt hour (energy) meters are examples of this category.

2.3 PRINCIPLE OF OPERATION

Analog instruments may be classified according to the principle of operation they utilise. The effects they utilise are

1. Magnetic effect
2. Heating effect
3. Electrostatic effect
4. Electromagnetic effect
5. Hall effect

The majority of analog instruments including moving coil, moving iron and electrodynamic utilise the magnetic effect. The effect of the heat produced by a current in a conductor is used in thermocouple and hotwire instruments. Electrostatic effect is used in electrostatic voltmeters. The electromagnetic induction effect is used in induction wattmeters and induction energy meters.

2.4 OPERATING TORQUES

Three types of torques are needed for satisfactory operation of any indicating instrument. These are

- Deflecting torque
- Controlling torque
- Damping torque

2.4.1 Deflecting Torque/Force

Any instrument's deflection is found by the total effect of the deflecting torque/force, control torque/ force and damping torque/force. The deflecting torque's value is dependent upon the electrical signal to be measured; this torque/force helps in rotating the instrument movement from its zero position. The system producing the deflecting torque is called the *deflecting system*.

2.4.2 Controlling Torque/Force

The act of this torque/force is opposite to the deflecting torque/force. When the deflecting and controlling torques are equal in magnitude then the movement will be in definite position or in equilibrium. Spiral springs or gravity is usually given to produce the controlling torque. The system which produces the controlling torque is called the *controlling system*. The functions of the controlling system are

- To produce a torque equal and opposite to the deflecting torque at the final steady position of the pointer in order to make the deflection of the pointer definite for a particular magnitude of current
- To bring the moving system back to its zero position when the force causing the instrument moving system to deflect is removed

The controlling torque in indicating instruments is almost always obtained by a spring, much less commonly, by gravity.

2.4.3 Damping Torque/Force

A damping force generally works in an opposite direction to the movement of the moving system. This opposite movement of the damping force, without any oscillation or very small oscillation brings the moving system to rest at the final deflected position quickly. Air friction, fluid friction and eddy currents provide the damping torque/force to act. It must also be noted that not all damping force affects the steady-state deflection caused by a given deflecting force or torque. With the angular velocity of the moving system, the intensity of the damping force rises; therefore, its effect is greatest when it rotates rapidly and zero when the system rotation is zero. In the description of various types of instruments, detailed mathematical expressions for the damping torques are taken into consideration.

When the deflecting torque is much greater than the controlling torque, the system is called underdamped. If the deflecting torque is equal to the controlling torque, it is called *critically damped*. When deflecting torque is much less than the controlling torque, the system is under overdamped condition. Figure 2.1 shows the variation of deflection (d) with time for underdamped, critically damped and overdamped systems.

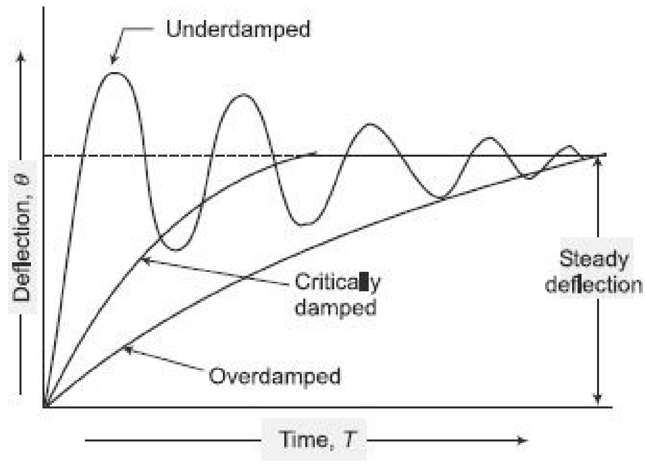


Figure 2.1 *Damping torque curve*

2.5 CONSTRUCTIONAL DETAILS

2.5.1 Moving System

The moving system should have the following properties:

- The moving parts should be light.
- The frictional force should be minimum.

These requirements should be fulfilled in order that power required by the instrument for its operation is small. The power is proportional to the weight of the moving parts and the frictional forces opposing the movement. The moving system can be made light by using aluminium as far as possible. The frictional forces are reduced by using spindle-mounted jewel bearings and by carefully balancing the system.

The force or torque developed by the moving element of an electrical instrument is necessarily small in order that the power consumption of the instrument is kept low so that the introduction of the instrument into a circuit may cause minimum change in the existing circuit conditions. Because of low power levels, the considerations of various methods of supporting the moving elements becomes of vital importance. With the operating force being small, the frictional force must be kept to a minimum in order that the instrument reads correctly and is not erratic in action and is reliable. Supports may be of the following types:

- Suspension
- Taut suspension
- Pivot and jewel bearings

1. *Suspension*

It consist of a fine, ribbon-shaped metal filament for the upper suspension and a coil of fine wire for the lower part. The ribbon is made of a spring material like beryllium copper or μHosphor bronze. This coiling of lower part of suspension is done in order to give negligible restrain on the moving system. The type of suspension requires careful leveling of the instrument, so that the moving system hangs in correct vertical position. This construction is, therefore, not suited to field use and is employed only in those laboratory applications in which very great sensitivity is required. In order to prevent shocks to the suspension during transit, etc., a clamping arrangement is employed for supporting the moving system.

2. *Taut Suspension*

A suspension type of instrument can only be used in vertical position. The taut suspension has a flat ribbon suspension both above and below the moving element, with suspension kept under tension by a spring arrangement (Figure 2.2). The advantage of this type of suspension is that exact levelling is not required if the moving system is properly balanced.

Suspension and taut suspension are customarily used in instruments of galvanometer class which requires a low friction and high sensitivity mechanism. But actually there is no strict line of demarcation between a galvanometer and other indicating instruments. Some sensitive wattmeters and electrostatic voltmeters use flexible suspension.

Ribbon suspension, in addition to supporting the moving element, exerts a controlling torque when twisted. Thus, the use of suspension results in elimination of pivots, jewels, control springs and therefore, pivotless instruments are free from many defects.

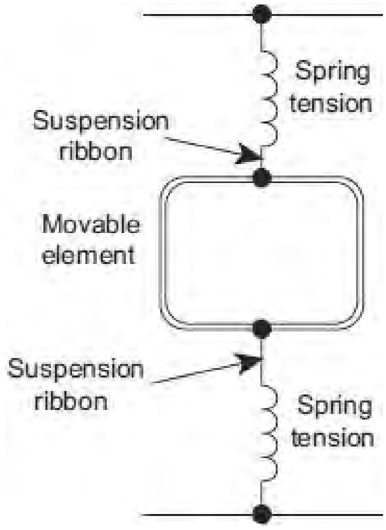


Figure 2.2 *Taut suspension*

3. *Pivot and Jewel Bearings*

The moving system is mounted on a spindle made of hardened steel. The two ends of the spindle are made conical and then polished to form pivots. These ends fit conical holes in jewels located in the fixed part of instruments (Figure 2.3). These jewels, which are preferably made of sapphire, form bearings.

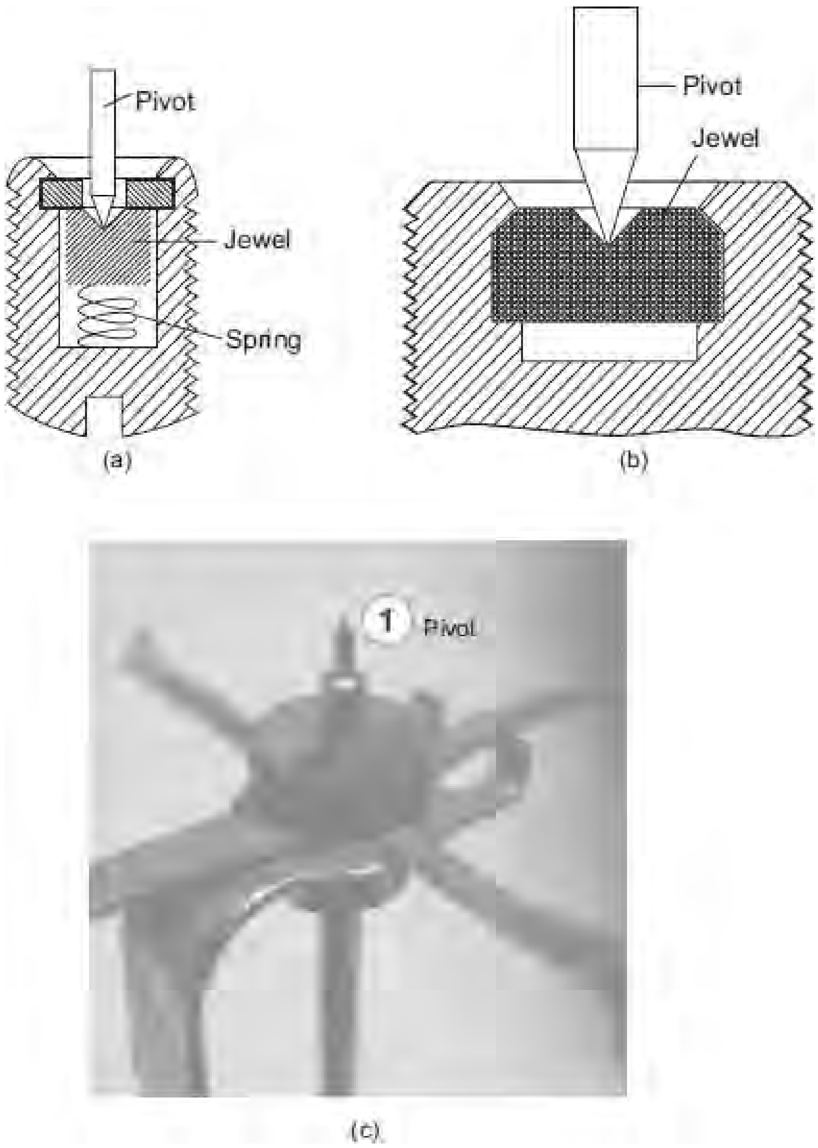


Figure 2.3 (*a) Spring-loaded jewel bearing (b) Jewel bearing (c) Pivot*

It has been found that the frictional torque, for jewel bearings, is proportional to the area of contact between the pivot and jewel. Thus, the contact area between a pivot and jewel should be small. The pivot is ground to a cone and its tip is rounded to a hemispherical surface of small area. The jewel is ground to a cone of somewhat larger angle.

4. *Torque/Weight Ratio*

The frictional torque in an instrument depends upon the weight of moving parts. If the weight of the moving parts is large, the frictional torque will be large. The frictional torque exerts a considerable influence on the performance of an indicating instrument. If the frictional torque is large and is comparable to a considerable fraction of the deflecting torque, the deflection of the moving system will depend upon the frictional torque to an appreciable extent. Also, the deflection will depend on the direction from which the equilibrium position is approached and will be uncertain. On the other hand, if the frictional torque is very small compared with the deflecting torque, its effect on deflection is negligible. Thus, the ratio of deflecting torque to frictional torque is a measure of reliability of the instrument indications and is the inherent quality of the design. Hence (deflecting) torque/weight ratio of an instrument is an index of its performance. The higher the ratio, the better will be its performance.

2.5.2 Controlling System

The controlling torque is provided by a spring or sometimes by gravity.

1. *Spring Control*

A hair-spring, usually of phosphor-bronze attached to the moving system, is used in indicating instruments for control purpose, the schematic arrangement being shown in Figure 2.4(a) and the actual controlling spring used in the instrument is shown in Figure 2.4(b).

To give a controlling torque which is directly proportional to the angle of deflection of the moving system, the number of turns on the spring should be fairly large, so that the deflection per unit length is small. The stress in the spring must be limited to such a value that there is no permanent set.

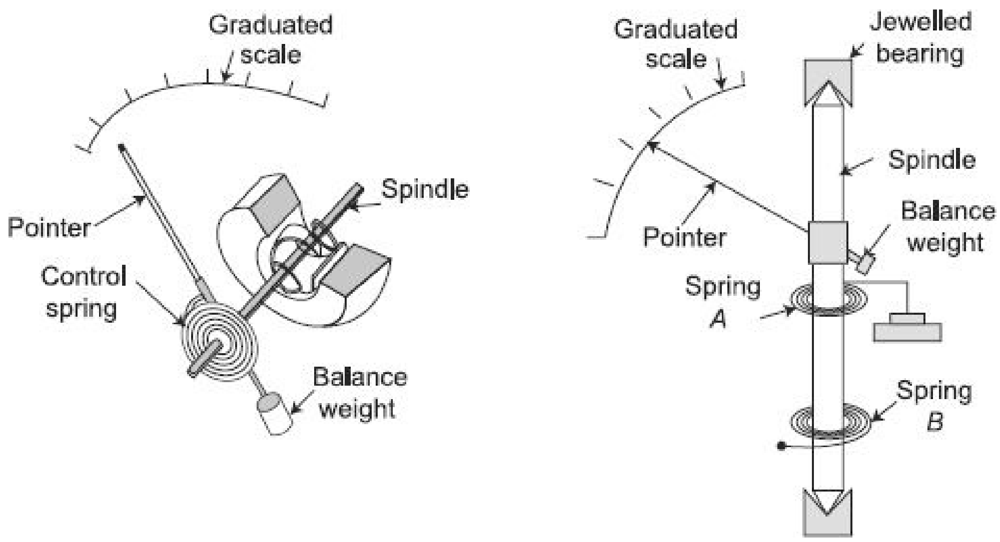


Figure 2.4(a) *Spring control*

Figure 2.4(b) *Spring control in an instrument*

Suppose that a spiral spring is made up of a total length L m of strip whose cross-section is rectangular, the radial thickness being t m and the depth b m. Let E be Young's modulus (N/m^2) for the material of the spring. Then, if θ radians be the deflection of the moving system to which one end of the spring is being attached, the expression for the controlling torque is

$$T_c = \frac{Ebt^3}{12L}\theta \quad (2.1)$$

Thus, controlling torque ∞ θ ∞ instrument deflection.

2. *Gravity Control*

In a gravity-controlled instrument, a small weight is attached to the moving system in such a way that it produces a restoring or controlling torque when the system is deflected. This is illustrated in Figure 2.5. The controlling torque, when the deflection is θ, is $\omega l \sin \theta$, where W is the control weight and l its distance from the axis of rotation of the moving system, and it is, therefore, proportional only to the *sine* of the angle of deflection, instead of, as with spring control, being directly proportional to the angle of deflection. Gravity-controlled instruments must obviously be used in a vertical position in order that the control may operate.

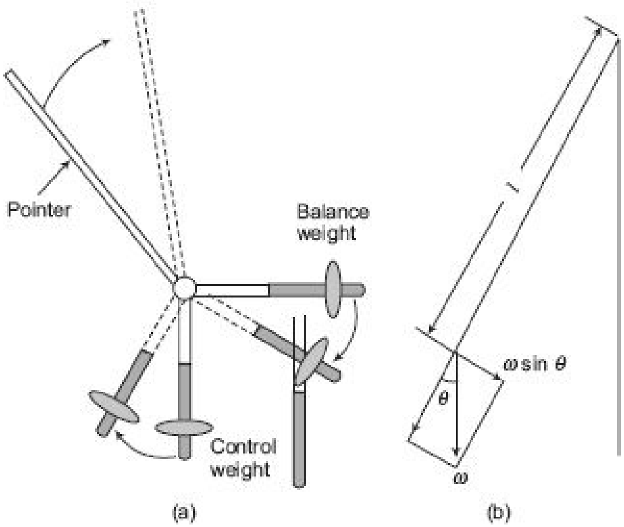


Figure 2.5 *Gravity control*

3. *Comparison of Spring and Gravity Control*

Gravity control has the following advantages when compared with spring control:

- It is cheaper
- Independent of temperature
- Does not deteriorate with time

Consider an instrument in which the deflecting torque T_D is directly proportional to the current (say) to be measured.

Thus, if I is the current,

$$T_D = kI, \text{ (where } k \text{ is a constant)} \quad (2.2)$$

If the instrument is spring-controlled, the controlling torque being T_C, when the deflection is θ,

$T_C = k_s\theta$ (k_s is spring constant)

Also, $T_C = T_D$

or $k_s\theta = kI$

$$\text{or} \qquad k_s\theta = kI$$

$$\therefore \qquad \theta = \frac{k}{k_s}\cdot I \tag{2.3}$$

Thus, the deflection is proportional to the current throughout the scale.

Now if the same instrument is gravity controlled,

$T_C = k_g \sin\theta$ (k_g is a constant that depends upon the control weight and its distance from the axis of rotation of the moving system).

And $T_C = T_D = kI$

$$\therefore \qquad kg \sin\theta = kI$$

$$\sin\theta = \frac{k}{k_g}\cdot I$$

$$\theta = \sin^{-1}\left(\frac{k}{k_g}\cdot I\right) \tag{2.4}$$

Thus, a gravity-controlled instrument would have a scale which is 'cramped' at its lower end instead of being uniformly divided, though the deflecting torque is directly proportional to the quantity to be measured.

2.5.3 Damping System

There are three systems of damping generally used. These are as follows:

- Air-friction damping
- Fluid-friction damping
- Eddy-current damping

1. *Air-Friction Damping*

In this method, a light aluminium piston is attached to the moving system and moves in an air chamber closed at one end, as shown in Figure 2.6. The cross-section of this chamber may be either circular or rectangular. The clearance between the piston and the sides of the chamber should be small and uniform. If the piston is moving rapidly into the chamber, the air in the closed space is compressed and the pressure opposes the motion of the piston (and, therefore, of the whole moving system). If the piston is moving out of the chamber rapidly, the pressure in the closed space falls, and the pressure on the open side of the piston is greater than that on the opposite side. Motion is thus again opposed. Sometimes instead of a piston, a vane, mounted on the spindle of the moving system, moves in a closed-sector-shaped box as shown in Figure 2.7.

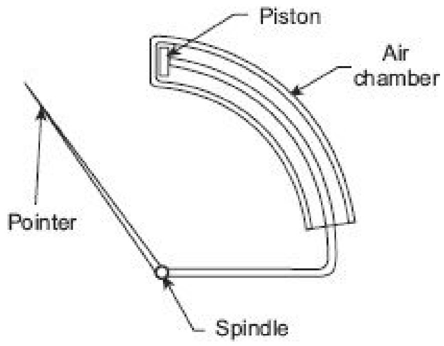


Figure 2.6 *Open-end air friction damping*

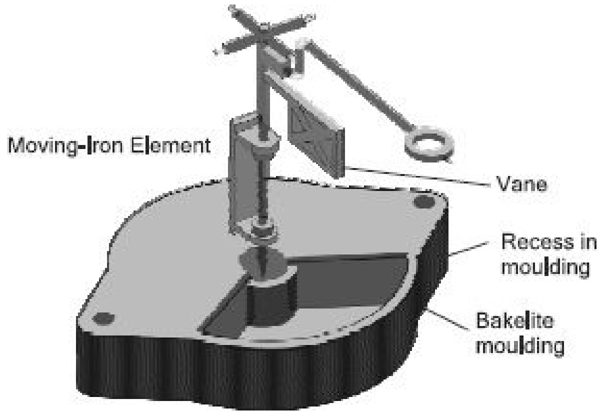


Figure 2.7 *Air-friction damping using vane*

2. *Fluid-Friction Damping*

In this type of damping, a light vane, attached to the spindle of the moving system, dips into a pot of damping oil and should be completely submerged by the oil. This is illustrated in Figure 2.8(a). The frictional drag in the disc is always in the direction opposing motion. There is no friction force when the disc is stationary. In the second system [Figure 2.8(b)], increased damping is obtained by the use of vanes.

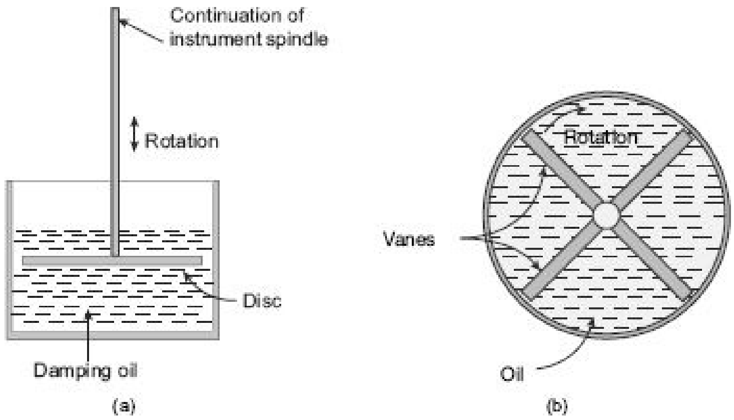


Figure 2.8 *Fluid-friction damping*

3. *Eddy-Current Damping*

When a sheet of conducting material moves in a magnetic field so as to cut through lines of force, eddy currents are set up in it and a force exists between these currents and the magnetic field, which is always in the direction opposing the motion. The force is proportional to the magnitude of the current and to the strength of the field. The magnitude of the current is proportional to the velocity of movement of the conductor, and thus, if the magnetic field is constant, the damping force is proportional to the velocity of the moving system and is zero when there is no movement of the system.

(i) Eddy-Current Damping Torque of Metal Former Figure 2.9 shows a metallic former moving in the field of a permanent magnet.

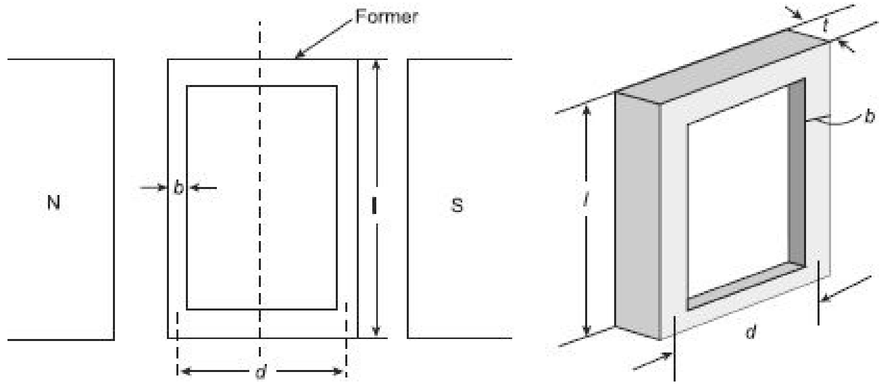


Figure 2.9 *Eddy-current damping on a metal former*

Let,

B = strength of magnetic field; (wb/m^2)

ω = asngular speed of former; (rad/s)

l = length of former; (m)

t = thickness of former; (m)

b = width of former; (m)

d = breadth of former; (m)

ρ = resistivity of material of former; (W m)

Linear velocity of former $v = \left(\frac{d}{2}\right)\omega$ (2.5)

[since linear velocity = radius × angular velocity]

Dynamically generated emf in the former

$$E_e = 2Blv = 2Bl\frac{d}{2}\omega = Bld\omega \quad (2.6)$$

Resistance of path of eddy current $R_e = \frac{\rho 2(d+l)}{bt}$ (2.7)

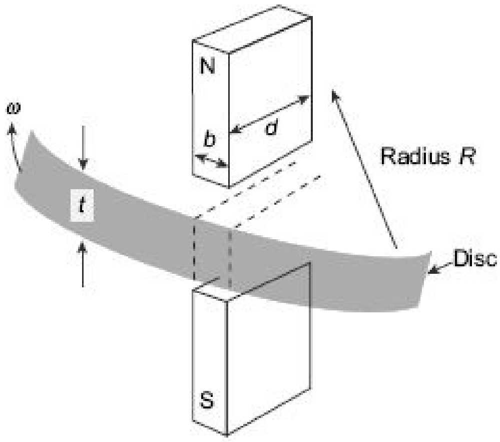
Eddy current $I_e = \frac{E_e}{R_e} = \frac{Blbtd\omega}{2\rho(d+l)}$ (2.8)

$\therefore$ damping force $F_d = B I_e l = \frac{B^2 l^2 btd\omega}{2\rho(d+l)}$ (2.9)

damping torque $T_d = F_d \times d = \frac{B^2 l^2 btd^2\omega}{2\rho(d+l)}$ (2.10)

damping constant $k_d = \frac{T_d}{\omega} = \frac{B^2 l^2 btd^2}{2\rho(d+l)}$ Nm/rad s^{-1} (2.11)

(ii) Eddy-Current Damping Torque of Metal Disc Figure 2.10 shows a metallic disc rotating in the field of a permanent magnet.



1. Disc, 2. Magnet

Figure 2.10 *Eddy-current damping on metallic disc* 2

Let, B = flux density of magnetic field; (wb/m^2)

ω = angular speed of disc; (rad/s)

t = thickness of disc; (m)

b = width of permanent magnet; (m)

d = length of permanent magnet; (m)

ρ = resistivity of material of disc; (Ω m)

R = radius measured from centre of pole to centre of disc; (m)

Considering the emf is induced in the disc under the pole face only, therefore, emf induces in the portion below the magnet

$$E_c = Blv = BdR\omega \tag{2.12}$$

[since 'l', length of the portion of the disc under the magnetic field = d]

$$\text{Resistance of eddy-current path under the pole} = \frac{\rho d}{bt} \tag{2.13}$$

Actual path for eddy current is not limited to the portion of the disc under the magnet but is greater than this. Therefore, to take this factor into account, the actual resistance is taken as k times of $\frac{\rho d}{bt}$.

$$\text{Therefore, resistance of eddy-current path } R_c = k\frac{\rho d}{bt} \tag{2.14}$$

where k is a constant which depends upon radial position of the disc and poles.

$$\text{Eddy current} \quad I_e = \frac{E_c}{R_c} = \frac{BRbt\omega}{k\rho} \tag{2.15}$$

$$\text{Damping force} \quad F_D = B \times I_e \times d = \frac{B^2 Rdbt\omega}{k\rho} (N) \tag{2.16}$$

$$\text{Damping torque} \quad T_D = F_D \times R = \frac{B^2 R^2 dbt\omega}{k\rho} (\text{N-m}) \tag{2.17}$$

$$\text{Damping constant} \quad K_D = \frac{T_D}{\omega} = \frac{B^2 R^2 dbt}{k\rho} (\text{N-m/rad s}^{-1}) \tag{2.18}$$

2.6 PERMANENT MAGNET MOVING COIL INSTRUMENT

Basic range: 10 µA-100 mA

Coil resistance: 10 Ω-1 kΩ

Usage:

- dc PMMC ammeters and voltmeters
- ac PMMC ammeters and voltmeters (with rectifiers)

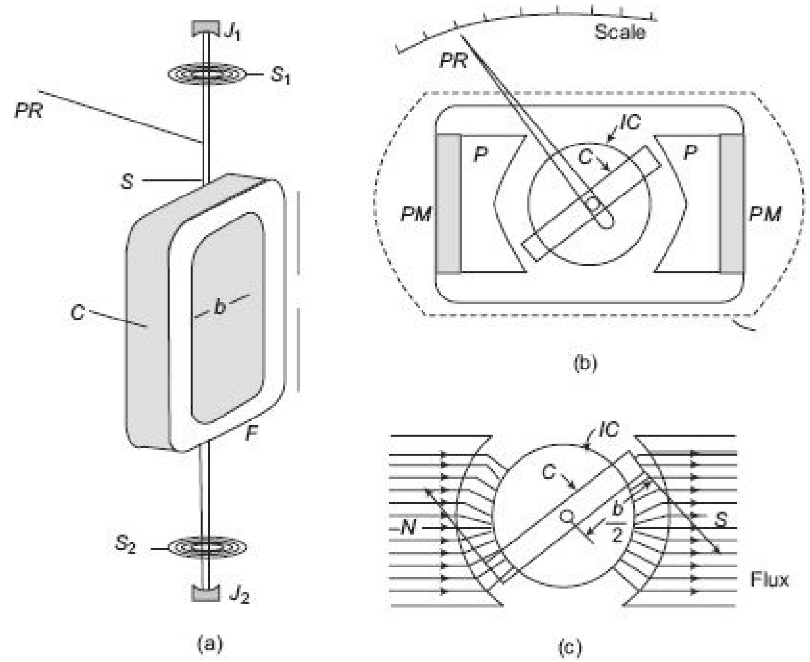
2.6.1 Principle of Operation

The principle on which a Permanent Magnet Moving Coil (PMMC) instrument operates is that a torque is exerted on a current-carrying coil placed in the field of a permanent magnet. A PMMC instrument is shown in Figure 2.11. The coil C has a number of turns of thin insulated wires wound on a rectangular aluminium former *F*. The frame is carried on a spindle S mounted in jewel bearings J_1, J_2. A pointer PR is attached to the spindle so that it moves over a calibrated scale. The whole of the moving system is made as light in weight as possible to keep the friction at the bearing to a minimum.

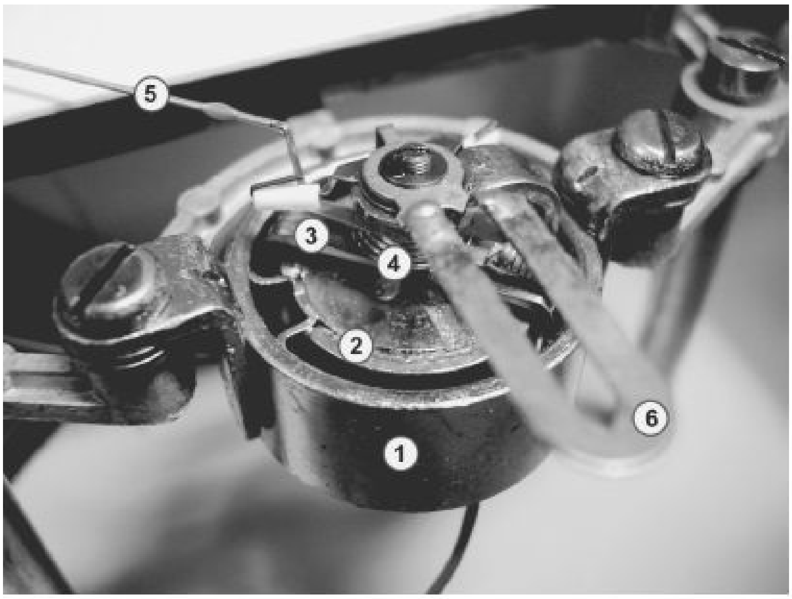
The coil is free to rotate in air gaps formed between the shaped soft-iron pole piece (pp) of a permanent magnet PM and a fixed soft-iron cylindrical core IC [Figure 2.11(b)]. The core serves two purposes; (a) it intensifies the magnetic field by reducing the length of the air gap, and (b) it makes the field radial and uniform in the air gap.

Thus, the coil always moves at right angles to the magnetic field [Figure 2.11(c)]. Modern permanent magnets are made of steel alloys which are difficult to machine. Soft-iron pole pieces (pp) are attached to the permanent magnet PM for easy machining in order to adjust the length of the air gap. Figure 2.11(d) shows the internal parts and Figure 2.11(e) shows schematic of internal parts of a moving-coil instrument.

A soft-iron yoke (*Y*) is used to complete the flux path and to provide shielding from stray external fields.



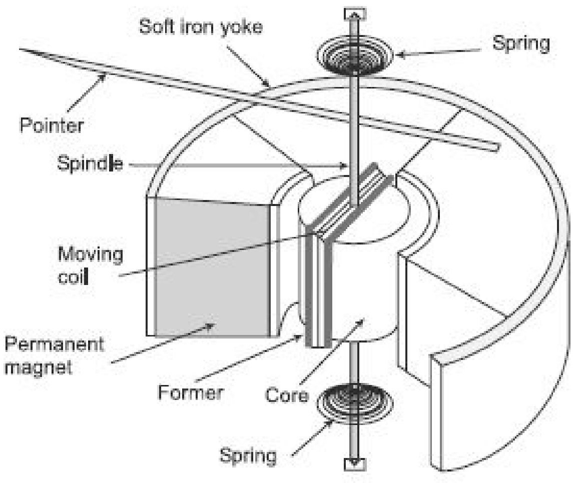
Permanent magnet moving coil instrument



1. External shield
2. Permanent magnet
3. Moving coil
4. Control spring
5. Pointer
6. Arrangement for zero balance of the pointer

(d)

Photograph of different components of a PMMC instrument



(e)
Internal construction of PMMC instruments

Internal construction of PMMC instruments **Figure 2.11**

2.6.2 Deflecting Torque Equation of PMMC Instrument

Let, B = flux density in the air gap (wb/m^2)

i = current in the coil (A)

l = effective axial length of the coil (m)

b = breadth of the coil (m)

n = number of turns of the coil.

Force on one side of the coil is

$$F = Biln \text{ (N)} \quad (2.19)$$

Torque on each side of the coil,

$$T = \text{force} \times \text{distance from axis of rotation}$$
$$= F \times b/2$$
$$= Biln \times b/2 \quad (2.20)$$

Total deflecting torque exerted on the coil,

$$T_d = 2 \times T = 2iln \times b/2$$
$$= Bilnb \text{ (N-m)} \quad (2.21)$$

For a permanent magnet, B is constant. Also, for a given coil l, b and n are constants and thus the product (*Blnb)* is also a constant, say k_1.

$$\text{Therefore,} \quad T_d = k_1 \times i \quad (2.22)$$

1. Control Torque The control on the movement of the pointer over the scale is provided by two spirally wound, phosphor-bronze springs S_1 and S_2, one at each end of the spindle S. Sometimes these springs also conduct the current into and out of the coil. The control torque of the springs is proportional to the angle θ turned through by the coil.

$$T_c = k_s \times \theta \quad (2.23)$$

where T_C is the control torque and k_s is the spring constant.

At final steady state position, Control torque = Deflecting torque

$$\therefore \quad T_c = T_d$$
$$k_s\theta = k_1 i$$
$$\text{or} \quad \boxed{\theta = \frac{k_1}{k_s} i = ki} \quad (2.24)$$
$$\text{where } k = \frac{k_1}{k_s} = \text{constant}$$

So, angular deflection of the pointer is directly proportional to the current. Thus the scale of the instrument is linear or uniformly divided.

2. *Damping Torque* When the aluminium former (F) moves with the coil in the field of the permanent magnet, a voltage is induced, causing eddy current to flow in it. These current exerts a force on the former. By Lenz's law, this force opposes the motion producing it. Thus, a damping torque is obtained. Such a damping is called eddy-current damping.

2.6.3 Swamping Resistor

The coil of the instrument is made of copper. Its resistance varies with temperature. A resistor of low temperature coefficients, called the swamping resistor, is connected in series with the coil. Its resistance practically remains constant with temperature. Hence the effect of temperature on coil resistance is swamped by this resistor.

Advantages of PMMC Instruments

1. Sensitive to small current
2. Very accurate and reliable
3. Uniform scale up to 270° or more
4. Very effective built in damping
5. Low power consumption, varies from 25 µW to 200 µW
6. Free from hysteresis and not effected by external fields because its permanent magnet shields the coil from external magnetic fields
7. Easily adopted as a multirange instrument

Disadvantages of PMMC Instruments

1. This type of instrument can be operated in direct current only. In alternating current, the instrument does not operate because in the positive half, the pointer experiences a force in one direction and in the negative half the pointer experiences the force in the opposite direction. Due to the inertia of the pointer, it retains it's zero position.
2. The moving system is very delicate and can easily be damaged by rough handling.
3. The coil being very fine, cannot withstand prolonged overloading.
4. It is costlier.
5. The ageing of the instrument (permanent magnet and control spring) may introduce some errors.

Example 2.1

The coil of a PMMC instrument has 60 turns, on a former that is 18 mm wide, the effective length of the conductor being 25 mm. It moves in a uniform field of flux density 0.5 Tesla. The control spring constant is 1.5×10^{-6} *Nm/degree. Calculate the current required to produce a deflection of 100 degree.*

Solution Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= 0.5 \times i \, x \, 25 \times 10^{-3} \times 60 \times 18 \times 10^{-3}$$

The control torque of the springs is

$T_C = k_s \times \theta$

$= 1.5 \times 10^{-6} \times 100$

At equilibrium, $T_d = T_C$

$= 0.5 \times i \times 18 \times 10^{-3} \times 25 \times 10^{-3} \times 60 = 1.5 \times 10^{-6} \times 100$ $1.5 \times 10^{-6} \times 100$

$$i = \frac{1.5 \times 10^{-6} \times 100}{0.5 \times 18 \times 10^{-3} \times 25 \times 10^{-3} \times 60} = 11.11 \text{ mA}$$

Example 2.2

A PMMC instrument has a coil of dimensions 15 mm × 12 mm. The flux density in the air gap is 1.8 × 10^{-3} wb/m^2 and the spring constant is 0.14 × 10^{-6} N-m/rad. Determine the number of turns required to produce an angular deflection of 90° when a current of 5 mA is flowing through the coil.

Solution Total deflecting torque exerted on the coil,

$T_d = Bilnb$ (N-m)

$= 1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3} \times n$

The control torque of the springs is

$T_C = k_s \times 6$

$= 0.14 \times 10^{-6} \times 90 \times \pi/180$

At equilibrium, $T_d = T_C$

$1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3} \times n = 0.14 \times 10^{-6} \times 90 \times \pi/180$

$$n = \frac{0.14 \times 10^{-6} \times 90 \times \pi/180}{1.8 \times 10^{-3} \times 5 \times 10^{-3} \times 15 \times 10^{-3} \times 12 \times 10^{-3}} = 136$$

Example 2.3

A PMMC voltmeter with a resistance of 20 Ω gives a full-scale deflection of 120° when a potential difference of 100 mV is applied across it. The moving coil has dimensions of 30 mm × 25 mm and is wound with 100 turns. The control spring constant is 0.375 × 10^{-6} N-m/degree. Find the flux density in the air gap. Find also the dimension of copper wire of coil winding if 30% of the instrument resistance is due to coil winding. The specific resistance of copper is 1.7 × 10^{-8} Ωm.

Solution Full-scale deflecting current

$$i = \frac{100}{20} \times 10^{-3} = 5 \times 10^{-3} \text{ A}$$

Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100$$

The control torque of the springs is

$$T_C = k_s \times \theta$$

$$= 0.375 \times 10^{-6} \times 120$$

At equilibrium, $T_d = T_C$

$$B \times 5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100 = 0.375 \times 10^{-6} \times 120$$

$$B = \frac{0.375 \times 10^{-6} \times 120}{5 \times 10^{-3} \times 30 \times 10^{-3} \times 25 \times 10^{-3} \times 100} = 0.12 \text{ wb/m}^2$$

Coil winding resistance = 20 × 0.3 = 6 Ω

If the copper wire has a cross-sectional area of a m then

$$n\rho \frac{l}{a} = R$$ [where n be the number of turns, ρ is the resistivity of the copper wire, l is the length of the wire and a is the cross-sectional area]

$$100 \times 1.7 \times 10^{-8} \times \frac{2 \times (30 + 25) \times 10^{-3}}{a} = 6$$

$$a = 100 \times 1.7 \times 10^{-8} \times \frac{2 \times (30 + 25) \times 10^{-3}}{6} = 31.16 \times 10^{-3} \text{ mm}^2$$

If d be the diameter of the copper wire then

$$d = \sqrt{\frac{4 \times 31.16 \times 10^{-3}}{\pi}} = 0.199 \text{ mm}$$

Example 2.4

The coil of a moving-coil voltmeter is 40 mm long and 30 mm wide and has 100 turns on it. The control spring exerts a torque of 240×10^{-6} N-m when the deflection is 100 divisions on full scale. If the flux density of the magnetic field in the air gap is 1 wb/m^2, estimate the resistance that must be put in series with the coil to give one volt per division. The resistance of the voltmeter coil may be neglected.

Solution Let the full scale deflecting current be I amp.

Total deflecting torque exerted on the coil,

$$T_d = Bilnb \text{ (N-m)}$$

$$= 1 \times I \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100$$

The control torque of the springs is

$$T_C = k_s \times \theta$$

$$= 240 \times 10^{-6}$$

At equilibrium, $T_d = T_C$

$$1 \times I \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100 = 240 \times 10^{-6}$$

$$I = \frac{240 \times 10^{-6}}{1 \times 40 \times 10^{-3} \times 30 \times 10^{-3} \times 100} = 0.002 \text{ A}$$

If R be the series resistance,

$$R = \frac{V}{I} = \frac{100 \times 1}{0.002} = 50 \times 10^3 \, \Omega$$

2.7 EXTENSION OF RANGE OF PMMC INSTRUMENTS

2.7.1 Ammeter Shunts

The moving-coil instrument has a coil wound with very fine wire. It can carry only few mA safely to give full-scale deflection. For measuring higher current, a low resistance is connected in parallel to the instrument to bypass the major part of the current. The low resistance connected in parallel with the coil is called a *shunt*. Figure 2.12 shows a shunt resistance R_{sh} connected in parallel with the basic meter.

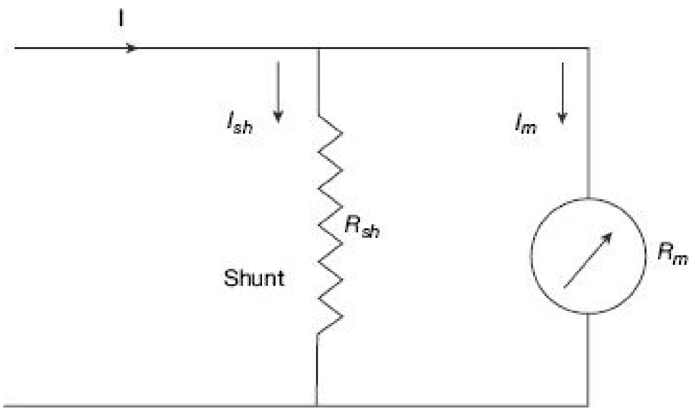


Figure 2.12 Extension of PMMC ammeter using shunt

The resistance of the shunt can be calculated using conventional circuit analysis.

R_{sh} = shunt resistance (Ω)

R_m = coil resistance (Ω)

$I_m = Ifs$ = full-scale deflection current (A)

1_{sh} = shunt current (A)

I = current to be measured (A)

The voltage drop across the shunt and the meter must be same as they are connected in parallel.

$$\therefore \quad I_{sh}R_{sh} = I_m R_m$$

$$\text{Again} \quad I = I_{sh} + I_m \tag{2.25}$$

$$\therefore \quad I_{sh} = I - I_m$$

From Eq. (2.25),

$$R_{sh} = \frac{I_m}{I_{sh}} R_m$$

$$\therefore \quad R_{sh} = \frac{I_m}{I - I_m} R_m \tag{2.26}$$

The ratio of the total current to the current in the meter is called *multiplying power of shunt*. Multiplying power,

$$m = \frac{I}{I_m} = 1 + \frac{R_m}{R_{sh}}$$

$$\therefore \quad R_{sh} = \frac{R_m}{m - 1}$$

2.7.2 Voltmeter Multipliers

For measuring higher voltages, a high resistance is connected in series with the instrument to limit the current in the coil to a safe value. This value of current should never exceed the current required to produce the full scale deflection. The high resistance connected in series with the instrument is called a *multiplier*. In Figure 2.13, R_{sc} is the multiplier.

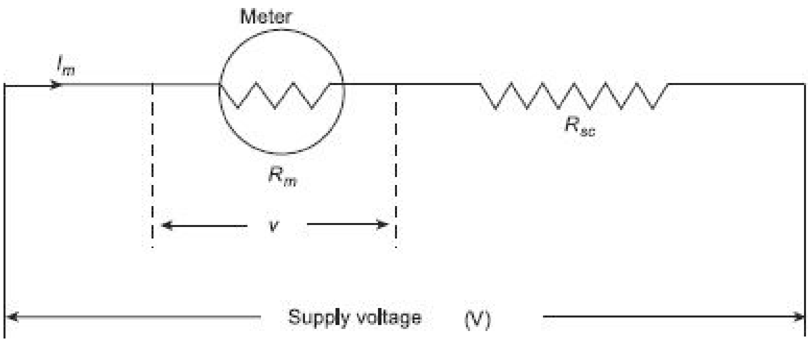


Figure 2.13 *Extension of PMMC voltmeter using multiplier*

The value of multiplier required to extend the voltage range, is calculated as under:

R_{sc} = multiplier resistance (Ω)

R_m = meter resistance (Ω)

I_m = Ifs = full scale deflection current (A)

v = voltage across the meter for producing current I_m (A)

V = voltage to be measured (A)

$$V = I_m R_m$$

$$V = I_m(R_m + R_{sc})$$

$$\therefore \quad R_{sc} = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

Now multiplying factor for multiplier

$$m = \frac{V}{v} = \frac{I_m(R_m + R_{sc})}{I_m R_m} = 1 + \frac{R_{sc}}{R_m}$$

$$\therefore \quad R_{sc} = (m-1)R_m$$

Sensitivity The moving-coil instrument is a very sensitive instrument. It is, therefore, widely used for measuring current and voltage.The coil of the instrument may require a small amount of current (in the range of μA) for full-scale deflection. The sensitivity is sometimes expressed in *ohm/volt.* The sensitivity of a voltmeter is given by

$$S = \frac{\text{Total voltmeter resistance in ohm}}{\text{Full scale reading in volts}}\,\Omega/v = \frac{R_m}{v} = \frac{1}{I_{fs}}\,\Omega/v$$

where I_{fs} is the full-scale deflecting current. Thus, the sensitivity depends upon on the current to give full-scale deflection.

Example 2.5

A moving-coil voltmeter has a resistance of 100 Ω. The scale is divided into 150 equal divisions. When a potential difference of 1 V is applied to the terminals of the voltmeter a deflection of 100 divisions is obtained. Explain how the instrument could be used for measuring up to 300 V.

Solution Let R_{sc}be the multiplier resistance that would be connected in series with the voltmeter.

Volt/division = 1/100

Voltage across the meter for producing the full-scale deflecting current v = 150 × 1/100 = 1.5 V

Full scale meter current I_m = 1.5/100 amp

Meter resistance R_m = 100 Ω

$$\therefore \quad R_{sc} = \frac{V - I_m R_m}{I_m} = \frac{V}{I_m} - R_m$$

$$= \frac{300 - 1.5/100 \times 100}{1.5/100} = 19.9\ \text{k}\Omega$$

Example 2.6

A moving coil instrument has a resistance of 5 Ω and gives a full scale deflection of 10 mv. Show how the instrument may be used to measure (a) voltage up to 50 v, and (b) *current up to 10 A.*

Solution Full scale deflection of 10 mv

Full scale deflection current = 10 × 10^{-3}/5 = 2 mA

(a) For measuring the voltage up to 50 V we need to connect a multiplier resistance R_{SC} in series with the instrument

$$\text{Thus, } R_{SC} = (m-1)\,R_m, \quad \text{where } m = \frac{V}{V_m}$$

$$\therefore \quad R_{SC} = \left(\frac{50}{10 \times 10^{-3}} - 1\right) \times 5 = 24995\,\Omega$$

(b) For measuring the current up to 10 A we need to connect a shunt resistance in parallel to the instrument.

Thus, $R_{Sh} = \frac{R_m}{m-1}$, where $m = \frac{I}{I_m} = \frac{10}{2\times10^{-3}} = 5\times10^3$

$$\therefore \quad R_{Sh} = \frac{5}{5\times10^{-3}-1} = 1.002\times10^{-3}\ \Omega$$

Example 2.7

A moving-coil ammeter has a fixed shunt of 0.02 Ω. With a coil resistance of R = 1000 Ω and a potential difference of 500 mV across it. Full-scale deflection is obtained. (a) To what shunted current does it correspond? (b) Calculate the value of R to give full-scale deflection when shunted current I is (i) 10 A, and (ii) 75 A, (c) With what value of R, 40% deflection obtained with I = 100 A.

Solution

(a) Current through shunt $I_{sh} = 500 \times 10^{-3}/0.02 = 25$ A.

(b) (i) Voltage across shunt for a current of 10 A = 0.02 × 10 = 0.2 V.

Therefore, resistance of meter for a current of 10 A to give full scale deflection = $0.2/(0.5 \times 10^{-3})$ = 400 Ω

(ii) Voltage across shunt for a current of 75 A = 0.02 × 75 = 1.5 V. Therefore, resistance of meter for a current of 75 A to give full scale deflection = $1.5/(0.5 \times 10^{-3})$ = 3000 Ω

(c) Now 40% deflection is obtained with 100 A.
Therefore, current to give full-scale deflection = 100/0.4 = 250 A
Voltage across shunt for a current of 250 A = 0.02 × 250 = 5 V

Resistance of meter for a current of 100 A to give 40% of full scale deflection = $5/(0.5 \times 10^{-3})$ = 10,000 Ω

Example 2.8

A simple shunted ammeter using a basic meter movement with an internal resistance of 1800 Ω and a full-scale deflection current of 100 pA is connected in a circuit and gives reading of 3.5 mA on it's 5 mA scale. The reading is checked with a recently calibrated dc ammeter which gives a reading of 4.1 mA. The implication is that the ammeter has a faulty shunt on it's 5 mA range. Calculate (a) the actual value of faulty shunt, and (b) the current shunt for the 5 mA range.

Solution

(a) 5 mA scale deflection corresponds to 100 µA.

Therefore, 3.5 mA corresponds to $\frac{100\times10^{-6}\times3.5}{5}=7\times10^{-5}$ A

As the shunt and the meter are connected in parallel, the drop across the shunt should be equal to the voltage drop across the meter. Meter current = 7×10^{-5} A Shunt current = $(4.1 \times 10^{-3} - 7 \times 10^{-5})$ A Therefore, actual value of faulty shunt,

$R_{sh} \times (4.1 \times 10^{-3} - 7 \times 10^{-5}) = 1800 \times 7 \times 10^{-5}$

$$R_{sh}=\frac{1800\times7\times10^{-5}}{(4.1\times10^{-3}-7\times10^{-5})}=31.26\ \Omega$$

(b) 5 mA scale deflection corresponds to 100 µA.

Therefore, 4.1 mA corresponds to $\frac{100\times10^{-6}\times4.1}{5}=82\times10^{-6}A$ $= 82 \times 10^{-6}$ A

As the shunt and the meter are connected in parallel, then the drop across the shunt should be equal to the voltage drop across the meter.

Meter current = 82×10^{-6} A

Shunt current = $(4.1 \times 10^{-3} - 82 \times 10^{-6})$ A
Therefore, actual value of faulty shunt,

$R_{sh} \times (4.1 \times 10^{-3} - 82 \times 10^{-6}) = 1800 \times 82 \times 10^{-6}$

$$R_{sh}=\frac{1800\times82\times10^{-6}}{(4.1\times10^{-3}-82\times10^{-6})}=36.73\ \Omega$$

Example 2.9

A moving-coil instrument gives the full-scale deflection of 10 mA when the potential difference across its terminals is 100 mV. Calculate (a) the shunt resistance for a full-scale deflection corresponding to 100 A, and (b) the series resistance for full scale reading with 1000 V. Calculate the power dissipation in each case.

Solution

(a) Meter resistance R_m = 100 mV/10 mA = 10Ω

The shunt resistance corresponds to 100 A full-scale deflection

$$R_{sh}=\frac{R_m}{m-1},\ \text{where } m=\frac{I}{I_m}=\frac{100}{10\times10^{-3}}=10\times10^{3}$$

$$=\frac{10}{10\times10^{3}-1}=0.001\ \Omega$$

Now R_m and R_{sh} are connected in parallel, the equivalent resistance is

$$R = \frac{R_{sh} \times R_m}{R_{sh} + R_m} = \frac{0.001 \times 10}{0.001 + 10} = 0.00099\ \Omega$$

Power dissipation P = I^2R = 100^2 × 0.00099 = 9.9 W

(b) The series resistance corresponds to 1000 V,

$$R_{sc} = (m-1)R_m, \text{ where } m = \frac{V}{v} = \frac{1000}{100 \times 10^{-3}} = 10^4$$

$$= (10^4 - 1) \times 10 = 99{,}990\ \Omega$$

Now R_m and R_{sc} are connected in series, the equivalent resistance is

R = R_{sh} + R_m = 99,990 +10 = 100,000 Ω

Power dissipation P = V^2 /R = 1000^2 /100,000 = 10 W

Example 2.10

A moving-coil instrument has a resistance of 75 Ω and gives a full-scale deflection of 100-scale divisions for a current of 1 mA. The instrument is connected in parallel with a shunt of 25 Ω resistance and the combination is then connected in series with a load and a supply. What is the current in the load when the instrument gives an indication of 80 scale divisions?

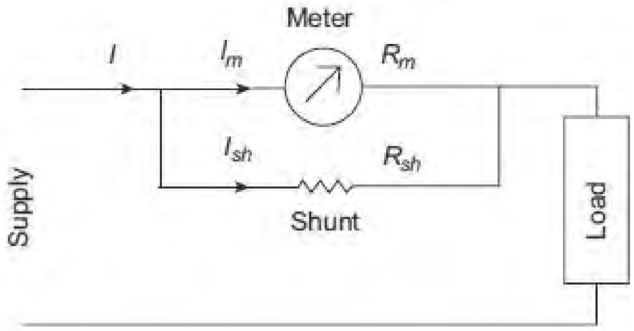
Solution For 80 scale divisions, current through the meter is × 1 = 0.8 mA

Now,

$$I \times \frac{R_{sh}}{R_{sh} + R_m} = 0.8$$

$$I \times \frac{25}{25 + 75} = 0.8$$

$$I = \frac{0.8 \times 100}{25} = 3.2 \text{ mA}$$



So, current through the load is 3.2 mA.

2.8 MOVING-IRON INSTRUMENTS

Basic range: 10 mA-100 A

Usage:

- dc MI ammeters and voltmeters
- ac MI ammeters and voltmeters

Moving-Iron or MI instruments can be classified as

- Attraction-type moving-iron instruments

- Repulsion-type moving-iron instruments

The current to be measured, in general, is passed through a coil of wire in the moving-iron instruments. In case of voltage measurement, the current which is proportional to the voltage is measured. The number of turns of the coil depends upon the current to be passed through it. For operation of the instrument, a certain number of ampere turns is required. These ampere turns can be produced by the product of few turns and large current or reverse.

2.8.1 Attraction-type Moving-Iron Instruments

The attraction type of MI instrument depends on the attraction of an iron vane into a coil carrying current to be measured. Figure 2.14 shows a attraction-type MI instrument. A soft iron vane IV is attached to the moving system. When the current to be measured is passed through the coil C, a magnetic field is produced. This field attracts the eccentrically mounted vane on the spindle towards it. The spindle is supported at the two ends on a pair of jewel bearings. Thus, the pointer PR, which is attached to the spindle S of the moving system is deflected. The pointer moves over a calibrated scale.

The control torque is provided by two hair springs S_1 and S_2 in the same way as for a PMMC instrument; but in such instruments springs are not used to carry any current. Gravity control can also be used for vertically mounted panel type MI meters. The damping torque is provided by the movement of a thin vane V in a closed sector-shaped box B, or simply by a vane attached to the moving system. Eddy current damping can not be used in MI instruments owing to the fact that any permanent magnet that will be required to produce Eddy current damping can distort the otherwise weak operating magnetic field produced by the coil.

If the current in the fixed coil is reversed, the field produced by it also reverses. So the polarity induced on the vane reverses. Thus whatever be the direction of the current in the coil the vane is always be magnetized in such a way that it is attracted into the coil. Hence such instrument can be used for both direct current as well as alternating current.

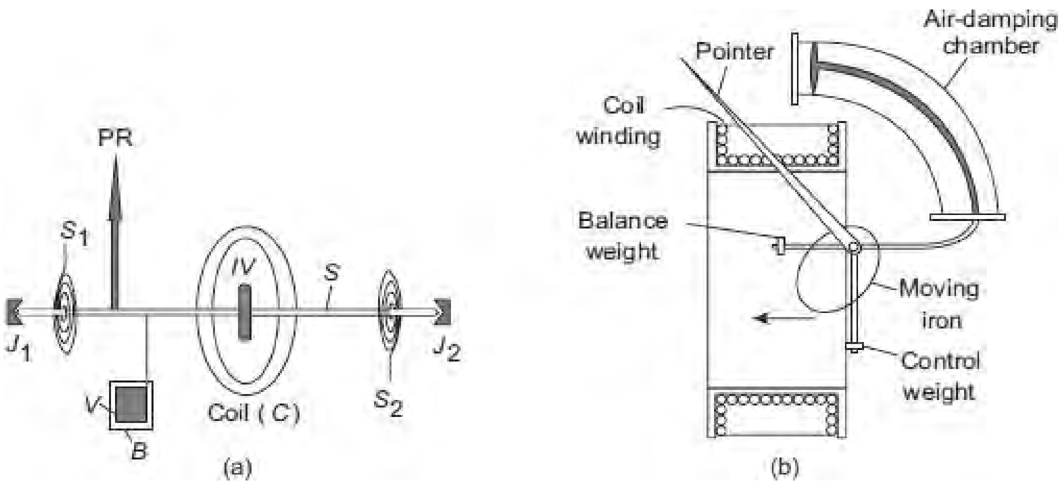


Figure 2.14 *Attraction-type moving iron (MI) instrument*

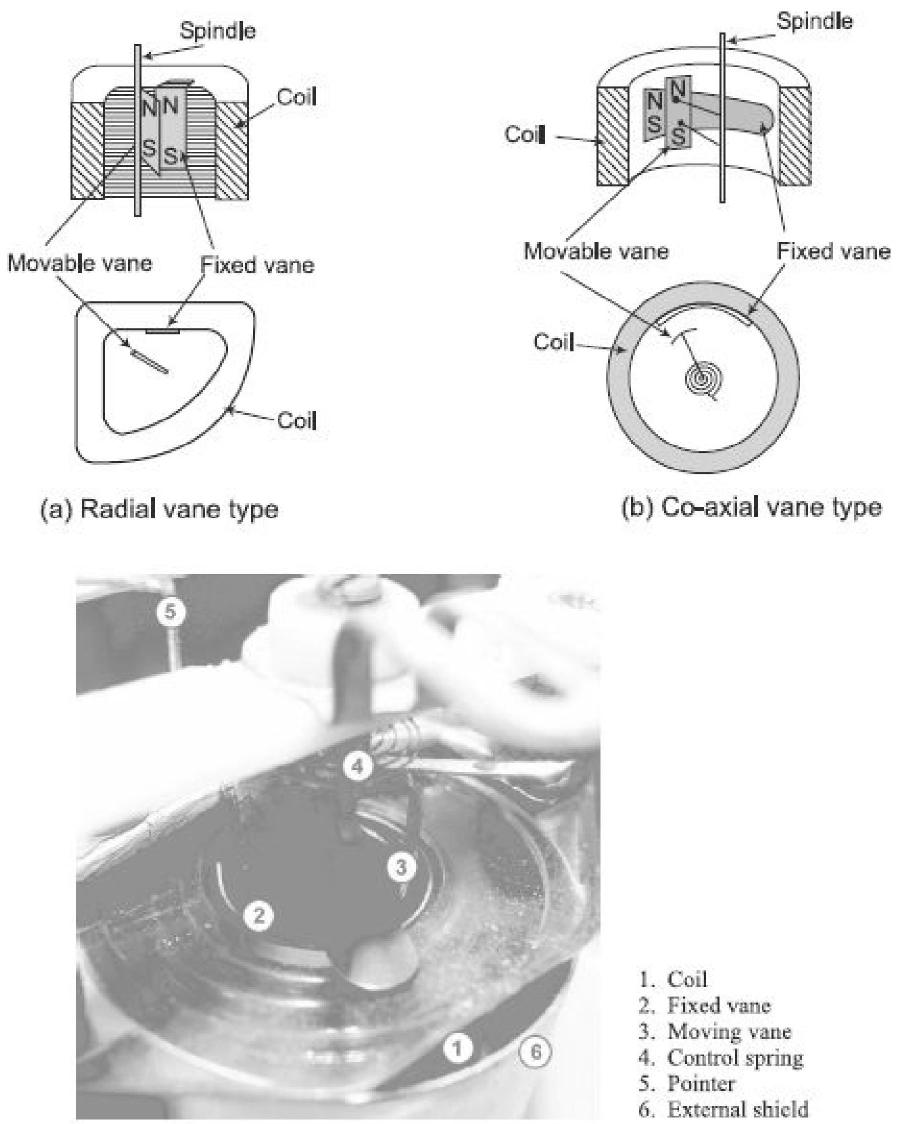
2.8.2 Repulsion-type Moving-Iron Instruments

In the repulsion type, there are two vanes inside the coil. One is fixed and the other is movable. These are similarly magnetised when the current flows through the coil and there is a force of repulsion between the two vanes resulting in the movement of the moving vane.

Two different designs for moving iron instruments commonly used are as follows:

1. Radial Vane Type In this type, the vanes are radial strips of iron. The strips are placed within the coil as shown in Figure 2.15(a). The fixed vane is attached to the coil and the movable one to the spindle of the instrument. The instrument pointer is attached to the moving vane spindle.

As current flows through the coil, the generated magnetic field induces identical polarities on both the fixed and moving vane. Thus, even when the current through the coil is alternating (for AC measurement), there is always a repulsion force acting between the like poles of fixed and moving vane. Hence deflection of the pointer is always in the same direction irrespective of the polarity of current in the coil. The amount of deflection depends on the repulsion force between the vanes which in turn depends on the amount of current passing through the coil. The scale can thus be calibrated to read the current or voltage directly.



(a) Radial vane type

(b) Co-axial vane type

(c) Photograph of a Co-axial vane-type MI instrument

Figure 2.15 *Re-pulsion-type Moving Iron (MI) instruments*

2. *Co-axial Vane Type* I In these type of instruments, the fixed and moving vanes are sections of coaxial cylinders as shown in Figure 2.15(b). Current in the coil magnetizes both the vanes with similar polarity. Thus the movable vane rotates along the spindle axis due to this repulsive force. Coaxial vane type instruments are moderately sensitive as compared to radial vane type instruments that are more sensitive.

Moving iron instruments have their deflection is proportional to the square of the current flowing through the coil. These instruments are thus said to follow a square law response and have non-uniform scale marking. Deflection being proportional to square of the current, whatever be the polarity of current in the coil, deflection of a moving iron instrument is in the same direction. Hence, moving iron instruments can be used for both DC and AC measurements.

2.8.3 Torque Equation of Moving-Iron Instruments

To deduce the expression for torque of a moving iron instrument, energy relation can be considered for a small increment in current supplied to the instrument. This result in a small deflection $d\theta$ and some mechanical work will be done. Let T_d be the deflecting torque.

Therefore mechanical work done = torque × angular displacement

$$= T_d \cdot d\theta \tag{2.27}$$

Due to the change in inductance there will be a change in the energy stored in the magnetic field.

Let I be the initial current, L be the instrument inductance and θ is the deflection. If the current increases by *dl* then it causes the change in deflection $d\theta$ and the inductance by *dL*. In order to involve the increment *dI* in the current, the applied voltage must be increase by:

$$e = \frac{d\phi}{dt} = \frac{d}{dt}(LI) = I\frac{dL}{dt} + L\frac{dI}{dt} \tag{2.28}$$

The electrical energy supplied is $eIdt = I^2 dL + ILdI$ (2.29)

[substitute the value of *edt* from equation (2.28)]

The current is changes from I to (I + *dI)*, and the inductor L to (L + *dL)*

Therefore the stored energy changes from $= \frac{1}{2}I^2L$ to $\frac{1}{2}(I + dI)^2\ (L + dL)$

Hence the change in stored energy $= \frac{1}{2}(I + dI)^2\ (L + dL) - \frac{1}{2}I^2L$ (2.30)

As *dI* and *dL* are very small, neglecting the second and higher order terms in small quantities, this

becomes $ILdL + \frac{1}{2}I^2 dL$

From the principle of conservation of energy,

Electrical energy supplied = Increase in stored energy + Mechanical work done.

$$I^2 dL + ILdI = ILdI + \frac{1}{2} I^2 dL + T_d d\theta$$

$$\therefore \qquad T_d d\theta = \frac{1}{2} I^2 dL \tag{2.31}$$

$$\text{or deflecting torque } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta} \tag{2.32}$$

where T_d is in newton-metre, I is in ampere, L is in henry and θ is in radians.

The moving system is provided with control springs and in turn the deflecting torque T_d is balanced by the controlling torque $T_C = k\,\theta$

where k is the control spring constant (N-m/rad) and θ is the deflection in radians.

At final steady position, $T_C = T_d$

$$\text{or} \qquad k\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\boxed{\therefore \text{ deflection} \quad \theta = \frac{1}{2} \frac{I^2}{k} \frac{dL}{d\theta}} \tag{2.33}$$

Hence, the deflection is proportional to square of the rms value of the operating current. The deflection torque is, therefore, unidirectional whatever may be the polarity of the current.

Advantages of MI Instruments

1. Robust construction and relatively cheap
2. Suitable for measuring both dc and ac
3. Can withstand overload momentarily

Disadvantages of MI Instruments

1. As the deflection is proportional to I^2, hence the scale of the instrument is not uniform. It is cramped in the lower end and expanded in the upper portion.
2. It is affected by stray magnetic fields.
3. There is hysteresis error in the instrument. The hysteresis error may be minimized by using the vanes of nickel-iron alloy.
4. When used for measuring ac the reading may be affected by variation of frequency due to the change in reactance of the coil, which has some inductance. With the increase in frequency iron loses and coil impedance increases.
5. Since large amount of power is consumed to supply I^2R loss in the coil and magnetic losses in the vanes, it is not a very sensitive instrument.

Example 2.11

The inductance of a moving-iron ammeter with a full-scale deflection of 90° at 1.5 A is given by $L = (200 + 40\theta - 4\theta^2 - \theta^3)$ μH where θ is the deflection in radian from the zero position. Estimate the angular deflection of the pointer for a current of 1 A.

Solution For an MI instrument,

$$\text{deflection } \theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

Here, $L = (200 + 40\theta - 4\theta^2 - \theta^3)$ μH

Then $\frac{dL}{d\theta} = (40 - 8\theta - 3\theta^2)$

For a deflection, $\theta = 90° = \frac{\pi}{2}$, current $I = 1.5$ A

$$\frac{\pi}{2} = \frac{1}{2} \times \frac{(1.5)^2}{k} \times \left[40 - 8\left(\frac{\pi}{2}\right) - 3\left(\frac{\pi}{2}\right)^2\right]$$

$$k = \frac{(1.5)^2}{\pi} \times \left[40 - 8\left(\frac{\pi}{2}\right) - 3\left(\frac{\pi}{2}\right)^2\right]$$

$$= 14.348 \text{ N-m/rad}$$

Now for a current of 1 A, the angular deflection θ is

$$\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2} \times \frac{1}{14.348} \times (40 - 8\theta - 3\theta^2)$$

$$40 - 8\theta - 3\theta^2 = 28.696\,\theta$$

$$3\theta^2 + 36.696\,\theta - 40 = 0$$

After solving for θ and taking only the positive value

$$\theta = 1.00712 \text{ rad} = 57.7°$$

Example 2.12

The law of deflection of a moving-iron ammeter is given by $I = 4\theta^n$ ampere, where θ is the deflection in radian and n is a constant. The self-inductance when the meter current is zero is 10 mH. The spring constant is 0.16 N-m/rad.

(*a*) *Determine an expression for self-inductance of the meter as a function of θ and n.*

(*b*) *With n = 0.75, calculate the meter current and the deflection that corresponds to a self-inductance of 60 mH.*

Solution

(a) $$\theta = \frac{1}{2}\frac{I^2}{k}\frac{dL}{d\theta}$$

$$\theta = \frac{1}{2}\times\frac{(4\theta^n)^2}{0.16}\times\frac{dL}{d\theta}$$

$$2\theta\cdot d\theta = 100\cdot\theta^{2n}dL$$

$$dL = \frac{1}{50}\theta^{1-2n}d\theta$$

Integrating both sides,

$$L = \frac{1}{100}\frac{\theta^{2-2n}}{(1-n)}+C; \qquad C = \text{Integration constant}$$

at $I = 0$, i.e. $\theta = 0$, $L = 10 \times 10^{-13}$

Substituting the value of θ and L, we get

$$C = 10 \times 10^{-3}$$

So, $$L = \frac{1}{100}\frac{\theta^{2-2n}}{(1-n)}+10\times10^{-3}$$

(b) Now $n = 0.75$ and $L = 60 \times 10^{-3}$

$$60 \times 10^{-3} = \frac{1}{100}\frac{\theta^{2-2\times 0.75}}{(1-0.75)}+10\times10^{-3}$$

$$\theta^{0.5} = 25(60 \times 10^{-3}) = 1250 \times 10^{-3}$$

$$\theta = (1250 \times 10^{-3}) = 1.56 \text{ rad} = 89.38 \text{ degree}$$

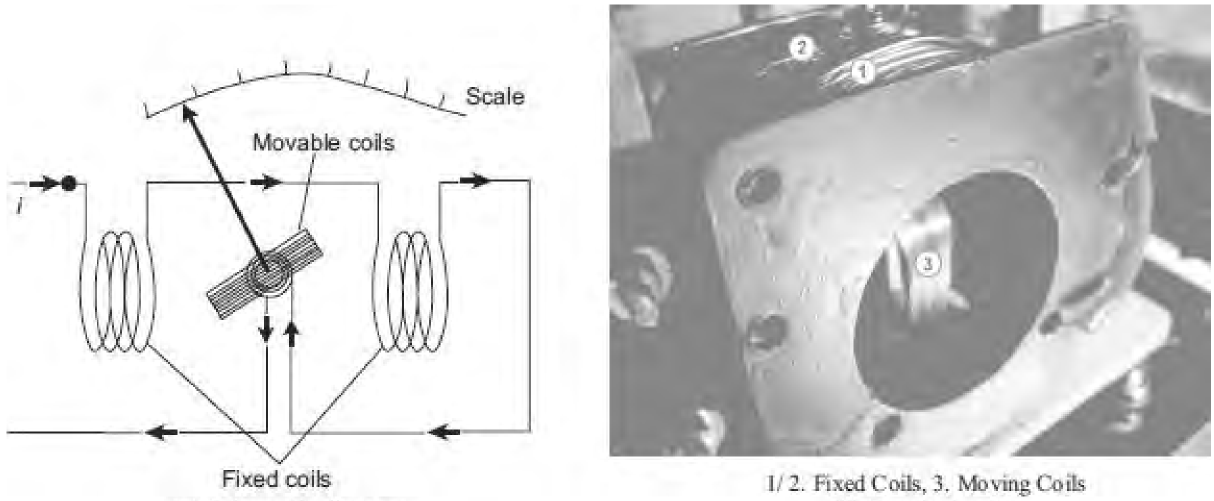
So meter current $I = 4 \times (1.56)^{0.75} = 5.58$ Amp

2.9 ELECTRODYNAMOMETER-TYPE INSTRUMENTS

The electrodynamometer is a transfer-type instrument. A transfer-type instrument is one that may be calibrated with a dc source and then used without modification to measure ac. This requires the transfer type instruments to have same accuracy for both dc and ac.

The electrodynamic or dynamometer-type instrument is a moving-coil instrument but the magnetic field, in which the coil moves, is provided by two fixed coils rather than by permanent magnets. The schematic diagram of electrodynamic instrument is shown in Figure 2.16(a) and a practical meter is shown in Figure 2.16(b). It consists of two fixed coils, which are symmetrically situated. It would have a torque in one direction during one half of the cycle and an equal effect in opposite direction during the other half of the cycle. If, however, we were to reverse the direction of the flux each time the current through the movable coil reverses, a unidirectional torque would be produced for both positive half and negative half of the cycle. In electrodynamic instruments, the field can be made to reverse simultaneously with the current in the movable coil if the fixed coil is connected in series with the movable coil.

1. *Controlling Torque* The controlling torque is provided by two control springs. These springs act as leads to the moving coil.



(a) Schematic diagram (b) Practical meter

Figure 2.16 Electrodynamometer-type instrument

2. *Damping* Air-friction damping is employed for these instruments and is provided by a pair of aluminium vanes, attached to the spindle at the bottom. These vanes move in a sector-shaped chamber.

2.9.1 Torque Equation of Electrodynamometer-type Instruments

Let, i_1 = instantaneous value of current in the fixed coils, (A)

i_2 = instantaneous value of current in the moving coils, (A)

L_1 = self-inductance of fixed coils, (H)

L_2 = self-inductance of moving coil, (H)

M = mutual inductance between fixed and moving coils (H)

Flux linkage of Coil 1, $\psi_1 = L_1 i_1 + M i_2$

Flux linkage of Coil 2, $\psi_2 = L_2 i_2 + M i_1$

Electrical input energy,

$$= e_1 i_1 dt + e_2 i_2 dt = i_1 d\psi_1 + i_2 d\psi_2$$

As $e_1 = \dfrac{d\psi_1}{dt}$ and $e_2 = \dfrac{d\psi_2}{dt}$

$$= i_1 d(L_1 i_1 + M i_2) + i_2 d(L_2 i_2 + M i_1)$$

$$= i_1 L_1 di_1 + i_1^2 dL_1 + i_1 i_2 dM + i_1 M di_2 + i_2 L_2 di_2 + i_2^2 dL_2 + i_1 i_2 dM + i_2 M di_1 \quad (2.34)$$

Energy stored in the magnetic field $= \dfrac{1}{2} i_1^2 L_1 + \dfrac{1}{2} i_2^2 L_2 + i_1 i_2 M$

Change in energy stored $= d\left(\dfrac{1}{2} i_1^2 L_1 + \dfrac{1}{2} i_2^2 L_2 + i_1 i_2 M\right)$

$$= i_1 L_1 di_1 + \frac{1}{2} i_1^2 dL_1 + i_2 L_2 di_2 + \frac{1}{2} i_2^2 dL_2 + i_1 M di_2 + i_2 M di_1 + i_1 i_2 dM \quad (2.35)$$

From the principle of conservation of energy,

Total electrical input energy = Change in energy in energy stored + mechanical energy The mechanical energy can be obtained by subtracting Eq. (2.35) from Eq. (2.34).

Therefore, mechanical energy = $\frac{1}{2}i_1^2 dL_1 + \frac{1}{2}i_2^2 dL_2 + i_1 i_2 dM$

Now, the self-inductances L_1 and L_1 are constants and, therefore, dL_2 and dL_2 both are equal to zero. Hence, mechanical energy = $i_1 i_2\ dM$

Suppose T_i is the instantaneous deflecting torque and $d\theta$ is the change in deflection, then, Mechanical energy = work done = $T_i\ d\theta$

Thus we have

$$T_i d\theta = i_1 i_2 dM \quad \text{or} \quad T_i = i_1 i_2 \frac{dM}{d\theta} \tag{2.36}$$

1. ***Operation with dc*** Let, I_1 = current in the fixed coils, I_2 = current in the moving coil

So deflecting torque T_d = $I_1 I_2 \frac{dM}{d\theta}$. This shows that the deflecting torque depends in general on the product of current I_1 and I_2 and the rate of change of mutual inductance.

This deflecting torque deflects the moving coil to such a position where the controlling torque of the spring is equal to the deflecting torque. Suppose θ be the final steady deflection.

Therefore controlling torque $T_C = k\theta$ where k = spring constant (N-m/rad)

At final steady position $T_d = T_C$

$$I_1 I_2 \frac{dM}{d\theta} = k\theta$$

or, the deflection

$$\boxed{\theta = \frac{I_1 I_2}{k}\frac{dM}{d\theta}} \tag{2.37}$$

If the two coils are connected in series for measurement of current, the two currents I_1 and I_2 are equal.

Say, $I_1 = I_2 = I$

Thus, deflection of the pointer is $\theta = \frac{I^2}{k}\frac{dM}{d\theta}$

For dc use, the deflection is thus proportional to square of the current and hence the scale non-uniform and crowded at the ends.

2. ***Operation with ac*** Let, i_1 and i_2 be the instantaneous values of current carried by the coils. Therefore, the instantaneous deflecting torque is:

$$T_i = i_1 i_2 \frac{dM}{d\theta}$$

If the two coils are connected in series for measurement of current, the two instantaneous currents i_1 and i_2 are equal.

Say, $i_i = i_2 = i$

Thus, instantaneous torque on the pointer is $T_i = i_2 \frac{dM}{d\theta}$

Thus, for ac use, the instantaneous torque is proportional to the square of the instantaneous current. As the quantity i^2 is always positive, the current varies and the instantaneous torque also varies. But the moving system due to its inertia cannot follow such rapid variations in the instantaneous torque and responds only to the average torque.

The average deflecting torque over a complete cycle is given by:

$$T_d = \frac{1}{T}\int_0^T T_i\, dt = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2\, dt$$

where T is the time period for one complete cycle.

At final steady position $T_d = T_C$

$$\text{or,} \qquad k\theta = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2\, dt$$

Thus, deflection of the pointer is $\theta = \frac{1}{k}\frac{dM}{d\theta}\frac{1}{T}\int_0^T i^2\, dt$

Deflection is thus a function of the mean of the square of the current. If the pointer scale is calibrated in terms of square root of this value, i.e. square root of the mean of the square of current value, then rms value of the ac quantity can be directly measured by this instrument.

3. ***Sinusoidal Current*** If currents i_1 and i_2 are sinusoidal and are displaced by a phase angle j, i.e.

$$i_1 = i_{m1} \sin \omega t \text{ and } i_2 = I_{m1} \sin(wt - j)$$

∴ The average deflecting torque

$$T_d = \frac{dM}{d\theta}\frac{1}{T}\int_0^T i_1 i_2\, dt = \frac{dM}{d\theta}\frac{1}{2\pi}\int_0^{2\pi} I_{m1} \sin \omega t \cdot I_{m2} \sin(\theta t - \varphi)\, d\omega t$$

$$\frac{I_{m1} I_{m2}}{2} \cos\varphi \frac{dM}{d\theta} = I_1 I_2 \cos\varphi \frac{dM}{2\theta}$$

where I_1 and I_2 are the rms values of the currents flowing through the coils. At equilibrium, $T_d = T_C$

$$\text{or} \qquad I_1 I_2 \cos\varphi \frac{dM}{d\theta} = k\theta \qquad (2.38)$$

$$\therefore \qquad \boxed{\theta = \frac{I_1 I_2}{k} \cos\varphi \frac{dM}{d\theta}}$$

As was in the case with ac measurement, with sinusoidal current also the deflection is a function of the mean of the square of the current. If the pointer scale is calibrated in terms of square root of this value, i.e. square root of the mean of the square of current value, then RMS value of the ac quantity can be directly measured by this instrument.

1. *Electrodynamic Ammeter* In an electrodynamic ammeter, the fixed and moving coils are connected in series as shown in Figure 2.17. A shunt is connected across the moving coil for limiting the current. The reactance–resistance ratio of the shunt and the moving coil is kept nearly same for independence of the meter reading with the supply frequency. Since the coil currents are the same, the deflecting torque is proportional to the mean square value of the current. Thus, the scale is calibrated to read the rms value.

2. *Electrodynamic Voltmeter* The electrodynamic instrument can be used as a voltmeter by connecting a large noninductive resistance (R) of low temperature coefficient in series with the instrument coil as shown in Figure 2.18.

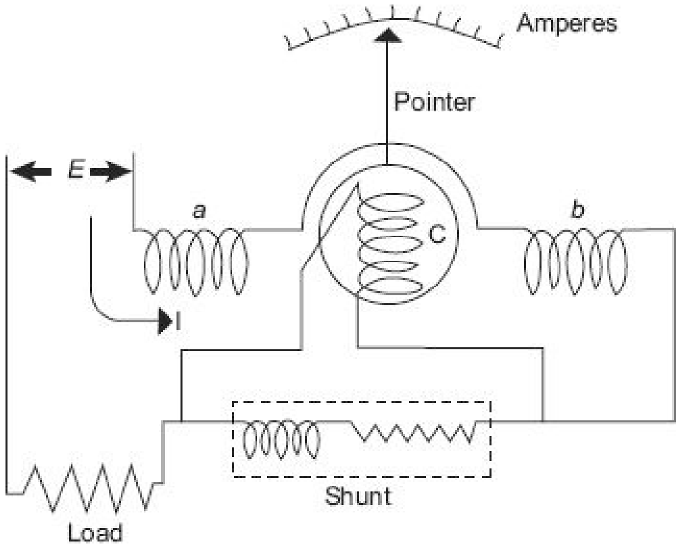


Figure 2.17 *Electrodynamometer ammeter*

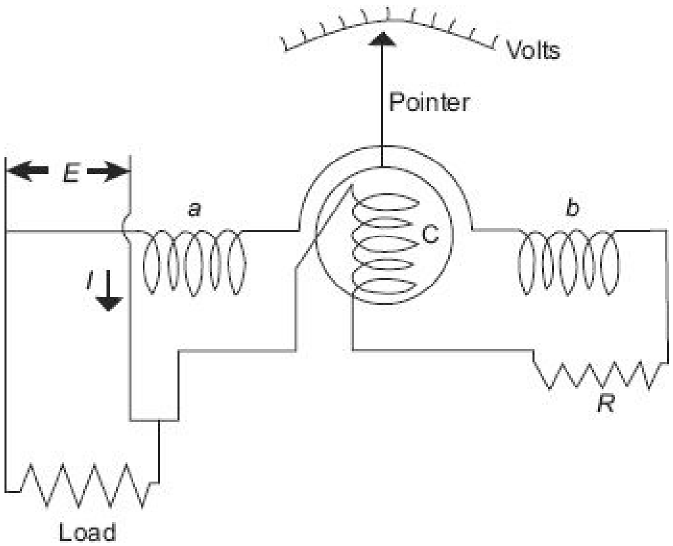


Figure 2.18 *Electrodynamometer voltmeter*

3. *Electrodynamic Wattmeter* The electrodynamic wattmeter consist of two fixed coils *'a'* and *'b'* placed symmetrical to each other and producing a uniform magnetic field. They are connected in series with the load and are called the Current Coils (CC). The two fixed coils can be connected in series or parallel to give two different current ratings. The current coils carry the full-load current or a fraction of full load current. Thus the current in the current coils is proportional to the load current. The moving coil '*c*', in series with a high non inductive resistance R_v is connected across the supply. Thus the current flowing in the moving coil is proportional to, and practically in phase with the supply voltage. The moving coil is also called the voltage coil or Pressure Coil (PC). The voltage coil is carried on a pivoted spindle which carries the pointer, the pointer moved over a calibrated scale.

Two hair springs are used for providing the controlling torque and for leading current into and out of the moving coil. Damping is provided by air friction. Figure 2.20 shows the basic arrangement of a electrodynamic wattmeter.

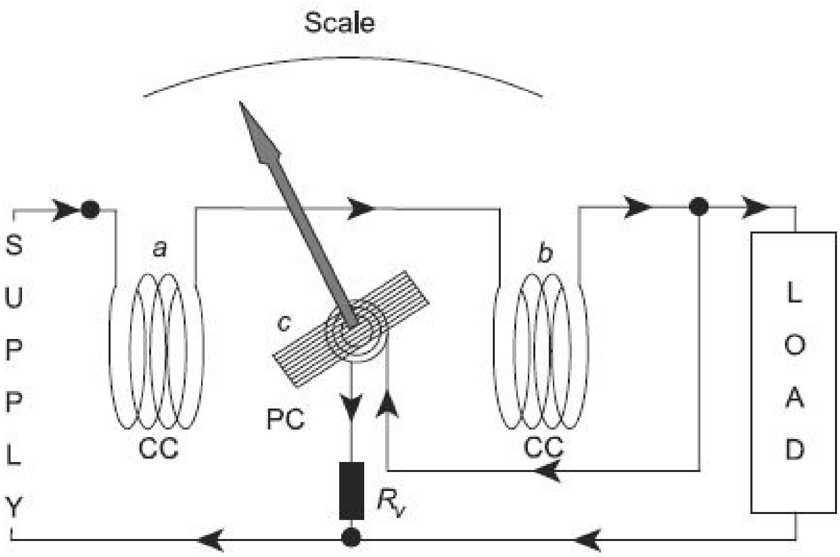


Figure 2.19 *Electrodynamic wattmeter*

4. ***Torque Equation***

Let, i_f = current in the fixed coil

i_m = current in the moving coil

i = load current

v = load voltage

T_{in} = instantaneous value of the deflecting torque

p = instantaneous power

$T_{in} \times i_f i_m$

$$T_{in} \propto i_f i_m \tag{2.39}$$

But since $i_f \propto i$ and $i_m \propto v$

$$T_{in} \propto vi \propto p \tag{2.40}$$

Thus, the instantaneous value of the deflecting torque is proportional to the instantaneous power. Owing to the inertia of the moving system, the pointer reads the average power. In dc circuits, the power is given by the product of voltage and current, and hence the torque is directly proportional to the power. Thus, the instrument indicates the power.

For ac, the instrument indicates the average power. This can be proved as follows:

$T_{in} \propto Vi$

Average deflecting torque × average power

Let, $v = V_m \sin d$

$I = I_m \sin(\theta - \Phi)$

Average deflecting torque ∞ average value of $V_m \sin d \times I_m \sin(\theta - \Phi) \propto VI \cos\theta$ If T_d be the average torque, then

$$T_d \propto VI \cos\Phi \propto \text{true power} = kP \qquad (2.41)$$

where P is the true power and k is the constant.

For spring control $T_C = k_s\theta_1$

where T_C is the control torque, k_s is the spring constant and θ_1 is the angle of deflection of the pointer.

For steady deflection,

$$T_c = T_d$$

$$k_s\theta_1 = kP$$

$$\theta_1 = \frac{k}{k_s}P$$

$$\theta_1 \propto P$$

Hence, in case of ac also the deflection is proportional to the true power in the circuit. The scale of the electrodynamometer wattmeter is therefore uniform.

Advantages of Electrodynamometer-type Instruments

1. They can be used on ac as well as dc measurements.
2. These instruments are free from eddy current and hysteresis error.
3. Electrodynamometer-type instruments are very useful for accurate measurement of rms values of voltages irrespective of waveforms.
4. Because of precision grade accuracy and same calibration for ac and dc measurements these instruments are useful as transfer type and calibration instruments.

Disadvantages of Electrodynamometer-type Instruments

1. As the instrument has square law response, the scale is non-uniform.
2. These instruments have small torque/weight ratio, so the frictional error is considerable.
3. More costly than PMMC and MI type of instruments.
4. Adequate screening of the movements against stray magnetic fields is essential.
5. Power consumption is comparably high because of their construction.

The inductance of a 25 A electrodynamic ammeter changes uniformly at the rate of 0.0035 mH/radian. The spring

Example 2.13 *constant is 10^{-6}N-m/radian. Determine the angular deflection at full scale.*

Solution $\frac{dM}{d\theta} = 0.0035 \times 10^{-6}$ H/rad

Now the deflection $\theta = \frac{I^2}{k}\frac{dM}{d\theta}$

Angular deflection at full scale current of I = 25 A is given by:

$$\theta = \frac{25^2}{10^{-6}} \times 0.0035 \times 10^{-6} \times \frac{180°}{\pi} = 125°$$

Example 2.14

In an electrodynamic instrument the total resistance of the voltage coil circuit is 8200 Ω and the mutual inductance changes uniformly from -173 μH at zero deflection to +175 μH at full scale. The angle of full scale being 95°. If a potential difference of 100 V is applied across the voltage circuit and a current of 3 A at a power factor of 0.75 is passed through the current coil, what will be the deflection. Spring constant of the instrument is 4.63 × 10^{-6} N-m/rad.

Solution Change in mutual inductance *dM* = 175 – (–173) = 348 μH

Deflection θ = 95° = 1.66 rad

Rate of change of mutual inductance

$$\frac{dM}{d\theta} = \frac{348}{1.66} = 209.63 \ \mu\text{H/rad}$$

Current through the current coil I_1= 3 A

Current through the voltage coil $I_2 = \frac{100}{8200} = 0.0122 \text{ A}$

Power factor cos *6* = 0.75

Deflection $\theta = \frac{I_1 I_2}{k}\cos\phi\frac{dM}{d\theta}$

$$\theta = \frac{3 \times 0.0122}{4.63 \times 10^{-6}} \times 0.75 \times 209.63 \times 10^{-6}$$
$$= 1.242 \text{ rad} = 71.2°$$

Example 2.15

A 50 V range spring-controlled electrodynamic voltmeter has an initial inductance of 0.25 H, the full scale deflection torque of 0.4 × 10^{-4} Nm and full scale deflection current of 50 mA. Determine the difference between dc and 50 Hz ac reading at 50 volts if the voltmeter inductance increases uniformly over the full scale of 90°.

Solution

Full-scale deflection *6* = 90°

Full-scale deflecting torque, T_d = 0.4 × 10^{-4} Nm

Full-scae deflection current, I = 50 mA = 0.05 A

Initial induction, M = 0.25 H

Since deflecting torque, $T_d = I^2 \frac{dM}{d\theta}$

So for full-scale deflection 0 4 × 10^{-4} = $(0\ 05)^2$ $\frac{dM}{d\theta}$

or, $\frac{dM}{d\theta} = \frac{0.4 \times 10^{-4}}{(0.05)^2} = 0.016 \text{ H/rad}$

Total change in inductance for full-scale deflection,

$$dM = 0.016 \times 90 \times \frac{\pi}{180} = 0.0251\ H$$

Total mutual inductance, M = 0.25 + 0.0251 = 0.2751 H

The resistance of voltmeter, $R = \frac{\text{Voltage}}{\text{Current}} = \frac{50}{0.05} = 1000\ \Omega$

The impedance while measuring the voltage of 50 V at 50 Hz AC

$$Z = \sqrt{(1000)^2 + (2\pi \times 50 \times 0.2751)^2} = 1004\ \Omega$$

And voltmeter reading, = $\frac{50}{1004} \times 1000 = 49.8\ V$

Therefore, difference in reading = 50 – 49.8 = 0.2 V

2.10 ELECTROSTATIC INSTRUMENTS

In electrostatic instruments, the deflecting torque is produced by action of electric field on charged conductors. Such instruments are essentially voltmeters, but they may be used with the help of external components to measure the current and power. Their greatest use in the laboratory is for measurement of high voltages.

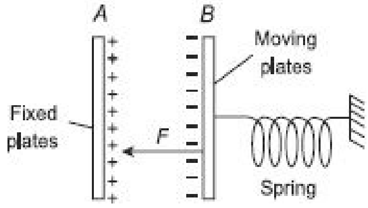


Figure 2.20 Linear motion of electrostatic instruments

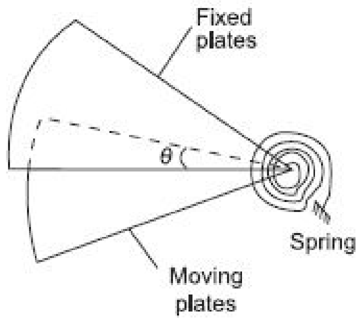


Figure 2.21 Rotary motion of electrostatic instruments

Rotary motion of electrostatic instruments

There are two ways in which the force acts:

1. One type involves two oppositely charged electrodes. One of them is fixed and the other is movable. Due to force of attraction, the movable electrode is drawn towards the fixed one.
2. In the other type, there is force of attraction or repulsion between the electrodes which causes rotary motion of the moving electrode.

In both the cases, the mechanism resembles a variable capacitor and the force or torque is due to the fact that the mechanism tends to move the moving electrode to such a position where the energy stored is maximum.

2.10.1 Force and Torque Equation

1. *Linear Motion* Referring to Figure 2.20, plate *A* is fixed and *B* is movable. The plates are oppositely charged and are restrained by a spring connected to the fixed point. Let a potential difference of V volt be applied to the plates; then a force of attraction F Newton exists between them. Plate B moves towards A until the force is balanced by the spring. The capacitance between the plates is then C farad and the stored energy is ½ CV joules.

Now let there be a small increment *dV* in the applied voltage, then the plate *B* moves a small distance *dx* towards *A*. when the voltage is being increased a capacitive current flows. This current is given by

$$i = \frac{dq}{dt} = \frac{d}{dt}(CV) = C\frac{dV}{dt} + V\frac{dC}{dt} \quad (2.42)$$

The input energy is $Vidt = V^2 dC + CVdV$ (2.43)

Change in stored energy = $= \frac{1}{2}(C + dC)(V + dV)^2 - \frac{1}{2}CV^2$

$$= \frac{1}{2}V^2 dC + CVdV \quad (2.44)$$

(neglecting the higher order terms as they are small quantities)

From the principle of conservation of energy,

Input electrical energy = increase in stored energy + mechanical work done

$$V^2dC + CVdV = \frac{1}{2}V^2dC + CVdV + Fdx$$

$$\therefore \quad \boxed{F = \frac{1}{2}V^2\frac{dC}{dx}} \tag{2.45}$$

2. ***Rotational Motion*** The forgoing treatment can be applied to the rotational motion by writing an angular displacement θ in place of linear displacement x and deflecting torque T_d instead of force F (Figure 2.21).

Deflecting torque $T_d = \frac{1}{2}V^2\frac{dC}{d\theta}$ (2.46)

If the instrument is spring controlled or has a suspension then

Controlling torque T_C = k&, where k = spring constant

θ = deflection

Hence, deflection

$$\boxed{\theta = \frac{1}{2}\frac{V^2}{k}\frac{dC}{d\theta}} \tag{2.47}$$

Since the deflection is proportional to the square of the voltage to be measured, the instrument can be used on both ac and dc. The instrument exhibits a square law response and hence the scale is non-uniform.

Advantages of Electrostatic Instrumentss

1. These instruments draws negligible amount of power from the mains.
2. They may be used on both ac and dc.
3. They have no frequency and waveform errors as the deflection is proportional to square of voltage and there is no hysteresis.
4. There are no errors caused by the stray magnetic field as the instrument works on the electrostatic principle.
5. They are particularly suited for high voltage.

Disadvantages of Electrostatic Instruments

1. The use of electrostatic instruments is limited to certain special applications, particularly in ac circuits of relatively high voltage, where the current drawn by other instruments would result in erroneous indication. A protective resistor is generally used in series with the instrument in order to limit the current in case of a short circuit between plates.
2. These instruments are expensive, large in size and are not robust in construction.
3. Their scale is not uniform.
4. The operating force is small.

2.11 INDUCTION-TYPE INSTRUMENTS

Induction-type instruments are used only for ac measurement and can be used either as ammeter, voltmeter or wattmeter. However, the induction principle finds its widest application as a watt-hour or energy meter (for details, refer Chapter 8). In such instruments, the deflecting torque is produced due to the reaction between the flux of an ac magnet and the eddy currents induced by another flux.

2.11.1 Principle of Operation

The operations of induction-type instruments depend on the production of torque due to the interaction between a flux Φ_1 (whose magnitude depends on the current or voltage to be measured) and eddy current induced in a metal disc or drum by another flux Φ_2 (whose magnitude also depends on the current or voltage to be measured). Since the magnitude of eddy current also depends on the flux producing them, the instantaneous value of the torque is proportional to the square of current or voltage under measurement and the value of mean torque is proportional to the mean square value of this current or voltage.

Consider a thin aluminium or copper disc D free to rotate about an axis passing through its centre as shown in Figure 2.22. Two electromagnets P_1 and P_2 produce alternating fluxes Φ_1 and Φ_2 respectively which cuts this disc. Consider any annular portion of the disc around P_1 with centre of the axis of P_l. This portion will be linked by flux $\Phi_1$and so an alternating emf Φ_1 be induced in it. Φ_2 will induce an emf e_2 which will further induce an eddy current i_2 in an annular portion of the disc around P_l. This eddy currents i_2 flows under the pole P_l.

Let us take the downward directions of fluxes as positive and further assume that at the instant under consideration, both Φ_1 and Φ_2 are increasing. By applying Lenz's law, the direction of the induced currents i_1 and i_2 can be found as indicated in Figure 2.22(b).

The portion of the disc which is traversed by flux Φ_1 and carries eddy currents i_2 experiences a force F_1 along the direction as indicated. As $F = Bil$, force $F_1 \infty \Phi_1 i_2$. Similarly, the portion of the disc lying under flux Φ_2 and carrying eddy current i_1 experiences a force $\Phi_2 \infty F_2$ij.

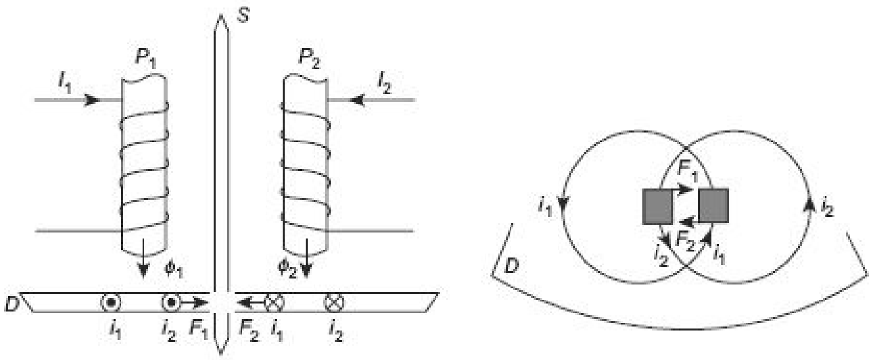
$$\therefore \quad F_1 \propto \varphi_1 i_2 = k\varphi_1 i_2 \qquad (2.48)$$

$$F_2 \propto \varphi_2 i_1 = k\varphi_2 i_1 \qquad (2.49)$$

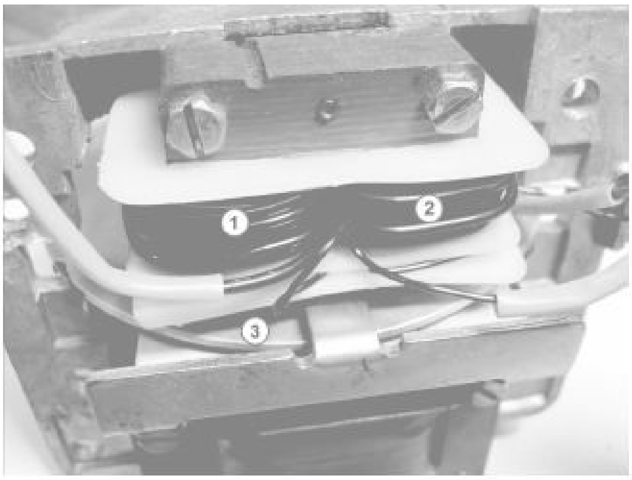
It is assumed that the constant k is the same in both the cases due to the symmetrical position of P_1 and P_2 with respect to the disc.

If r be the effective radius at which these forces acts, then net instantaneous torque T acting on the disc being equal to the different of the two torques, it is given by

$$T = r(k\varphi_1 i_2 - k\varphi_2 i_1) = k_1(\varphi_1 i_2 - \varphi_2 i_1) \qquad (2.50)$$



(a) (b)



1/2 - Electromagnetic coils, 3 – Aluminium rotating disc

(c) Photograph of Induction type instrument

Figure 2.22 *Principle of operation of induction-type instrument*

Let the alternating flux φ_1 be given by $\varphi_1 = \varphi_{1m} \sin \omega t$. The flux $\varphi_2$which is assumed to lag φ_1 by an angle a radian is given by $\varphi_2 = \varphi_{2m} \sin (\omega t - a)$

Induced emf $e_1 = \frac{d\varphi_1}{dt} = \frac{d}{dt} (\varphi_{1m} \sin \omega t) = \omega\varphi_{1m} \cos \omega t$

Assuming the eddy current path to be purely resistive and of value R, then the value of eddy current is

$$i_1 = \frac{e_1}{R} = \frac{\omega \varphi_{1m}}{R} \cos \omega t$$

similarly, $e_2 = \omega \varphi_{2m} \cos(\omega t - \alpha)$ and $i_2 = \frac{e_2}{R} = \frac{\omega\varphi 2m}{R} \cos(\omega t - \alpha)$

Substituting these values of i_1 and i_2 in Eq. (2.48), we get *kw*

$$T = \frac{k_1\omega}{R}\left[\varphi_{1m} \sin\omega t \cdot \varphi_{2m} \cos(\omega t - \alpha) - \varphi_{2m} \sin(\omega t - \alpha) \cdot \varphi_{1m} \cos\omega t\right]$$

$$= \frac{k_1\omega}{R}\varphi_{1m}\varphi_{2m}\left[\sin\omega t \cdot \cos(\omega t - \alpha) - \sin(\omega t - \alpha) \cdot \cos\omega t\right]$$

$$= \frac{k_1\omega}{R}\varphi_{1m}\varphi_{2m} \sin\alpha = k_2\omega\varphi_{1m}\varphi_{2m} \sin\alpha \quad \left[\text{putting } \frac{k_1}{R} = k_2\right] \qquad (2.51)$$

The following is observed:

1. If $\alpha = 0$, i.e., if two fluxes are in phase, then net torque is zero. If, on the other hand, $a = 90°$, the net torque is maximum for a given values of φ_{1m} and φ_{2m}.

2. The net torque is such a direction as to rotate the disc from the pole with leading flux, towards the pole with lagging flux.

3. Since the expression for torque does not involve t, it is independent of time, i.e., it has a steady value at all times.

4. The torque T is inversely proportional to R; the resistance of the eddy current path. Hence, it is made of copper or more often, of aluminium.

2.12 ELECTROTHERMAL INSTRUMENTS

Mainly there are two types of thermal instruments:

- Hot-wire type
- Thermocouple instrument

Hot-wire and thermocouple meter movements use the heating effect of current flowing through a resistance to cause meter deflection. Each uses this effect in a different manner. Since their operation depend only on the heating effect of current flow, they may be used to measure both direct and alternating currents of any frequency on a single scale.

2.12.1 Hot-wire Instrument

The hot-wire meter movement deflection depends on the expansion of a high resistance wire caused by the heating effect of the wire itself as current flows through it. A resistance wire is stretched between the two meter terminals, with a thread attached at a right angles to the centre of the wire. A spring connected to the opposite end of the thread exerts a constant tension on the resistance wire. Current flow heats the wire, causing it to expand. This motion is transferred to the meter pointer through the thread and a pivot. Figure 2.23 shows the basic arrangement of a hot wire type instrument.

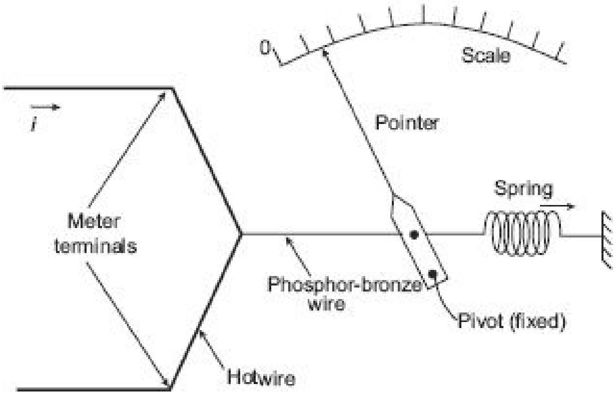


Figure 2.23 *Hot-wire instruments*

Advantages of Hot-wire-type Instruments

1. The deflection depends upon only the rms value of the current flowing through the wire, irrespective if its waveform and frequency. Hence, the instrument can used for ac as well as dc system.

2. The calibration is same for ac as well as dc measurement. So it is a transfer-type instrument.

3. They are free from stray magnetic fields because no magnetic field is used to cause their operation.

4. It is cheap in cost and simple in construction.

5. With suitable adjustments, error due to temperature variation can be made negligible.

6. This type of instruments are quite suitable for very high frequency measurement.

Disadvantages of Hot-wire-type Instruments

1. Power consumption is relatively high.
2. Nonuniform scale.
3. These are very sluggish in action as time is taken in heating up the wire.
4. The deflection of the instrument is not the same for ascending and descending values.
5. The reading depends upon the atmospheric temperature.

2.12.2 Thermocouple-Type Instrument

When two metals having different work functions are placed together, a voltage is generated at the junction which is nearly proportional to the temperature of the junction. This junction is called a thermocouple. This principle is used to convert heat energy to electrical energy at the junction of two conductors as shown in Figure 2.24.

The heat at the junction is produced by the electrical current flowing in the heater element while the thermocouple produces an emf at its output terminals, which can be measured with the help of a PMMC meter. The emf produced is proportional to the temperature and hence to the rms value of the current. Therefore, the scale of the PMMC instrument can calibrate to read the current passing through the heater. The thermocouple type of instrument can be used for both ac and dc applications. The most effective feature of a thermocouple instrument is that they can be used for measurement of current and voltages at very high frequency. In fact, these instruments are very accurate well above a frequency of 50 MHz.

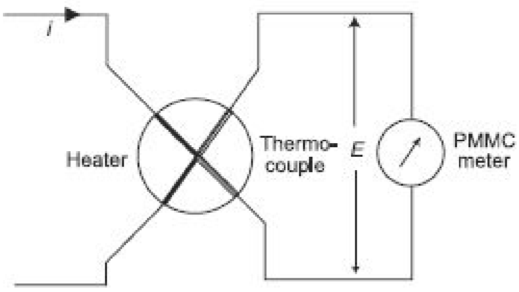


Figure 2.24 *Circuit diagram of thermocouple instrument*

Advantages of Thermocouple-type Instruments

1. These are not affected by stray magnetic fields.
2. They have very high sensitivity.
3. The indication of these instruments are practically unaffected by the frequency and waveform of the measuring quantity. Hence these instruments can be used for measurement of currents upto frequencies of 50 MHz and give accuracy as high as 1%.
4. These instruments are very useful as transfer instruments for calibration of dc instruments by potentiometer and a standard cell.

Disadvantages of Thermocouple-Type Instruments

1. Considerable power losses due to poor efficiency of thermal conversion.

2. Low accuracy of measurement and sensitivity to overloads, as the heater operates at temperatures close to the limit values. Thus, the overload capacity of such instrument is approximately 1.5 times of full-scale current.

3. The multi-voltmeters used with thermo-elements must be necessarily more sensitive and delicate than those used with shunts, and therefore, requires careful handling.

2.13 RECTIFIER-TYPE INSTRUMENTS

The basic arrangement of a rectifier type of instrument using a full-wave rectifier circuit is shown in Figure 2.25. If this instrument is used for measuring ac quantity then first the ac signal is converted to dc with the help of the rectifier. Then this dc signal is measured by the PMMC meter. The multiplier resistance R_s, is used to limit the value of the current in order that it does not exceed the current rating of the PMMC meter.

These types of instruments are used for light current work where the voltage is low and resistances high.

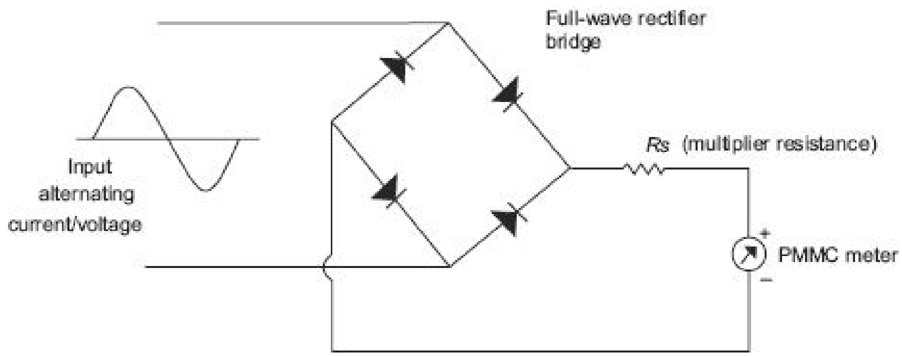


Figure 2.25 *Rectifier-type instrument*

2.13.1 Sensitivity of Rectifier-Type Instrument

The dc sensitivity of a rectifier-type instrument is

$S_{dc} = \frac{1}{I_{fs}}\ \Omega/v$ where I_{fs} is the current required to produce full-scale deflection.

1. *Sensitivity of a Half-wave Rectifier Circuit*

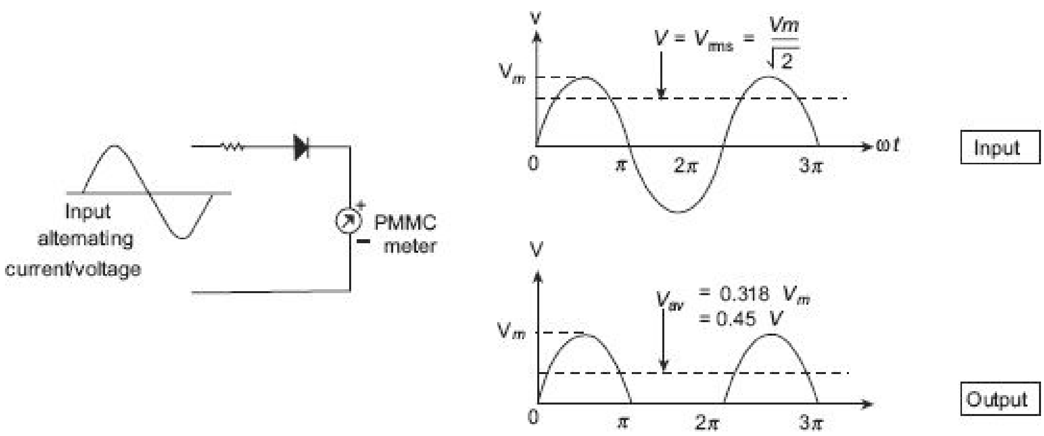


Figure 2.26 *Half-wave rectifier*

Figure 2.26 shows a simple half-wave rectifier circuit along with the input and output waveform. The average value of voltage/current for half-wave rectifier,

$$V_{av} = \frac{1}{2\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{V_m}{\pi} = 0.318 V_m = 0.45 \text{ V} \quad (2.52)$$

Hence, the sensitivity of a half-wave rectifier instrument with ac is 0.45 times its sensitivity with dc and the deflection is 0.45 times that produces with dc of equal magnitude V.

$$S_{ac} = 0.45 S_{dc} \quad (2.53)$$

2. *Sensitivity of a Full-wave Rectifier Circuits*

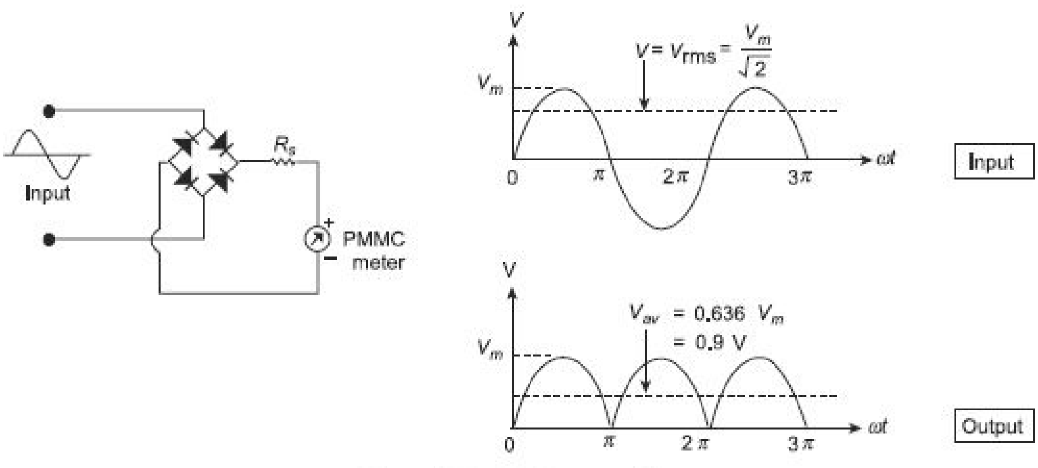


Figure 2.27 *Full-wave rectifier*

Figure 2.27 *Full-wave rectifier*

Figure 2.27 shows a full-wave rectifier circuit along with the input and output waveform. Average value of voltage/current for full-wave rectifier,

$$V_{av} = \frac{1}{\pi}\int_0^{\pi} V_m \sin\omega t \, d\omega t = \frac{2V_m}{\pi} = 0.636 V_m = 0.9 \text{ V} \quad (2.54)$$

So the deflection is 0.9 times in a full-wave rectifier instrument with an ac than that produced with dc of equal magnitude V.

Sensitivity of a full-wave rectifier instrument with an ac is 0.9 times its sensitivity with dc.

$$S_{ac} = 0.9 S_{dc} \quad (2.55)$$

2.13.2 Extension of Range of Rectifier Instrument as Voltmeter

Suppose it is intended to extend the range of a rectifier instrument which uses a PMMC instrument having a dc sensitivity of S_{dc}.

Let, v = voltage drop across the PMMC instrument

V = applied voltage

Therefore, for dc operation, the values of series resistance (multiplier) needed can be calculated from Figure 2.28 as

$$V = R_S \cdot I_{fs} + R_d \cdot I_{fs} + R_m \cdot I_{fs}$$

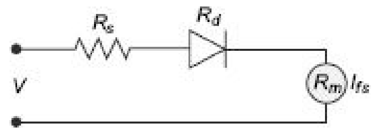


Figure 2.28 *Range extension of rectifier voltmeter*

$$R_s = \left(\frac{V}{I_{fs}}\right) - R_m - R_d$$
$$= S_{dc}V - R_m - R_d \text{ (for half-wave rectification)} \quad (2.56)$$
$$= S_{dc}V - R_m - 2R_d \text{ (for full-wave rectification)}$$

where R_m = meter resistance

R_d = diode forward resistance

For ac voltmeter,

$$R_s = S_{ac}V - R_m - R_d = 0.45S_{dc}V - R_m - R_d \text{ (for half-wave)}$$
$$= S_{ac}V - R_m - 2R_d = 0.9\,S_{dc}V - R_m - R_d \text{ (for full-wave)} \quad (2.57)$$

Limitations

1. Rectifier instruments are only accurate on the waveforms on which they are calibrated. Since calibration assumes pure sine waves, the presence of harmonics gives erroneous readings.
2. The rectifier is temperature sensitive, and therefore, the instrument readings are affected by large variations of temperature.

Applications

1. The rectifier instrument is very suitable for measuring alternating voltages in the range of 50–250 V.
2. The rectifier instrument may be used as a micrometer or low milliammeter (up to 10–15 mA). It is not suitable for measuring large currents because for larger currents the rectifier becomes too bulky and providing shunts is impracticable due to rectifier characteristics.
3. Rectifier instruments find their principal application in measurement in high-impedance circuits at low and audio frequencies. They are commonly used in communications circuits because of their high sensitivity and low power consumption.

2.14 TRUE rms VOLTMETER

The commonly available multimeters are average or peak reading instruments, and the rms values they display are based on the signal mean value. They multiply the average value with some factor to convert it to the rms reading. For this reason, conventional multimeters are only suited for sinusoidal signals. For measuring rms value of a variety of signals over a wide range of frequencies, a new kind of voltmeter—called the True RMS (TRMS) voltmeter has been developed. Since these voltmeters do not measure rms value of a signal based on its average value, they are suited for any kind of waveforms (such as sine wave, square wave or sawtooth wave).

The conventional moving-iron voltmeter has its deflection proportional to the square of the current passing through its coil. Thus, if the scale is calibrated in terms of square root of the measured value, moving-iron instruments can give true rms value of any signal, independent of its wave shape. However, due to large inertia of the mechanical moving parts present in such a moving iron instrument, the frequency bandwidth of such a true rms voltmeter is limited. Similar is the case for electrodynamometer type instruments which once again have their deflecting torque proportional to the current through their operating coil. But once again, though electrodynamometer-type instruments can give true rms indication of a signal of any waveform, their frequency bandwidth is also limited due to their mechanical moving parts.

Modern-day true rms reading voltmeters are made to respond directly to the heating value of the input signal. To measure rms value of any arbitrary waveform signal, the input signal is fed to a heating element and a thermocouple is placed very close to it. A thermocouple is a junction of two dissimilar metals whose contact potential is a function of the temperature of the junction. The heating value is proportional to the square of the rms value of the input signal. The heater raises the temperature of the heater and the thermocouple produces an output voltage that is proportional to the power delivered to the heater by the input signal. Power being proportional to the square of the current (or voltage) under measurement, the output voltage of the thermocouple can be properly calibrated to indicate true rms value of the input signal. This way, such a thermal effect instrument permits the determination of true rms value of an unknown signal of any arbitrary waveform. Bandwidth is usually not a problem since this kind of principle can be used accurately even beyond 50 MHz. Figure 2.28 shows such an arrangement of thermocouple based true rms reading voltmeter.

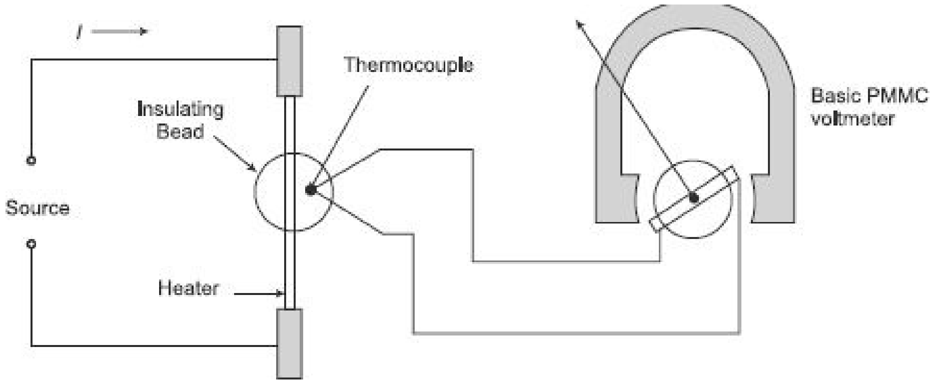


Figure 2.28 *Thermocouple based true rms reading voltmeter*

2.15 COMPARISON BETWEEN DIFFERENT TYPES OF INSTRUMENTS

Sl. No.	Type of Instruments	Suitability for type of measurement	Type of control	Type of damping	Specialty
1.	*Moving Coil* (i) PMMC	dc measurement (current and voltage only)	Spring	Eddy current	It is most accurate type for dc measurements and most widely used for measurement of dc voltage, current and resistance.
	(ii) Dynamometer	dc or ac measurement (current, voltage and power)	Spring	Air friction	Mainly used as wattmeter. Also used as standard meter for calibration and as transfer instrument.
2.	*Moving Iron*	dc or ac measurement (current, voltage)	Spring or gravity control	Air friction	It is cheaper to manufacture and mostly used as an indicating instrument. It is very accurate for ac and dc, if properly designed.
3.	*Electrostatic*	dc or ac (voltage only)	Gravity or spring	Air friction	These instruments have very low power consumption and can be made to cover a large range of voltage. Usually, range is above 500 volts.
4.	*Induction*	ac measurement (current, voltage, Power and energy) only.	Spring	Eddy current	Ammeters and voltmeters of this type are expensive and not of high degree of accuracy. These instruments are mainly used for measurement of power and energy in ac circuits.
5.	*Thermal* (i) Hot wire	dc or ac measurement (current, voltage and power)	Spring	Eddy current	These instrument have same calibration for both ac and dc. These are free from errors due to frequency, wave form and external field when used on ac, therefore, these are particularly used for ac measurement.
	(ii) Thermocouple	dc or ac measurement (current and voltage)			These are free from errors due to frequency, wave form and external field when used on ac and are used for measurement of current and voltage at power frequencies upto 100 MHz.
6.	*Rectifier*	dc or ac measurement (current and voltage)	Spring	Eddy current	These instruments are nothing but permanent magnet moving coil instruments used in conjunction with rectifying device for AC measurements (current and voltage) from about 20 Hz to 20 kHz.

EXERCISE

Objective-type Questions

1. A spring produces a controlling torque of $16x10^{-6}$ Nm for a deflection of 120Y. If the width and length become two times their original values and the thickness is halved, the value of controlling torque for the same deflection will be

   (a) 16×10^{-6} N

   (b) $8x10^{-6}$ Nm (c)$2x10^{-6}$ Nm

   (d) $32x10^{-6}$ Nm

2. The shunt resistance in an ammeter is usually

   (a) less than meter resistance

   (b) equal to meter resistance

   (c) more than meter resistance

   (d) of any value

3. A voltage of 200 V produces a deflection of 90° in a PMMC spring-controlled instrument. If the same instrument is provided with gravity control, what would be the deflection?

(a) 45°

(b) 65°

(c) 90°

(d) cannot be determined by the given data

4. A current-carrying conductor is shown in Figure (a). If it is brought in a magnetic field shown in Figure (b)

   (a) it will experience a force from left to right.

   (b) it will experience a force from right to left.

   (c) it will experience a force from top to bottom.

   (d) it will experience no force.

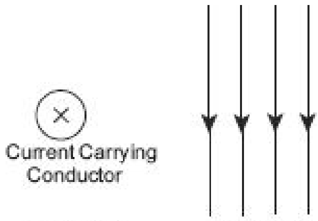


Figure (a) Figure (b)

5. The high torque to weight ratio in an analog indicating instrument indicates

   (a) high friction loss

   (b) nothing as regards friction loss

   (c) low friction loss

   (d) none of the above

6. Swamping resistance is connected

   (a) in series with the shunt to reduce temperature error in shunted ammeter

   (b) in series with the ammeters to reduce errors on account of friction

   (c) in series with meter and have a high resistance temperature coefficient in order to reduce temperature errors in ammeters.

   (d) in series with the meter and have a negligible resistance co-efficient in order to reduce temperature errors in shunted ammeters

7. Moving-iron instruments when measuring voltages or currents

   (a) indicate the same values of the measurement for both ascending and descending values

   (b) indicate higher value of measurand for ascending values

   (c) indicate higher value of measurand for descending values

   (d) none of the above

8. A moving-iron type of instrument can be used as

   (a) standard instruments for calibration of other instruments

   (b) transfer-type instruments

   (c) indicator-type instruments as on panels

   (d) all of the above

9. In spring-controlled moving iron instruments, the scale is

   (a) uniform

   (b) cramped at the lower end and expanded at the upper end

   (c) expanded at the lower end and cramped at the upper end

   (d) cramped both at the lower and the upper ends

10. Thermocouple instruments can be used for a frequency range

(a) up to 500 Hz

(b) up to 5 MHz

(c) up to 100 Hz

(d) up to 1 MHz

11. The reason why eddy-current damping cannot he used in a moving-iron instrument, is

(a) they have a strong operating magnetic field

(b) they are not normally used in vertical position

(c) they need a large damping force which can only he provided by air friction

(d) they have a very weak operating magnetic field and introduction of a permanent magnet required for eddy current damping would distort the operating magnetic field

12. An electrodynamometer type of instrument finds its major use as

(a) standard instrument only

(b) both as standard and transfer instrument

(c) transfer instrument only

(d) indicator-type instrument

13. The frequency range of moving-iron instruments is

(a) audio-frequency band 20 Hz to 20 kHz

(b) very low-frequency band 10 Hz to 30 kHz

(c) low-frequency band 30 Hz to 300 kHz

(d) power frequencies 0 to 125 Hz.

14. A voltage of 200 V at 5 Hz is applied to an electrodynamometer type of instrument which is spring controlled. The indication on the instruments is

(a) 200 V

(b) 0 V

(c) the instrument follows the variations in voltage and does not give a steady response

(d) none of the above

15. Spring-controlled moving-iron instruments exhibit a square law response resulting in a non-linear scale. The shape of the scale can be made almost linear by

(a) keeping rate of change of inductance, L, with deflection, θ, as constant

(b) keeping $\frac{1}{\theta}.\frac{dL}{d\theta}$ as constant

(c) keeping $\theta.\frac{dL}{d\theta}$ as constant

(d) keeping $\frac{1}{k\theta}$ as constant, where k is the spring constant

16. Electrostatic-type instruments are primarily used as

(a) ammeters

(b) voltmeters

(c) wattmeters

(d) ohmmeters

17. The sensitivity of a PMMC instrument is 10CkZ/V. If this instrument is used in a rectifier-type voltmeter with half wave rectification. What would be the sensitivity?

(a) 10 kΩ/V

(b) 4.5 kΩ/V

(c) 9 kΩ/V

(d) 22.2 kΩ/V

18. The heater wire of thermocouple instrument is made very thin in order

(a) to have a high value of resistance

(b) to reduce skin effects at high frequencies

(c) to reduce the weight of the instrument

(d) to decrease the over-ranging capacity of the instrument

19. Which instrument has the highest frequency range with accuracy within reasonable limits?

(a) PMMC

(b) Moving iron

(c) Electrodynamometer

(d) Rectifier

20. Which meter has the highest accuracy in the prescribed limit of frequency range?

(a) PMMC

(b) Moving iron

(c) Electrodynamometer

(d) Rectifier

Answers

1. (c)	2. (a)	3. (c)	4. (d)	5. (c)	6. (d)	7. (c)
8. (c)	9. (b)	10. (d)	11. (d)	12. (b)	13. (d)	14. (c)
15. (c)	16. (b)	17. (b)	18. (b)	19. (d)	20. (a)	

Short-answer Questions

1. Describe the various operating forces needed for proper operation of an analog indicating instrument.
2. Sketch the curves showing deflection versus time for analog indicating instruments for underdamping, critical damping and overdamping.
3. What are the difference between recording and integrating instruments? Give suitable examples in each case.
4. Derive the equation for deflection of a PMMC instrument if the instrument is spring controlled.
5. How can the current range of a PMMC instrument be extended with the help of shunts?
6. Derive the equation for deflection of a spring-controlled moving-coil instrument.
7. Describe the working principle of a rectifier-type instrument. What is the sensitivity of such an instrument?
8. What are the advantages and disadvantages of a PMMC instrument?
9. Describe the working principle and constructional details of an attraction-type moving iron instrument.
10. Derive the expression for deflection for a rotary-type electrostatic instrument using spring control.
11. What is swamping resistance? For what purpose is swamping resistance used?
12. How many ways can the damping be provided in an indicating instrument?

Long-answer Questions

1. (a) How many operating forces are necessary for successful operation of an indicating instrument? Explain the methods of providing these forces.

(b) A moving-coil instrument has the following data: number of turns = 100, width of coil = 20 mm, depth of coil = 30 mm, flux density in the gap = 0.1 Wb/m^2. Calculate the deflecting torque when carrying a current of 10 mA. Also calculate the deflection if the control spring constant is 2×10^{-6} N-m/degree.

[Ans. 60×10^{-6} Nm, 30°]

2. (a) What are the advantages and disadvantages of moving-coil instruments?

(b) A moving-coil voltmeter has a resistance of 200 W and the full scale deflection is reached when a potential difference of 100 mV is applied across the terminals. The moving coil has effective dimensions of 30 mm × 25 mm and is wound with 100 turns. The flux density in the gap is 0.2 Wb/m^2. Determine the control constant of the spring if the final deflection is 100° and a suitable diameter of copper wire for the coil winding if 20% of the total instrument resistance is due to the coil winding. Resistivity of copper is 1.7 × 10^{-8} Ωm.

[Ans. 0.075 × 10^{-6} Nm/degree; 0.077 mm]

3. (a) Derive the expression for the deflection of a spring controlled permanent magnet moving coil instrument. Why not this instrument able to measure the ac quantity?

(b) The coil of a moving coil voltmeter is 40 mm × 30 mm wide and has 100 turns wound on it. The control spring exerts a torque of 0.25 × 10^{-3} Nm when the deflection is 50 divisions on the scale. If the flux density of the magnetic field in the air-gap is 1 Wb/m^2, find the resistance that must be put in series with the coil to give 1 volt per division. Resistance of the voltmeter is 10000 Ω.

[Ans. 14000 Q]

4. (a) A moving-coil instrument has at normal temperature a resistance of 10 W and a current of 45 mA gives full scale deflection. If its resistance rises to 10.2 Ω due to temperature change, calculate the reading when a current of 2000 A is measured by means of a 0.225 × 10^{-3}. A shunt of constant resistance. What is the percentage error?

[Ans. 44.1 mA, -1.96%]

(b) The inductance of a certain moving-iron ammeter is $(8+4\theta-\frac{1}{2}\theta^2)$ pH, where *0* is the deflection in radian from the zero position. The control spring torque is 12 × 10^{-6} Nm/rad. Calculate the scale position in radian for current of 5 A.

[2.04 rad]

5. (a) Discuss the constructional details of a thermocouple-type instrument used at very high frequencies. Write their advantages and disadvantages.

(b) The control spring of a moving-iron ammeter exerts a torque of 0.5 × 10^{-} Nm/degree when the deflection is 52°. The inductance of the coil varies with pointer deflection according to

deflection (degree) 20 40 60 80

inductance (|mH) 659 702 752 792

Determine the current passing through the meter.

[0.63 A]

6. (a) Describe the constructional details of an attraction-type moving iron instrument with the help of a neat diagram. Derive the equation for deflection if spring control is used and comment upon the shape of scale.

(b) Derive a general equation for deflection for a spring-controlled repulsion-type moving-iron instrument. Comment upon the share of the scale. Explain the methods adopted to linearise the scale.

4 Measurement of Resistance

4.1 INTRODUCTION

Resistors are used in many places in electrical circuits to perform a variety of useful tasks. Properties of resistances play an important role in determining performance specifications for various circuit elements including coils, windings, insulations, etc. It is important in many cases to have reasonably accurate information of the magnitude of resistance present in the circuit for analysing its behaviour. Measurement of resistance is thus one of the very basic requirements in many working circuits, machines, transformers, and meters. Apart from these applications, resistors are used as standards for the measurement of other unknown resistances and for the determination of unknown inductance and capacitance.

From the point of view of measurement, resistances can be classified as follows:

1. Low Resistances

All resistances of the order less than 1 Ω may be classified as low resistances. In practice, such resistances can be found in the copper winding in armatures, ammeter shunts, contacts, switches, etc.

2. Medium Resistances

Resistances in the range 1 Ω to 100 kΩ may be classified as medium resistances. Most of the electrical apparatus used in practice, electronic circuits, carbon resistance and metal-film resistors are found to have resistance values lying in this range.

3. High Resistances

Resistances higher than 100 kΩ are classified as high resistances. Insulation resistances in electrical equipment are expected to have resistances above this range.

The above classifications are, however, not rigid, but only form a guideline for the method of measurement to be adopted, which may be different for different cases.

4.2 MEASUREMENT OF MEDIUM RESISTANCES

The different methods for measurement of medium range resistances are (i) ohmmeter method, (ii) voltmeter–ammeter method, (iii) substitution method, and (iv) Wheatstone-bridge method.

4.2.1 Ohmmeter Method for Measuring Resistance

Ohmmeters are convenient direct reading devices for measurement of approximate resistance of circuit components without concerning too much about accuracy. This instrument is, however, very popular in the sense that it can give quick and direct readings for resistance values without any precise adjustments requirements from the operator. It is also useful in measurement laboratories as an adjunct to a precision bridge. Value of the unknown resistance to be measured is first obtained by the ohmmeter, and this can save lot of time in bridge balancing for obtaining the final precision value using the bridge.

Series-type Ohmmeter

Figure 4.1 shows the elements of a simple single-range series-type ohmmeter.

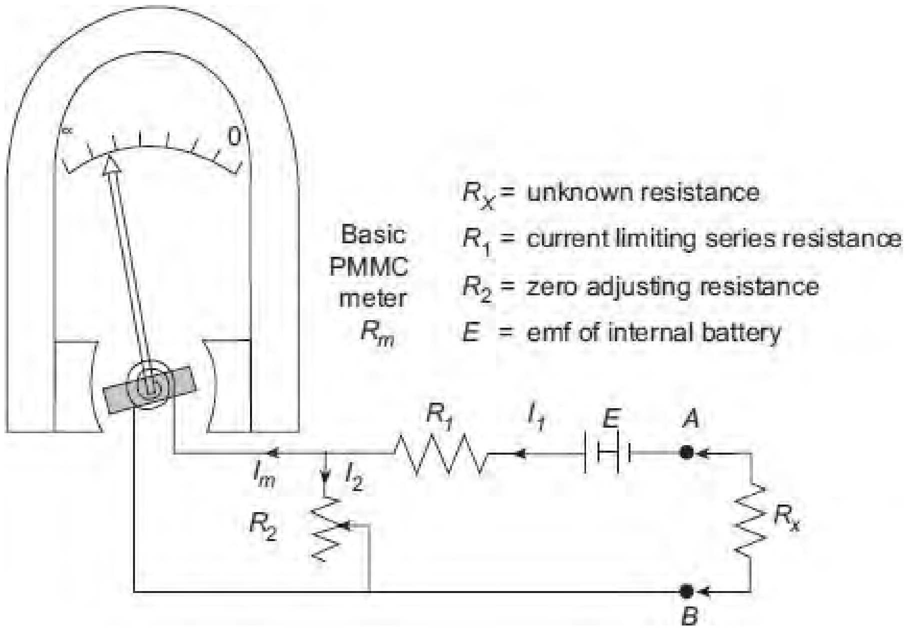


Figure 4.1 *Single-range series ohmmeter*

The series-type ohmmeter consists basically of a sensitive dc measuring PMMC ammeter connected in parallel with a variable shunt R_2. This parallel circuit is connected in series with a current limiting resistance R_1 and a battery of emf E. The entire arrangement is connected to a pair of terminals *(A–B)* to which the unknown resistance R_x to be measured is connected.

Before actual readings are taken, the terminals *A–B* must be shorted together. At this position with $R_x = 0$, maximum current flows through the meter. The shunt resistance R_2 is adjusted so that the meter deflects corresponding to its right most full scale deflection (FSD) position. The FSD position of the pointer is marked 'zero-resistance', i.e., 0 Ω on the scale. On the other hand, when the terminals A–B are kept open ($R_x \rightarrow \infty$), no current flows through the meter and the pointer corresponds to the left most zero current position on the scale. This position of the pointer is marked as '∞Ω' on the scale. Thus, the meter will read infinite resistance at zero current position and zero resistance at full-scale current position. Series ohmmeters thus have '0' mark at the extreme right and '∞' mark at the extreme left of scale (opposite to those for ammeters and voltmeters).

The main difficulty is the fact that ohmmeters are usually powered by batteries, and the battery voltage gradually changes with use and age. The shunt resistance R_2 is used in such cases to counteract this effect and ensure proper zero setting at all times.

For zero setting, $R_x = 0$, where R_m = internal resistance of the basic PMMC meter coil

$\therefore$ equivalent resistance of the circuit $R_{eq} = R_1 + \dfrac{R_2 R_m}{R_2 + R_m}$

And, total current $I_1 = \dfrac{E}{R_{eq}} = I_2 + I_m$

The current I_2 can be adjusted by varying R_2 so that the meter current I_m can be held at its calibrated value when the main current I_1 changes due to drop in the battery emf E.

> If R_2 were not present, then it would also have been possible to bring the pointer to full scale by adjustment of the series resistance R_1, But this would have changed the calibration all along the scale and cause large error..

(i) Design of R_1 and R_2 The extreme scale markings, i.e., 0 and ∞, in an ohmmeter do not depend on the circuit constants. However, distributions of the scale markings between these two extremes are affected by the constants of the circuit. It is thus essential to design for proper values of the circuit constants, namely, R_1 and R_2 in particular to have proper calibration of the scale. The following parameters need to be known for determination of R_1 and R_2.

- Meter current I_m at full scale deflection (= I_{FSD}.)
- Meter coil resistance, R_m
- Ohmmeter battery voltage, E
- Value of the unknown resistance at half-scale deflection, (R_h), i.e., the value of R_x when the pointer is at the middle of scale

With terminals A–B shorted, when $R_x = 0$

Meter carries maximum current, and current flowing out of the battery is given as $I_{1MAX} = \dfrac{E}{R_i}$

where Ri = internal resistance of the ohmmeter $= R_1 + \dfrac{R_2 R_m}{R_2 + R_m}$

At half-scale deflection, $R_x = R_h$, and $I_h = \dfrac{I_{1MAX}}{2} = \dfrac{E}{R_i + R_h}$

$$\therefore \qquad \frac{E}{R_i + R_h} = \frac{E}{2R_i}$$

$$\text{or,} \qquad R_i = R_h$$

$$\therefore \qquad I_h = \frac{E}{2R_h} \text{ and } R_h = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

For full-scale deflection,

$$I_h = I_{FSD} \text{ and } I_1 = 2I_h = 2 \times \frac{E}{2R_h} = \frac{E}{R_h}$$

Also, $$I_m R_m = I_2 R_2 \text{ and } I_2 = I_1 - I_m$$

$$\therefore \text{At FSD, } I_{FSD} R_m = R_2 (I_1 - I_m) = R_2 \left(\frac{E}{R_h} - I_{FSD} \right)$$

Thus,

$$R_2 = \frac{I_{FSD} R_m R_h}{(E - I_{FSD} R_h)} \quad (4.1)$$

Again, since $R_h = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$, putting the value of R_2 from (4.1), we get

$$R_1 = R_h - \frac{I_{FSD} R_m R_h}{E} \quad (4.2)$$

(ii) . Shape of Scale in Series Ohmmeters Electrical equivalent circuit of a series-type ohmmeter is shown in Figure 4.2.

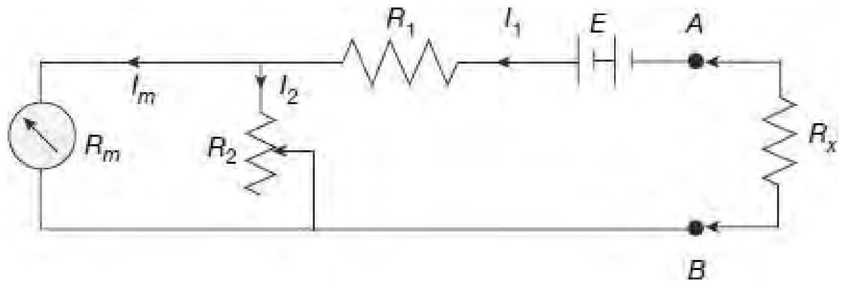


Figure 4.2 *Electrical equivalent circuit of a series-type ohmmeter*

Internal resistance of the ohmmeter

$$R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m}$$

Thus, $$I_1 = \frac{E}{R_i + R_x}$$

Meter current $$I_m = I_1 \times \frac{R_2}{R_2 + R_m} = \frac{E}{R_i + R_x} \times \frac{R_2}{R_2 + R_m}$$

With the terminals *A–B* short circuited $R_x = 0$; thus, from (4.3) we have

$$I_{FSD} = \frac{E}{R_i} \times \frac{R_2}{R_2 + R_m} \quad (4.4)$$

From (4.3) and (4.4), the meter can be related to the FSD as

$$\frac{I_m}{I_{FSD}} = \frac{\frac{E}{R_i + R_x} \times \frac{R_2}{R_2 + R_m}}{\frac{E}{R_i} \times \frac{R_2}{R_2 + R_m}} = \frac{R_i}{R_i + R_x}$$

Thus,

$$I_m = \frac{R_i}{R_i + R_x} \times I_{FSD} \quad (4.5)$$

From Eq. (4.5), it can be observed that the meter current I_m is not related linearly with the resistance R_x to be measured. The scale (angle of deflection) in series ohmmeter if thus non-linear and cramped.

The above relation (4.5) also indicates the fact that the meter current and hence graduations of the scale get changed from the initial calibrated values each time the shunt resistance R_2 is adjusted. A superior design is found in some ohmmeters where an adjustable soft-iron shunt is placed across the pole pieces of the meter, as indicated in Figure 4.3.

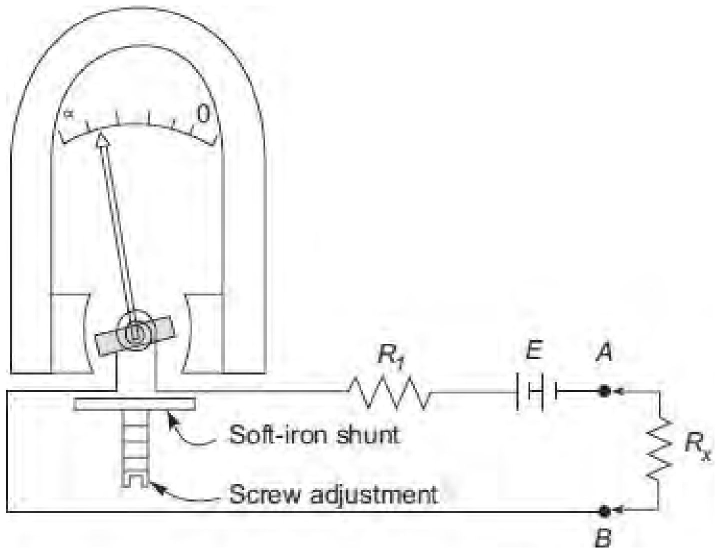


Figure 4.3 *Series ohmmeter with soft-iron magnetic shunt*

The soft-iron magnetic shunt, when suitably positioned with the help of screw adjustment, modifies the air gap flux of main magnet, and hence controls sensitivity of movement. The pointer can thus be set at proper full scale marking in compensation against changes is battery emf, without any change in the electrical circuit. The scale calibrations thus do not get disturbed when the magnetic shunt is adjusted.

1. Multi-range Series Ohmmeter

For most practical purposes, it is necessary that a single ohmmeter be used for measurement of a wide range of resistance values. Using a single scale for such measurements will lead to inconvenience in meter readings and associated inaccuracies. Multi-range ohmmeters, as shown schematically in Figure 4.4, can be used for such measurements. The additional shunt resistances R_3, R_4…, R_7 are used to adjust the meter current to correspond to 0 to FSD scale each time the range of the unknown resistance R_x is changed. In a practical multi-range ohmmeter, these shunt resistances are changed by rotating the range setting dial of the ohmmeter. The photograph of such a laboratory grade analog multi-range ohmmeter is provided in Figure 4.5.

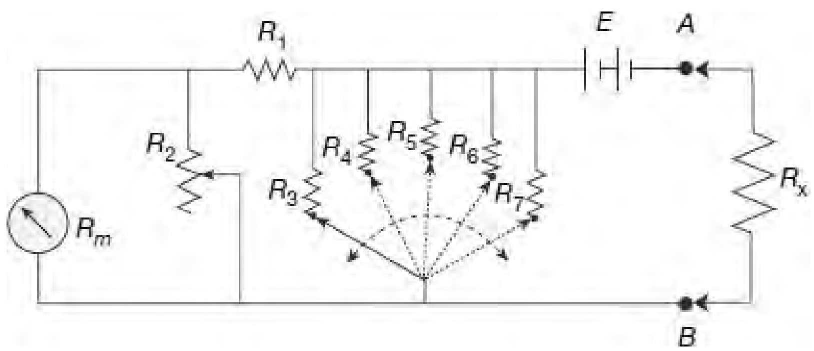


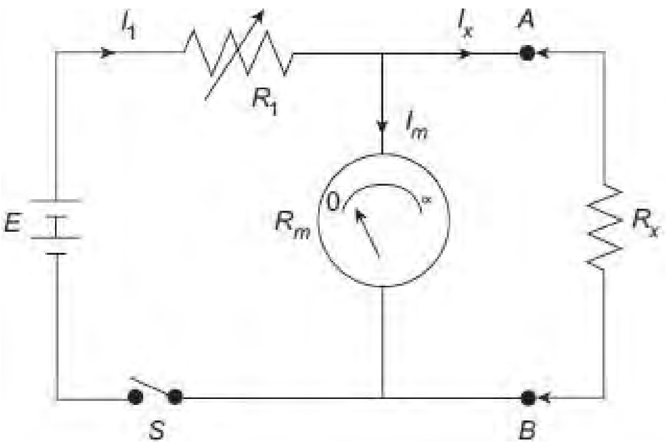
Figure 4.4 *Multi-range series-type ohmmeter*

Figure 4.5 *Photograph of multi-range ohmmeter (Courtesy, SUNWA)*

2. Shunt-type Ohmmeter

Figure 4.6 shows the schematic diagram of a simple shunt-type ohmmeter.

The shunt-type ohmmeter consists of a battery in series with an adjustable resistance R_1 and a sensitive dc measuring PMMC ammeter. The unknown resistance R_x to be measured is connected across terminals A–B and parallel with the meter.



R_x = unknown resistance
R_1 = scale adjusting resistance
E = emf of internal battery
R_m = PMMC meter resistance
S = switch

Figure 4.6 *Shunt-type ohmmeter*

When the terminals A–B are shorted (R_x = 0), the meter current is zero, since all the current in the circuit passes through the short circuited path A–B, rather than the meter. This position of the pointer is marked 'zero-resistance', i.e., '0 Ω' on the scale. On the other hand, when R_x is removed, i.e., the terminals *A–B* open circuited ($R_x \rightarrow \infty$), entire current flows through the meter. Selecting proper value of R_1, this maximum current position of the pointer can be made to read full scale of the meter. This position of the pointer is marked as '∞Ω' on the scale. Shunt type ohmmeters, accordingly, has '0 Ω' at the left most position corresponding to zero current, and '∞Ω' at the rightmost end of the scale corresponding to FSD current.

When not under measurement, i.e., nothing is connected across the terminals *A–B* ($R_x \rightarrow \infty$) the battery always drives FSD current through the meter. It is thus essential to disconnect the battery from rest of the circuit when the meter is idle. A switch S, as shown in Figure 4.6, is thus needed to prevent the battery from draining out when the instrument is not in use.

Shape of Scale in Shunt Ohmmeters Internal resistance of the ohmmeter

$$R_i = \frac{R_1 R_m}{R_1 + R_m}$$

With terminals A–B open, the full-scale current through the meter is

$$I_{FSD} = \frac{E}{R_1 + R_m} \quad (4.6)$$

With R_x connected between terminals *A-B*, the current out of the battery is

$$I_1 = \frac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}}$$

Thus, meter current $I_m = I_1 \times \dfrac{R_x}{R_x + R_m} = \dfrac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} \times \dfrac{R_x}{R_x + R_m}$ (4.7)

From Eqs (4.6) and (4.7), the meter can be related to the FSD as

$$\frac{I_m}{I_{FSD}} = \frac{\dfrac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} \times \dfrac{R_x}{R_x + R_m}}{\dfrac{E}{R_1 + R_m}} = \frac{R_x (R_1 + R_m)}{R_1 (R_m + R_x) + R_m R_x} = \frac{R_x (R_1 + R_m)}{R_1 R_m + R_x (R_1 + R_m)}$$

or, $$\frac{I_m}{I_{FSD}} = \frac{R_x}{\dfrac{R_1 R_m}{(R_1 + R_m)} + R_x} = \frac{R_x}{R_x + R_i}$$

$$I_m = \frac{R_x}{R_i + R_x} \times I_{FSD} \quad (4.8)$$

From Eq (4.8), it can be observed that the meter current I_m increases almost linearly with the resistance R_x to be measured for smaller values of R_x when $R_x << R_i$. The scale (angle of deflection) in shunt type ohmmeters is thus almost linear in the lower range, but progressively becomes more cramped at higher values of R_x. Shunt-type ohmmeters are thus particularly suitable for measurement of low resistances when the meter scale is nearly uniform.

Design a single-range series-type ohmmeter using a PMMC ammeter that has internal resistance of 50 Ω and requires a current of 1 mA for full-scale deflection.

Example 4.1 *The internal battery has a voltage of 3 V. It is desired to read half scale at a resistance value of 2000 Ω. Calculate (a) the values of shunt resistance and current limiting series resistance, and (b) range of values of the shunt resistance to accommodate battery voltage variation in the range 2.7 to 3.1 V.*

Solution Schematic diagram of the series ohmmeter with the given values is shown in the following figure

Given, R_m = meter internal resistance = 50 Ω

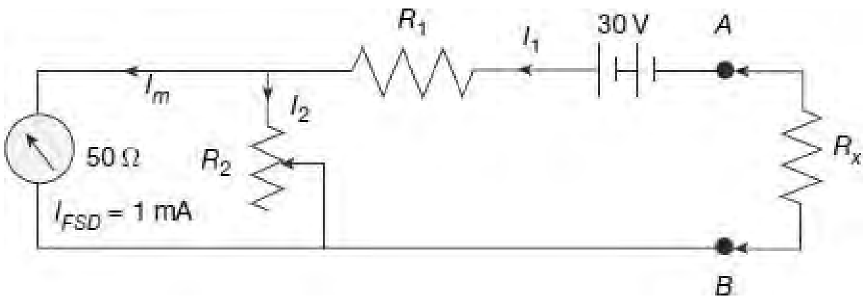
I_{FSD} = meter full scale deflection current = 1 mA

R_h = half-scale deflection resistance = 2000 W

E = battery voltage = 3 V

(a) With terminals A–B shorted, when $R_x = 0$

Meter carries maximum current, and current flowing out of the battery is given as $I_{1MAX} = \frac{E}{R_i}$,



where E = battery voltage = 3 V, and

R_i = internal resistance of the ohmmeter $= R_1 + \frac{R_2 R_m}{R_2 + R_m}$

At half-scale deflection, $R_x = R_h$, and battery current $I_h = \frac{I_{1MAX}}{2} = \frac{E}{R_i + R_h}$

$$\therefore \quad \frac{E}{R_i + R_h} = \frac{E}{2R_i}$$

$$\text{or,} \quad R_i = R_h$$

$$\therefore \quad I_h = \frac{E}{2R_h} = \frac{3}{2 \times 2000} = 0.75 \text{ mA}$$

$$\text{and} \quad R_h = 2000 = R_i = R_1 + \frac{R_2 R_m}{R_2 + R_m} = R_1 + \frac{50 R_2}{R_2 + 50}$$

For full-scale deflection,

$$\therefore \quad \text{At } FSD,\ R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times50}{0.5\times10^{-3}} = 100\ \Omega$$

Again, since $R_1 + \frac{50R_2}{R_2+50} = 2000$, putting the value of R_2

$$R_1 = 1966.7\ \Omega$$

(b) For a battery voltage of E = 2.7 V, battery current at half scale is

$$I_h = \frac{E}{2R_h} = \frac{2.7}{2\times2000} = 0.675\ \text{mA}$$

For full-scale deflection,

$$I_m = I_{FSD} = 1\text{mA and } I_1 = 2I_h = 1.35\ \text{mA}$$

Also, $I_mR_m = I_2R_2$ and $I_1 - I_m = (1.35\text{-}1)\ \text{mA} = 0.35\ \text{mA}$

$$\therefore \text{At } FSD,\ R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times50}{0.35\times10^{-3}} = 142.86\ \Omega$$

For a battery voltage of E = 3.1 V, battery current at half scale is

$$I_h = \frac{E}{2R_h} = \frac{3.1}{2\times2000} = 0.775\ \text{mA}$$

For full-scale deflection,

$$I_m = I_{FSD} = 1\text{mA and } I_1 = 2I_h = 1.55\ \text{mA}$$

Also, $I_mR_m = I_2R_2$ and $I_1 - I_m = (1.55\text{-}1)\ \text{mA} = 0.55\ \text{mA}$

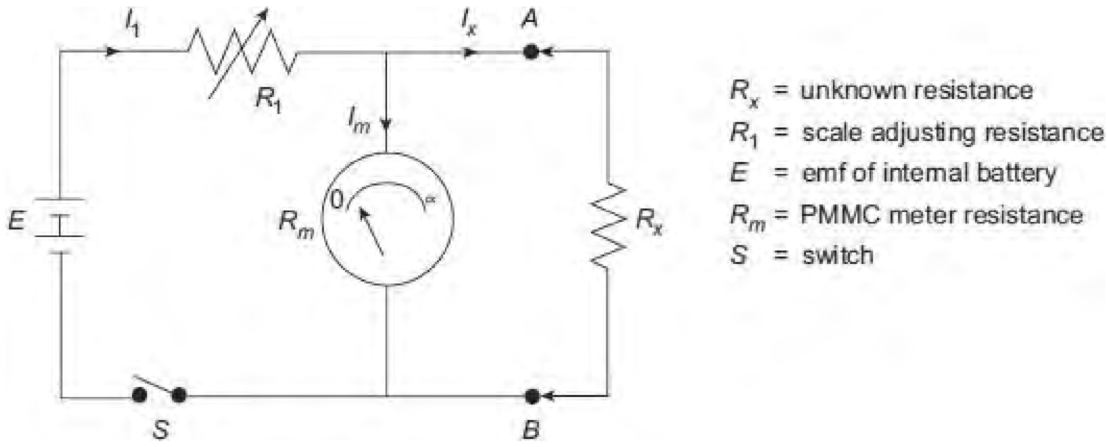
$$\therefore \text{At FSD, } R_2 = \frac{I_{FSD}R_m}{I_2} = \frac{1\times10^{-3}\times50}{0.55\times10^{-3}} = 90.0\ \Omega$$

∴ range of R_2 to accommodate the given change in battery voltage is 142.86 Ω > R_2 > 90.9 Ω.

Example 4.2

A shunt-type ohmmeter uses a 2 mA basic d'Arsonval movement with an internal resistance of 25 Ω. The battery emf is 1.5 V.
Calculate (a) value of the resistor in series with the battery to adjust the FSD, and (b) at what point (.percentage) of full-scale will 100 Ω be marked on the scale?

Solution Schematic diagram of a shunt type-ohmmeter under the condition as stated in Example 4.2 is shown below:



At FSD when terminals A–B is opened, meter FSD current is

$$I_m = I_{FSD} = \frac{E}{R_1 + R_m} = \frac{1.5}{R_1 + 25} = 2\times10^{-3}$$

$$\boxed{\text{Thus, } R_1 = 725\,\Omega}$$

When R_x = 100Ω, battery output current will be

$$I_1 = \frac{E}{R_1 + \dfrac{R_m R_x}{R_m + R_x}} = \frac{1.5}{725 + \dfrac{25\times100}{25+100}} = 2.013 \text{ mA}$$

∴ meter current is $I_m = I_1 \times \dfrac{R_x}{R_m + R_x} = 2.013 \times \dfrac{100}{25+100} = 1.6104 \text{ mA}$

Thus, percentage of full scale at which the meter would read 100 Ω is

$$\boxed{\frac{I_m}{I_{FSD}}\times100\% = \frac{1.6104}{2}\times100\% = 80.52\%}$$

4.2.2 Voltmeter–Ammeter Method for Measuring Resistance

The voltmeter–ammeter method is a direct application of ohm’s law in which the unknown resistance is estimated by measurement of current *(I)* flowing through it and the voltage drop (V) across it. Then measured value of the resistance is

$$R_m = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}} = \frac{V}{I}$$

This method is very simple and popular since the instruments required for measurement are usually easily available in the laboratory.

Two types of connections are employed for voltmeter–ammeter method as shown in Figure 4.7.

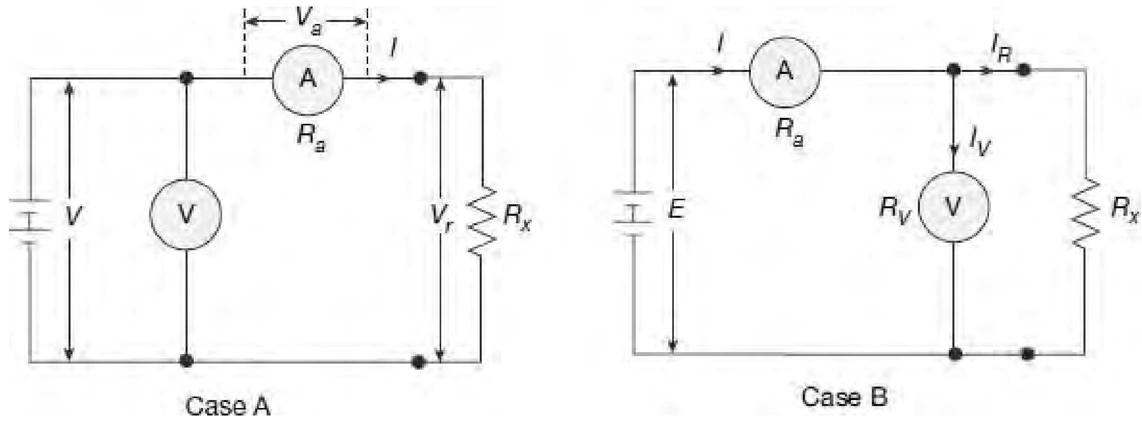


Figure 4.7 *Measurement of resistance by voltmeter–ammeter method*

R_x = true value of unknown resistance

R_m = measured value of unknown resistance

R_a = internal resistance of ammeter

R_V = internal resistance of voltmeter

It is desired that in both the cases shown in Figure 4.7, the measured resistance R_m would be equal to the true value R_x of the unknown resistance. This is only possible, as we will see, if the ammeter resistance is zero and the voltmeter resistance is infinite.

Case A

In this circuit, the ammeter is connected directly with the unknown resistance, but the voltmeter is connected across the series combination of ammeter and the resistance R_x. The ammeter measures the true value of current through the resistance but the voltmeter does not measure the true value of voltage across the resistance. The voltmeter measures the sum of voltage drops across the ammeter and the unknown resistance R_x.

Let, voltmeter reading = V

And, ammeter reading = I

∴ measured value of resistance $= R_m = \frac{V}{I}$

However, $\quad V = V_a + V_r$

or, $\quad V = I \times R_a + I \times R_x = I \times (R_a + R_x)$

Thus, $\frac{V}{I} = R_m = (R_a + R_x)$ (4.9)

The measured value R_m of the unknown resistance is thus higher than the true value R, by the quantity R_a, internal resistance of the ammeter. It is also clear from the above that true value is equal to the measured value only if the ammeter resistance is zero.

Error in measurement is $\varepsilon = \frac{R_m - R_x}{R_x} = \frac{R_a}{R_x}$

Equation (4.10) denotes the fact that error in measurement using connection method shown in Case A will be negligible only if the ratio $\frac{R_a}{R_x} \to 0$. In other words, if the resistance under measurement is much higher as compared to the ammeter resistance (R_x R_a.), then the connection method shown in Case A can be employed without involving much error.

Therefore, circuit shown in Case A should be used for measurement of high resistance values.

Case B

In this circuit, the voltmeter is connected directly across the unknown resistance, but the ammeter is connected in series with the parallel combination of voltmeter and the resistance R_x. The voltmeter thus measures the true value of voltage drop across the resistance but the ammeter does not measure the true value of current through the resistance. The ammeter measures the summation of current flowing through the voltmeter and the unknown resistance R_x.

Let, voltmeter reading = V

And, ammeter reading = I

Thus, $V = I_R \times R_X = I_V \times R_V$

However, $I = I_V + I_R$

∴ measured value of resistance

$$= R_m = \frac{V}{I} = \frac{V}{I_V + I_R} = \frac{V}{\frac{V}{R_V} + \frac{V}{R_x}} = \frac{R_V R_x}{R_V + R_x} = \frac{R_x}{1 + \frac{R_x}{R_V}}$$

or $$R_m = \frac{R_x}{1 + \frac{R_x}{R_V}} \qquad (4.11)$$

The measured value R_m of the unknown resistance is thus lower than the true value R_x by a quantity related to internal resistance of the voltmeter. It is also clear from Eq. (4.11) that true value is equal to the measured value only if the quantity $\frac{R_x}{R_V} \to 0$, i.e., if voltmeter resistance is infinite. In other words, if the voltmeter resistance is much higher as compared to the resistance under measurement ($R_V >> R_A$) then the connection method shown in Case B can be emploted without involving much error.

Therefore, circuit shown in Case B should be used for measurement of low resistance values.

Example 4.3

A voltmeter of 600 Ω resistance and a milliammeter of 0.8 Ω resistance are used to measure two unknown resistances by voltmeter–ammeter method.

If the voltmeter reads 40 V and milliammeter reads 120 mA in both the cases, calculate the percentage error in the values of measured resistances if (a) in the first case, the *voltmeter is put across the resistance and the milliammeter connected in series with the supply, and (b) in the second case, the voltmeter is connected in the supply side and milliammeter connected directly in series with the resistance.*

Solution The connections are shown in the following figure.

Voltmeter reading $V = 40$ V

Ammeter reading $I = 120$ mA

∴ measured resistance from voltmeter and I ammeter readings is given by

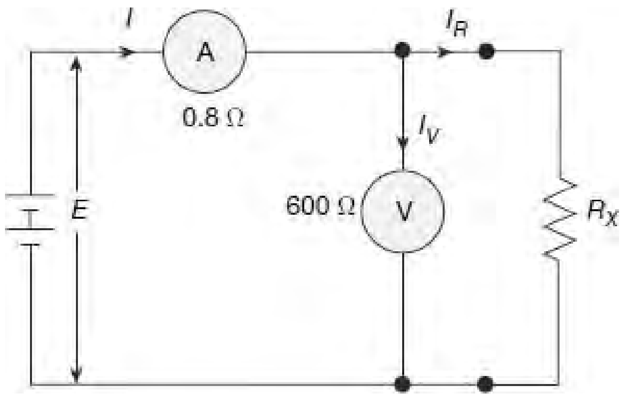
$$= R_m = \frac{V}{I} = \frac{40}{120 \times 10^{-3}} = 333\ \Omega$$

The ammeter reads the current flowing I_R through the resistance R_x and also the current I_V through the voltmeter resistance R_V.

Thus, $I = I_V + I_R$

Now, the voltmeter and the resistance R_x being in parallel, the voltmeter reading is given by

$$V = I_R \times R_X = I_V \times R_V$$



Current through voltmeter

$$I_V = \frac{V}{R_V} = \frac{40}{600} = 66.67\ \text{mA}$$

∴ true current throught resistance $I_R = I - I_V = 120\text{-}66.67 = 55.33$ mA

∴ true value of resistance $= R_x = \frac{V}{I_R} = \frac{40}{53.33 \times 10^{-3}} = 750\ \Omega$

Thus, percentage error $\varepsilon = \frac{R_m - R_x}{R_x} = \frac{333 - 750}{750} \times 100\% = 55.5\%$

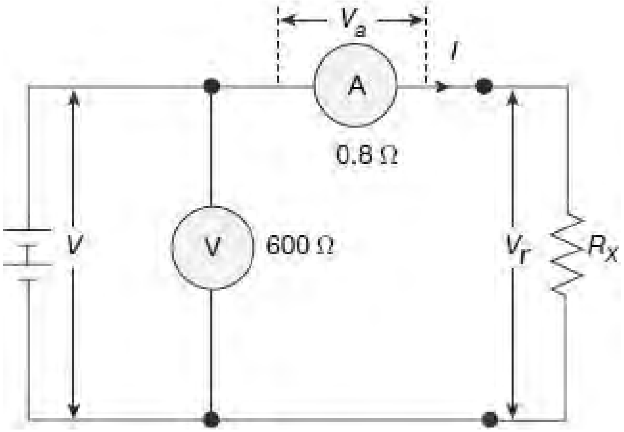
The connections are shown in the following fi gure.

Voltmeter reading V = 40V

Ammeter reading I = 120 mA

∴ measured resistance from voltmeter and ammeter readings is given by

$$= R_m = \frac{V}{I} = \frac{40}{120 \times 10^{-3}} = 333\ \Omega$$



Voltmeter reads the voltage drop V_r across the resistance R_x and also the voltage drop V_a across the ammeter resistance R_a.

Thus, $V = V_a + V_r$

Voltage drop across ammeter

$$V_a = I \times R_a = 120 \times 10^{-3} \times 0.8 = 0.096\ \text{V}$$

∴ true voltage drop across the resistance

$$V_r = V - V_a = 40 - 0.096 \times 39.904\ \text{V}$$

∴ true value of resistance $= R_x = \frac{V_r}{I} = \frac{39.904}{120 \times 10^{-3}} = 332.53\ \Omega$

Percentage error in measurement is $\varepsilon = \frac{333 - 332.53}{332.53} \times 100\% = 0.14\%$

4.2.3 Substitution Method for Measuring Resistance

The connection diagram for the substitution method is shown in Figure 4.8.

In this method the unknown resistance R_x is measured with respect to the standard variable resistance S. The circuit also contains a steady voltage source V, a regulating resistance r and an ammeter. A switch is there to connect R_x and S in the circuit alternately.

To start with, the switch is connected in position 1, so that the unknown resistance R_x gets included in the circuit. At this condition, the regulating resistance r is adjusted so that the ammeter pointer comes to a specified location on the scale. Next, the switch is thrown to position 2, so that the standard resistance S comes into circuit in place of R_x. Settings in the regulating resistance are not changed. The standard variable resistance S is varied till ammeter pointer reaches the same location on scale as was with R_x. The value of the standard resistance S at this position is noted from its dial. Assuming that the battery emf has not changed and also since the value of r is kept same in both the cases, the current has been kept at the same value while substituting one resistance with another one. The two resistances thus, must be equal. Hence, value of the unknown resistance R_x can be estimated from dial settings of the standard resistance S.

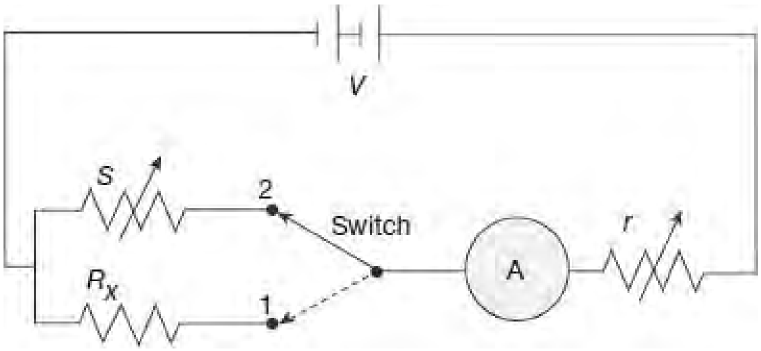


Figure 4.8 *Substitution method*

Accuracy of this method depends on whether the battery emf remains constant between the two measurements. Also, other resistances in the circuit excepting *R* and *S* should also not change during the course of measurement. Readings must be taken fairly quickly so that temperature effects do not change circuit resistances appreciably. Measurement accuracy also depends on sensitivity of the ammeter and also on the accuracy of the standard resistance *S*.

4.2.4 Wheatstone Bridge for Measuring Resistance

The Wheatstone bridge is the most commonly used circuit for measurement of medium-range resistances. The Wheatstone bridge consists of four resistance arms, together with a battery (voltage source) and a galvanometer (null detector). The circuit is shown in Figure 4.9.

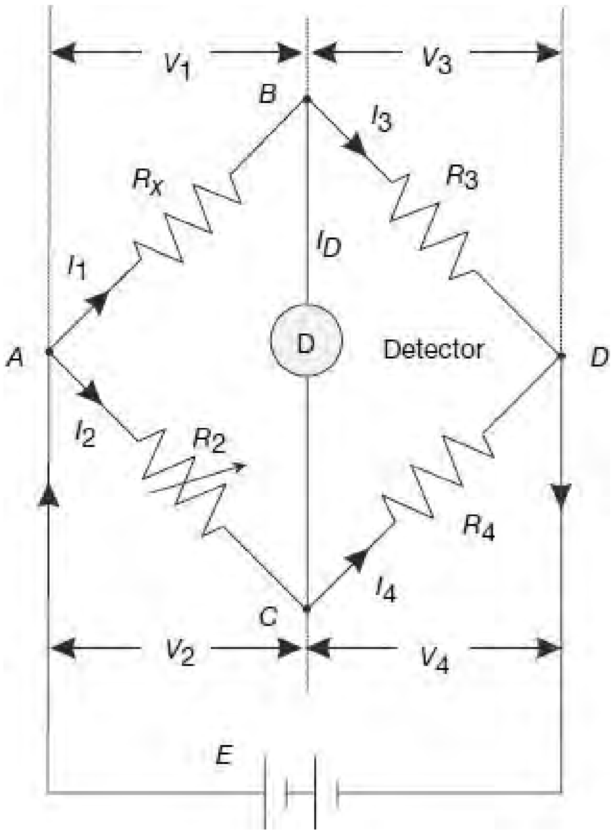


Figure 4.9 *Wheatstone bridge for measurement of resistance*

In the bridge circuit, R_3 and R_4 are two fixed known resistances, R_2 is a known variable resistance and R_X is the unknown resistance to be measured. Under operating conditions, current I_D through the galvanometer will depend on the difference in potential between nodes *B* and C. A bridge balance condition is achieved by varying the resistance R_2 and checking whether the galvanometer pointer is resting at its zero position. At balance, no current flows through the galvanometer. This means that at balance, potentials at nodes *B* and *C* are equal. In other words, at balance the following conditions are satisfied:

1. The detector current is zero, i.e., $1_D = 0$ and thus $I_t = I_3$ and $I_2 = I_4$
2. Potentials at node *B* and *C* are same, i.e., $V_B = V_C$, or in other words, voltage drop in the arm *AB* equals the voltage drop across the arm AC, i.e., $V_{AB} = V_{AC}$ and voltage drop in the arm *BD* equals the voltage drop across the arm *CD*, i.e., $V_{BD} = V_{CD}$

From the relation $V_{AB} = V_{AC}$ we have $I_1 \times R_x = I_2 \times R_2$ (4.12)

At balanced 'null' position, since the galvanometer carries no current, it as if acts as if open circuited, thus

$$I_1 = I_3 = \frac{E}{R_X + R_3} \text{ and } I_2 = I_4 = \frac{E}{R_2 + R_4}$$

Thus, from Eq. (4.12), we have

$$\frac{E}{R_X + R_3} \times R_X = \frac{E}{R_2 + R_4} \times R_2$$

or, $$\frac{R_X + R_3}{R_X} = \frac{R_2 + R_4}{R_2}$$

or, $$\frac{R_X + R_3}{R_X} - 1 = \frac{R_2 + R_4}{R_2} - 1$$

or, $$\frac{R_X + R_3 - R_X}{R_X} = \frac{R_2 + R_4 - R_2}{R_2}$$

or, $$\frac{R_3}{R_X} = \frac{R_4}{R_2}$$

or, $$\frac{R_X}{R_2} = \frac{R_3}{R_4}$$

or, $$R_X = R_2 \times \frac{R_3}{R_4} \quad (4.13)$$

Thus, measurement of the unknown resistance is made in terms of three known resistances. The arms *BD* and *CD* containing the fixed resistances R_3 and R_4 are called the **ratio arms**. The arm *AC* containing the known variable resistance R_2 is called the **standard arm**. The range of the resistance value that can be measured by the bridge can be increased simply by increasing the ratio R_3/R_4.

Errors in a Wheatstone Bridge

A Wheatstone bridge is a fairly convenient and accurate method for measuring resistance. However, it is not free from errors as listed below:

1. Discrepancies between the true and marked values of resistances of the three known arms can introduce errors in measurement.

2. Inaccuracy of the balance point due to insufficient sensitivity of the galvanometer may result in false null points.

3. Bridge resistances may change due to self-heating (I^2R) resulting in error in measurement calculations.

4. Thermal emfs generated in the bridge circuit or in the galvanometer in the connection points may lead to error in measurement.

5. Errors may creep into measurement due to resistances of leads and contacts. This effect is however, negligible unless the unknown resistance is of very low value.

6. There may also be personal errors in finding the proper null point, taking readings, or during calculations.

Errors due to inaccuracies in values of standard resistors and insufficient sensitivity of galvanometer can be eliminated by using good quality resistors and galvanometer.

Temperature dependent change of resistance due to self-heating can be minimised by performing the measurement within as short time as possible.

Thermal emfs in the bridge arms may cause serious trouble, particularly while measuring low resistances. Thermal emf in galvanometer circuit may be serious in some cases, so care must be taken to minimise those effects for precision measurements. Some sensitive galvanometers employ all-copper systems (i.e., copper coils as well as copper suspensions), so that there is no junction of dissimilar metals to produce thermal emf. The effect of thermal emf can be balanced out in practice by adding a reversing switch in the circuit between the battery and the bridge, then making the bridge balance for each polarity and averaging the two results.

Example 4.4

Four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 10 Ω, CD = 4 Ω, DA = 50 Ω. A galvanometer with internal resistance of 20 Ω is connected between BD, while a battery of 10-V dc is connected between AC. Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out Thevenin equivalent voltage across nodes *BD* and also the Thevenin equivalent resistance between terminals *BD*.

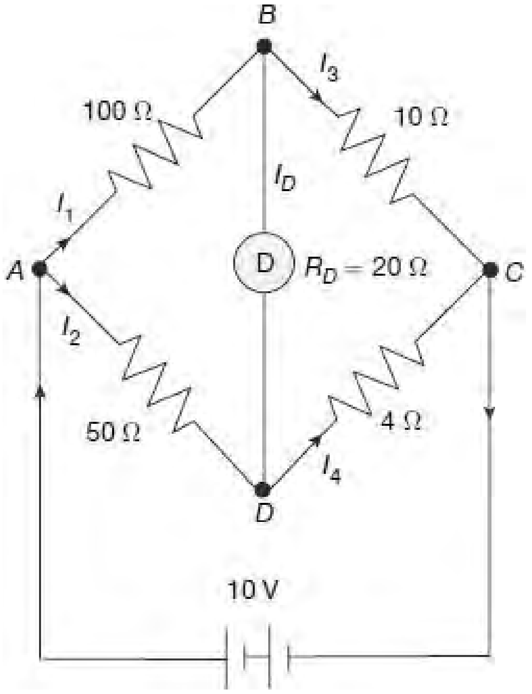
To find out Thevenin's equivalent voltage across *BD*, the galvanometer is open circuited, and the circuit then looks like the figure given below.

At this condition, voltage drop across the arm BC is given by

$$V_{BC} = 10 \times \frac{10}{100+10} = 0.91 \text{ V}$$

Voltage drop across the arm DC is given by:

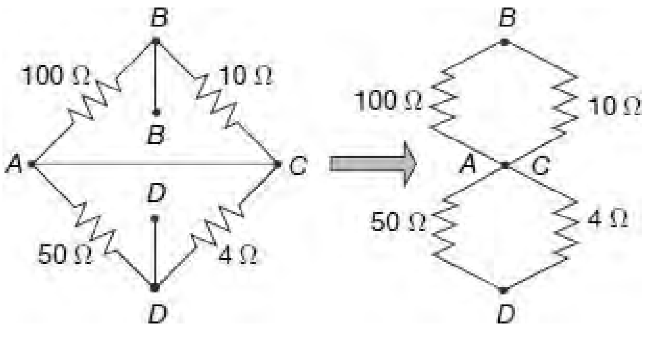
$$V_{DC} = 10 \times \frac{4}{50+4} = 0.74 \text{ V}$$



Hence, voltage difference between the nodes *B* and *D*, or the Thevenin equivalent voltage between nodes *B* and *D* is

$$V_{TH} = V_{BD} = V_B - V_D = V_{BC} - V_{DC} = 0.91 - 0.74 = 0.17 \text{ V}$$

To obtain the Thevenin equivalent resistance between nodes *B* and D, the 10 V source need to be shorted, and the circuit looks like the figure given below.



The Thevenin equivalent resistance between the nodes B and D is thus

$$R_{Th} = \frac{100 \times 10}{100+10} + \frac{50 \times 4}{50+4} = 12.79\ \Omega$$

Hence, current through galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.17}{20+12.79} = 5.18 \text{ mA}$$

In order to balance the bridge, there should be no current through the galvanometer, or in other words, nodes *B* and *D* must be at the same potential.

Balance equation is thus

$$\frac{100}{10} = \frac{R_{DA}}{4} \quad \text{or, } R_{DA} = 40\ \Omega$$

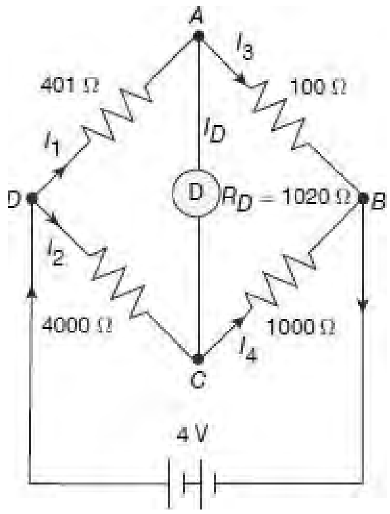
Example 4.5

The four arms of a Wheatstone bridge are as follows: AB = 100 Ω, BC = 1000 Ω, CD = 4000 Ω, DA = 400 Ω. A galvanometer with internal resistance of 100 Ω and sensitivity of 10 mm/μA is connected between AC, while a battery of 4 V dc is connected between BD. Calculate the current through the galvanometer and its deflection if the resistance of arm DA is changed from 400 Ω to 401 Ω.

Solution Configuration of the bridge with the values given in the example is as shown below:

To find out current through the galvanometer, it is required to find out the Thevenin equivalent voltage across nodes *AC* and also the Thevenin equivalent resistance between terminals AC.

To find out Thevenin equivalent voltage across AC, the galvanometer is open circuited. At this condition, voltage drop across the arm *AB* is given by



$$V_{AB} = 4 \times \frac{100}{100+401} = 0.798\ \text{V}$$

Voltage drop across the arm *CB* is given by

$$V_{CB} = 4 \times \frac{1000}{4000+1000} = 0.8\ \text{V}$$

Hence, voltage difference between the nodes *A* and *C*, or the Thevenin equivalent voltage between nodes *A* and *C* is

$$V_{TH} = V_{AC} = V_A - V_C = V_{AB} - V_{CB} = 0.798 - 0.8 = -0.002\ \text{V}$$

To obtain the Thevenin equivalent resistance between nodes *A* and C, the 10 V source need to be shorted. Under this condition, the Thevenin equivalent resistance between the nodes *A* and *C* is thus

$$R_{Th} = \frac{100 \times 401}{100+401} + \frac{1000 \times 4000}{1000+4000} = 880.04\ \Omega$$

Hence, current through the galvanometer is

$$I_D = \frac{V_{Th}}{R_D + R_{Th}} = \frac{0.002}{100+880.04} = 2.04\ \mu A$$

Deflection of the galvanometer

= Sensitivity × Current = 10 mm/μA = 2.04 μA = 20.4 mm

4.3 MEASUREMENT OF LOW RESISTANCES

The methods used for measurement of medium resistances are not suitable for measurement of low resistances. This is due to the fact that resistances of leads and contacts, though small, are appreciable in comparison to the low resistances under measurement. For example, a contact resistance of 0.001 Ω causes a negligible error when a medium resistance of value say, 100 Ω is being measured, but the same contact resistance would cause an error of 10% while measuring a low resistance of value 0. 01 Ω. Hence special type of construction and techniques need to be used for measurement of low resistances to avoid errors due to leads and contacts. The different methods used for measurement of low range resistances are (i) voltmeter–ammeter method, (iii) Kelvin's double-bridge method, and (iv) potentiometer method.

4.3.1 Voltmeter–Ammeter Method for Measuring Low Resistance

In principle, the voltmeter–ammeter method for measurement of low resistance is very similar to the one used for measurement of medium resistances, as described in Section 4.2.2. This method, due to its simplicity, is very commonly used for measurement of low resistances when accuracy of the order of 1% is sufficient. The resistance elements, to be used for such measurements, however, need to of special construction. Low resistances are constructed with four terminals as shown in Figure 4.10.

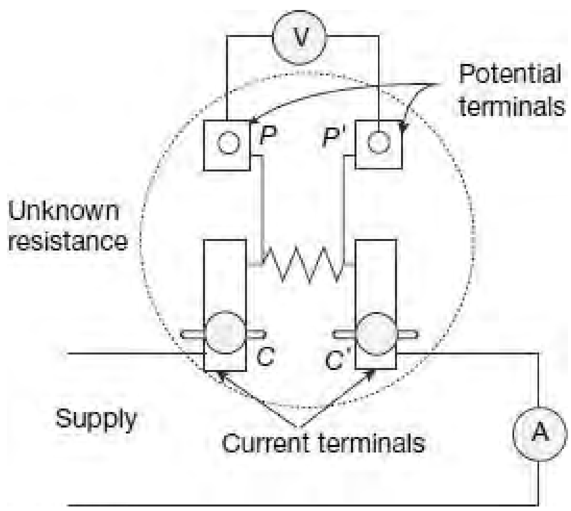


Figure 4.10 *Voltmeter–ammeter method for measuring*

One pair of terminals *CC'*, called the current terminals, is used to lead current to and from the resistor. The voltage drop across the resistance is measured between the other pair of terminals *PP'*, called the potential terminals. The voltage indicated by the voltmeter is thus simply the voltage drop of the resistor across the potential terminals *PP'* and does not include any contact resistance drop that may be present at the current terminals *CC'*.

Contact drop at the potential terminals PP' are, however, less itself, since the currents passing through these contacts are extremely small (even zero under 'null' balance condition) owing to high resistance involved in the potential circuit. In addition to that, since the potential circuit has a high resistance voltmeter in it, any contact resistance drop in the potential terminals PP' will be negligible with respect to the high resistances involved in the potential circuit.

Value of the unknown resistance R_X in this case is given by

$$R_X = \frac{\text{Voltmeter reading}}{\text{Ammeter reading}}$$

Precise measurement in this method requires that the voltmeter resistance to be appreciably high, otherwise the voltmeter current will be an appreciable fraction of the current actually flowing through the ammeter, and a serious error may be introduced in this account.

4.3.2 Kelvin's Double-Bridge Method for Measuring Low Resistance

Kelvin's double-bridge method is one of the best available methods for measurement of low resistances. It is actually a modification of the Wheatstone bridge in which the errors due to contacts and lead resistances can be eliminated. The connections of the bridge are shown in Figure 4.11.

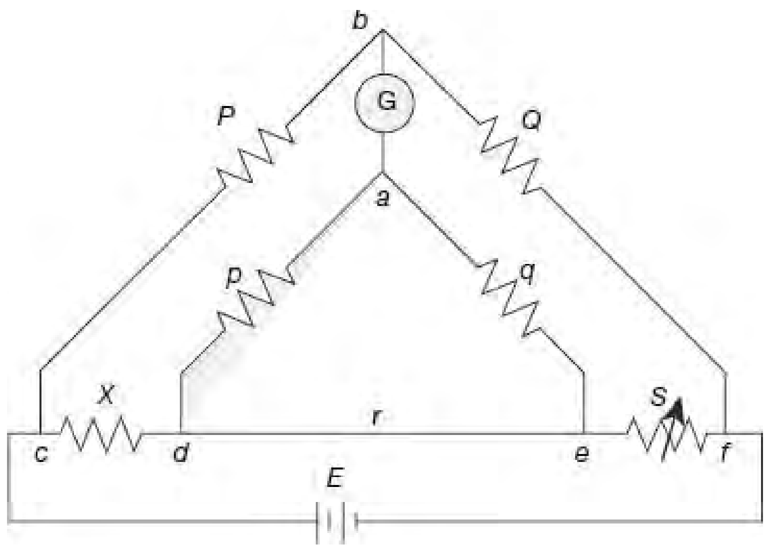


Figure 4.11 *Kelvin's double bridge*

Kelvin's double bridge incorporates the idea of a second set of ratio arms, namely, *p* and q, and hence the name **'double bridge'**.

X is the unknown low resistance to be measured, and *S* is a known value standard low resistance. ‘r’ is a very low resistance connecting lead used connect the unknown resistance *X* to the standard resistance S. All other resistances P, *Q*, *p*, and *q* are of medium range. Balance in the bridge is achieved by adjusting S.

Under balanced condition, potentials at the nodes *a* and *b* must be equal in order that the galvanometer *G* gives “null” deflection. Since at balance, no current flows through the galvanometer, it can be considered to be open circuited and the circuit can be represented as shown in Figure 4.12.

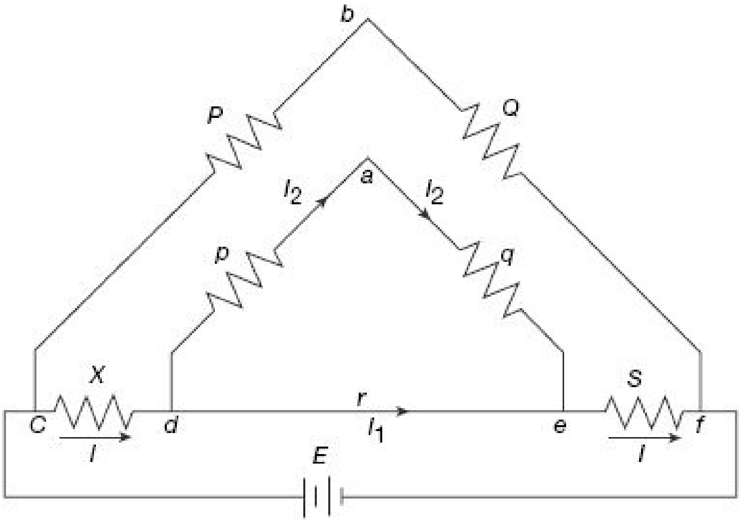


Figure 4.12 *Kelvin’s double-bridge under balanced condition*

Since under balanced condition, potentials at the nodes *a* and *b* are equal, the we must have

$$V_{cb} = V_{cda}$$

Now, $$V_{cb} = E \times \frac{P}{P+Q} \qquad (4.14)$$

and $$V_{cda} = V_{cd} + V_{da} = X \times I + p \times I_2$$

where, $I_2 = I \times \frac{r}{r+p+q}$

$$\therefore \qquad V_{cda} = I \times X + I \times \frac{pr}{r+p+q} = I\left(X + \frac{pr}{r+p+q}\right) \qquad (4.15)$$

Supply voltage $E = V_{cd} + V_{de} + V_{ef} = I \times X + I \times \frac{(p+q)}{p+q+r} \times r + I \times S$

or, $$E = I\left(X + S + \frac{(p+q)}{p+q+r} \times r\right) \qquad (4.16)$$

From (4.14) and (4.16), we have

$$V_{cb} = \frac{P}{P+Q} \times I\left(X + S + \frac{(p+q)}{p+q+r} \times r\right)a \qquad (4.17)$$

∴ the balance equation $V_{cb} = V_{cda}$ can now be re-written as

$$\frac{P}{P+Q}\times I\left(X+S+\frac{(p+q)}{p+q+r}\times r\right)=I\left(X+\frac{pr}{r+p+q}\right)a$$

or, $$\left(X+S+\frac{(p+q)}{p+q+r}\times r\right)=\left(1+\frac{Q}{P}\right)\times\left(X+\frac{pr}{r+p+q}\right)$$

or, $$X+S+\frac{(p+q)}{p+q+r}\times r=X+\frac{pr}{r+p+q}+\frac{Q}{P}\times X+\frac{Q}{P}\times\frac{pr}{r+p+q}a$$

or, $$S+\frac{(p+q)}{p+q+r}\times r=\frac{pr}{r+p+q}+\frac{Q}{P}\times X+\frac{Q}{P}\times\frac{pr}{r+p+q}a$$

or, $$\frac{Q}{P}\times X= S+\frac{(p+q)}{p+q+r}\times r-\frac{pr}{r+p+q}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X= S+\frac{pr}{p+q+r}+\frac{qr}{p+q+r}-\frac{pr}{r+p+q}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X=S+\frac{qr}{p+q+r}-\frac{Q}{P}\times\frac{pr}{r+p+q}$$

or, $$\frac{Q}{P}\times X=S+\frac{qr}{p+q+r}\left(1-\frac{Q}{P}\times\frac{p}{q}\right)a$$

or, $$X=\frac{P}{Q}\times S+\frac{qr}{p+q+r}\left(\frac{P}{Q}-\frac{p}{q}\right) \tag{4.18}$$

The second quantity of the Eq. (4.18), $\frac{qr}{p+q+r}\left(\frac{P}{Q}-\frac{p}{q}\right)$, can be made very small by making the ratio P/Q as close as possible to p/q. In that case, there is no effect of the connecting lead resistance ‘r’ on the expression for the unknown resistance. Thus, the expression for the unknown resistance *X* can now be simply written as

or, $$X=\frac{P}{Q}\times S \tag{4.19}$$

However, in practice, it is never possible to make the ratio *p/q* exactly equal to P/Q. Thus, there is always a small error.

$\Delta=\left(\frac{P}{Q}-\frac{p}{q}\right)$ and hence, the resistance value becomes

or, $$X=\frac{P}{Q}\times S+\frac{q}{p+q+r}\times\Delta\times r \tag{4.20}$$

It is thus always better to keep the value of ‘r’ as small as possible, so that the product Δ × *r* is extremely small and therefore the error part can be neglected, and we can assume, under balanced condition,

$$\boxed{X=\frac{P}{Q}\times S}$$

In order to take into account the effects of thermoelectric emf, two measurements are normally done with the battery connections reversed. The final value of resistance is taken as the average of these two readings.

A 4-terminal resistor was measured with the help of a Kelvin’s double bridge having the following components: Standard resistor = 98.02 nW, inner ratio arms = 98.022 Ω

Example 4.6 *and 202 W, outer ratio arms = 98.025 Ω and 201.96 W, resistance of the link connecting the standard resistance and the unknown resistance = 600 nW. Calculate the value of the unknown resistance.*

Solution From Eq. (4.18), value of the unknown resistance is

$$X = \frac{P}{Q} \times S + \frac{qr}{p+q+r}\left(\frac{P}{Q} - \frac{p}{q}\right)$$

or, $$X = \frac{98.025}{201.96} \times 98.02 \times 10^{-6} + \frac{202 \times 600 \times 10^{-6}}{98.022 + 202 + 600 \times 10^{-6}}\left(\frac{98.025}{201.96} - \frac{98.022}{202}\right)$$

or, $$X = 47.62\ \mu\Omega$$

4.3.3 Potentiometer Method for Measuring Low Resistance

The circuit for measurement of low value resistance with a potentiometer is shown in Figure 4.13.

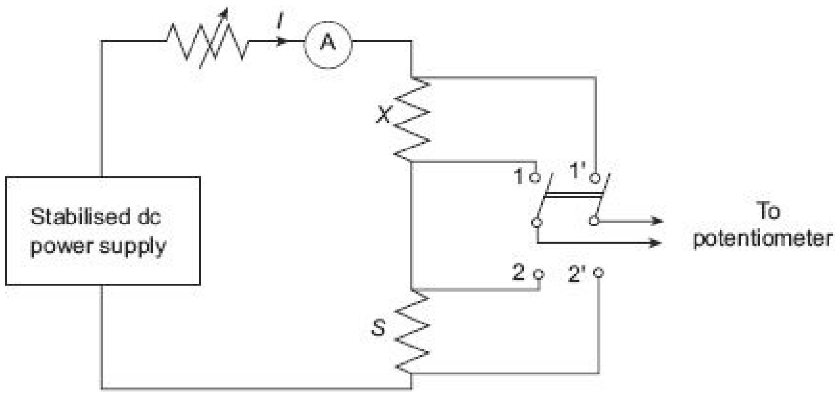


Figure 4.13 *Measurement of low resistance using potentiometer*

The unknown resistance *X* is connected in series with a standard known resistance S. Current through the ammeter in the circuit is controlled by a rheostat. A two-pole double throw switch is used. When the switch is in the position 1-1', the unknown resistance *X* gets connected to the potentiometer, whereas when the switch is at position 2-2', the standard resistance *S* gets connected to the potentiometer.

Potentiometers are believed to give reasonably accurate values of potentials.

Thus, with the switch in position 1-1', the potentiometer reading is the voltage drop across the unknown resistance, given by

$$V_X = I \times X \tag{4.21}$$

Without changing any of the circuit parameters, now if the switch is thrown to position 2-2', potentiometer now reads the voltage drop across the standard resistance, given by

$$V_S = I \times S \tag{4.22}$$

From Eqs (4.21) and (4.22), we get

or, $$X = \frac{V_R}{V_S} \times S \tag{4.23}$$

Knowledge of accurate value of the standard resistance *S* can thus give reasonably accurate values of the unknown resistance *X*.

Accuracy of this method however, depends on the assumption that the value of current remains absolutely constant during the two sets of measurements. Therefore, an extremely stabilised dc power supply is required in this method.

Value of the standard resistor *S* should be of the same order as the unknown resistance X. The ammeter inserted in the circuit has no other function rather than simply indicating whether there is any current is flowing in the circuit is not. Exact value of the current is not required for final calculations. It is however, desired that the current flowing through the circuit be so adjusted that the voltage drop across each resistor is of the order of 1 V to be suitable for accurate measurement by commercially available potentiometers.

4.4 MEASUREMENT OF HIGH RESISTANCES

High resistances of the order of several hundreds and thousands of meghoms (MW) are often encountered in electrical equipments in the form of insulation resistance of machines and cables, leakage resistance of capacitors, volume and surface resistivity of different insulation materials and structures.

4.4.1 Difficulties in Measurement of High Resistance

1. Since the resistance under measurement has very high value, very small currents are encountered in the measurement circuit. Adequate precautions and care need to be taken to measure such low value currents.

2. Surface leakage is the main difficulty encountered while measurement of high resistances. The resistivity of the resistance under measurement may be high enough to impede flow of current through it, but due to moisture, dust, etc., the surface of the resistor may provide a lower resistance path for the current to pass between the two measuring electrodes. In other words, there may thus be a leakage through the surface. Leakage paths not only pollute the test results, but also are generally variable from day to day, depending on temperature and humidity conditions.
   The effect of leakage paths on measurements can be eliminated by the use of guard circuits as described by Figure 4.14.
   Figure 4.14(a) shows a high resistance R_X being mounted on a piece of insulation block. A battery along with a voltmeter and a micro-ammeter are used to measure the resistance by voltmeter–ammeter method. The resistance R_X under measurement is fitted on the insulating block at the two binding posts *A* and *B*. I_X is the actual current flowing through the high resistance and I_L is the surface leakage current flowing over the body of the insulating block. The micro-ammeter, in this case, thus reads the actual current through the resistor, and also the leakage current ($I = I_X + I_L$.). Measured value of the resistance, thus computed from the ratio E/I, will not be the true value of R_X, but will involve some error. To avoid this error, a guard arrangement has been added in Figure 4.14(b). The guard arrangement, at one end is connected to the battery side of the micro-ammeter, and the other end is wrapped over the insulating body and surrounds the resistance terminal *A*. The surface leakage current now, flows through this guard and bypasses the micro-ammeter. The

micro-ammeter thus reads the true of current I_X through the resistance R_X. This arrangement thus allows correct determination of the resistance value from the readings of voltmeter and micro-ammeter.

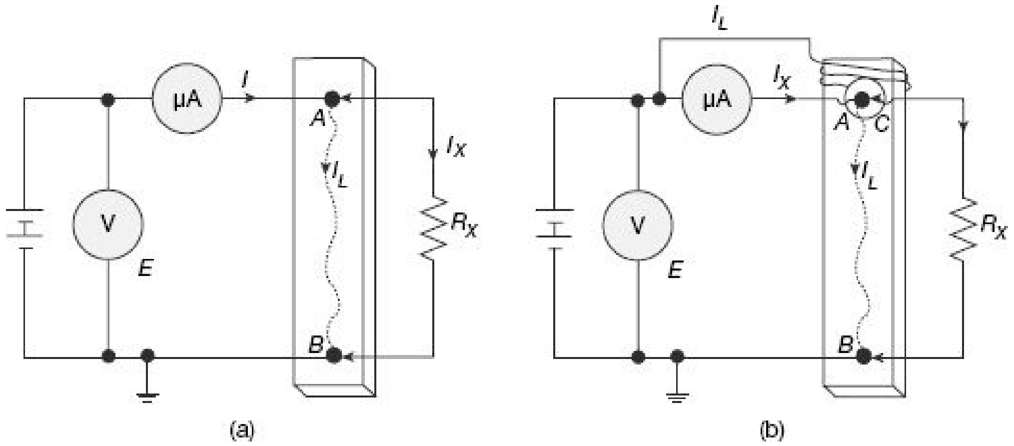


Figure 4.14 *Guard circuit for measurement of high resistance: (a) Circuit without guard (b) Circuit with guard*

3. Due to electrostatic effects, stray charges may be induced in the measuring circuit. Flow of these stray charges can constitute a current that can be comparable in magnitude with the low value current under measurement in high resistance circuits. This may thus, cause errors in measurement. External alternating electromagnetic fields can also affect the measurement considerably. Therefore, the measuring circuit needs to be carefully screened to protect it against such external electrostatic or electromagnetic effects.

4. While measuring insulation resistance, the test object often has considerable amount of capacitance as well. On switching on the dc power supply, a large charging current may flow initially through the circuit, which gradually decays down. This initial transient current may introduce errors in measurement unless considerable time is provided between application of the voltage supply and reading the measurement, so that the charging current gets sufficient time to die down.

5. High resistance measurement results are also affected by changes in temperature, humidity and applied voltage inaccuracies.

6. Reasonably high voltages are used for measurement of high resistances in order to raise the current to substantial values in order to be measured, which are otherwise extremely low. So, the associated sensitive galvanometers and micro-ammeters need to be adequately protected against such high voltages.

Taking these factors into account, the most well-known methods of high resistance measurements are (i) direct deflection method, (ii) loss of charge method, and (iii) megohmmeter or meggar.

4.4.2 Direct Deflection Method for High Resistance Measurement

The direct deflection method for measuring high resistances is based on the circuit described in Figure 4.14, which in effect is the voltmeter–ammeter method. For measurement of high resistances, a sensitive galvanometer is used instead of a micro-ammeter as shown in Figure 4.14. A schematic diagram for describing the direct deflection method for measurement of insulation resistance of a metal sheathed cable is given in Figure 4.15.

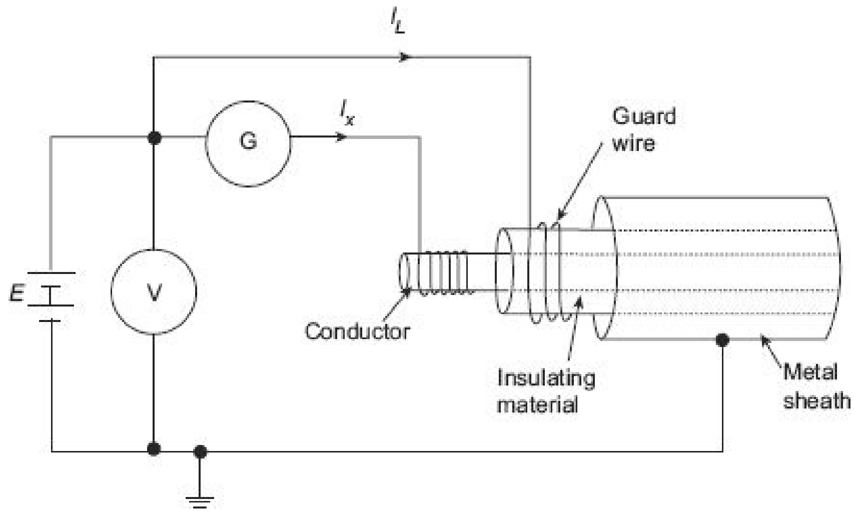


Figure 4.15 *Measurement of cable insulation resistance*

The test specimen, cable in this case, is connected across a high voltage stable dc source; one end of the source being connected to the inner conductor of the cable, and the other end, to the outer metal sheath of the cable. The galvanometer G, connected in series as shown in Figure 4.15, is intended to measure the current I_X flowing through the volume of the insulation between the central conductor

and the outer metal sheath. Any leakage current I_L flowing over the surface of the insulating material is bypassed through a guard wire wound on the insulation, and therefore does not flow through the galvanometer.

A more detailed scheme for measurement of insulation resistance of a specimen sheet of solid insulation is shown in Figure 4.16.

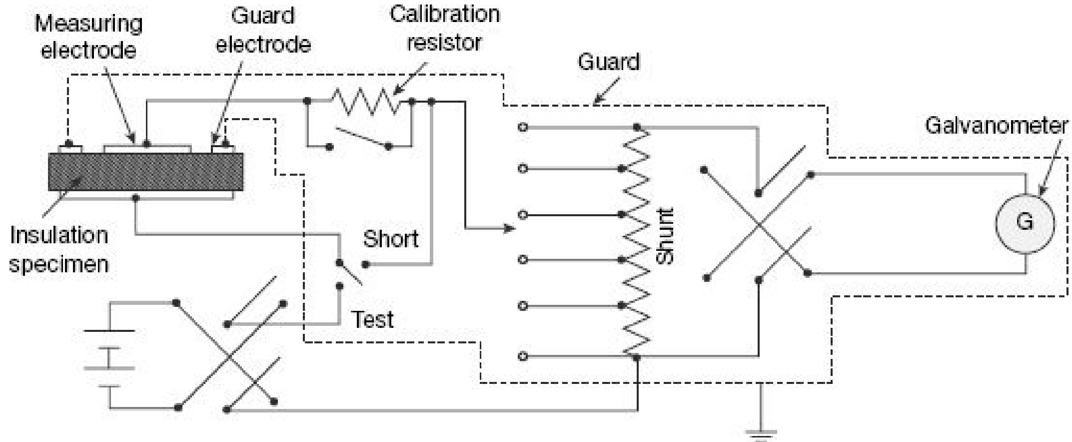


Figure 4.16 *Measurement of high resistance by direct deflection method*

A metal disk covering almost the entire surface is used as electrode on one side of the insulation sheet under measurement. On the other side of the insulating sheet, the second electrode is made of a smaller size disk. A guard ring is placed around the second electrode with a small spacing in between them. This guarding arrangement bypasses any surface leakage current on the insulator or any other parts of the circuit from entering the actual measuring circuit. The galvanometer thus reads specifically the volume resistance of the insulation specimen, independent of any surface leakage.

A calibrated Ayrton shunt is usually included along with the galvanometer to provide various scale ranges and also to protect it.

The galvanometer scale is graduated directly in terms of resistance. After one set of measurement is over, the galvanometer is calibrated with the help of a high value (≈ 1 MΩ) calibrating resistor and the shunts.

In case the insulation under measurement has high inherent capacitance values (like in a cable), there will be an initial inrush of high capacitive charging current when the dc source is first switched on. This charging current will, however, decay down to a steady dc value with time. To protect the galvanometer from such initial rush of high current, the Ayrton shunt connected across the galvanometer should be placed at the highest resistance position (lower most point in Figure 4.16). Thus, initially the galvanometer is bypassed from the high charging current.

After the test is complete, it is required that the test specimen should be discharged, especially if it is of capacitive in nature. The 'test-short' switch is placed in the 'short' position so that any charge remaining in the insulation specimen is discharged through the short circuited path.

The change-over switch across the battery enables tests at different polarities. The switch across the galvanometer enables reversal of the galvanometer connections.

A special technique, Price's guard-wire method is employed for measurement of insulation resistance of cables which do not have metal sheath outside. The schematic diagram of such a measurement system is provided in Figure 4.17.

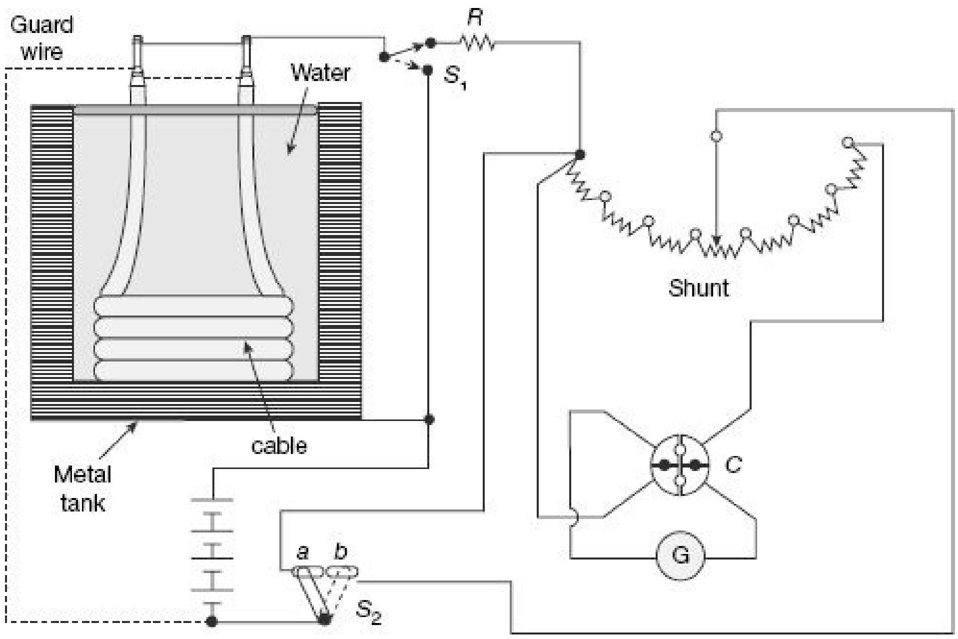


Figure 4.17 *Measurement of high resistance by Price's guard-wire method*

The unsheathed cable, except at the two ends where connections are made, is immersed in water in a tank. For testing of the cable insulation, the cable core conductor acts as one electrode and in the absence of the metal sheath outside, the water and the tank act as the other electrode for measurement. The cable is immersed in slightly saline water for about a day and at nearly constant ambient temperature.

The two ends of the cables are trimmed as shown in Figure 4.17, thus exposing the core conductor as well as some portion of the insulation. The core conductors are connected together to form one electrode of the measuring system. A guard circuit is formed by twisting a bare wire around the exposed portion of the insulation at the two stripped ends of the cable. This guard wire is connected to the negative terminal of the supply battery. The positive terminal of the battery is connected to the metal tank. This enables any surface leakage current to bypass the galvanometer and pass directly to the battery. Thus, the galvanometer will read only true value of the current flowing through volume of the insulation, and not the additional surface leakage current.

The D'Arsonval galvanometer to be used is normally of very high resistance and very sensitive to record the normally extremely low insulation currents. An Ayrton universal shunt is usually included along with the galvanometer to provide various scale ranges and also to protect it. The galvanometer scale is graduated directly in terms of resistance. After one set of measurement is over, the galvanometer is calibrated with the help of the high value (≈ 1 MΩ) calibrating resistor *R* and the shunt. The resistance *R* and the shunt also serve the purpose of protecting the galvanometer from accidental short circuit current surges. The 4-terminal commutator C, as shown in Figure 4.17 is used for reversal of galvanometer connections.

Since the cable will invariably have high capacitance value, there will be an initial inrush of high capacitive charging current when the dc source is first switched on. This charging current will, however, decay down to a steady dc value with time. To protect the galvanometer from such initial rush of high current, the switch S_2 is placed on position *a* so that initially the galvanometer is bypassed from the high charging current. Once the capacitor charging period is over and the current settles down, the switch S_2 is pushed over to position *b* to bring the galvanometer back in the measurement circuit. The contacts *a* and *b* are sufficiently close enough to prevent the circuit from breaking while the switch S_2 is moved over.

After the test is complete, it is required that the test specimen should be discharged. The switch S_1 is used for this purpose, so that any charge remaining in the insulation specimen is discharged through itself.

4.4.3 Loss of Charge Method for High Resistance Measurement

In this method, the resistance to be measured is connected directly across a dc voltage source in parallel with a capacitor. The capacitor is charged up to a certain voltage and then discharged through the resistance to be measured. The terminal voltage across the resistance-capacitance parallel combination is recorded for a pre-defined period of time with a help of a high-resistance voltmeter (electrostatic voltmeter or digital electrometers). Value of the unknown resistance is calculated from the discharge time constant of the circuit. Operation of the loss of charge method can be described by the schematic circuit diagram of Figure 4.18.

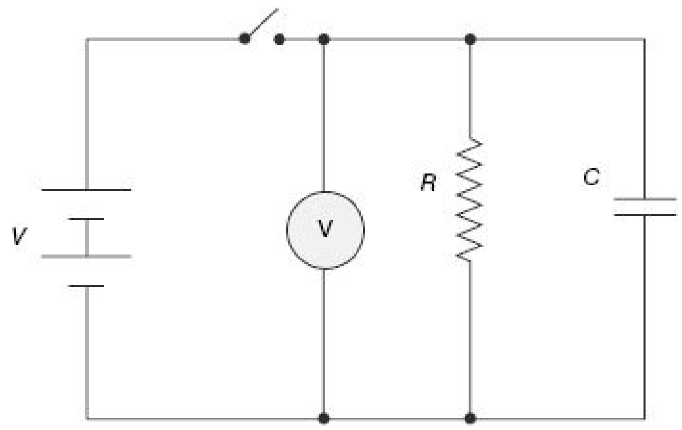


Figure 4.18 *Loss of charge method for measurement of high resistance*

In Figure 4.18, the unknown resistance R to be measured is connected across the capacitor C and their parallel combination is connected to the dc voltage source.

Let the capacitor is initially charged up to a voltage of V while the switch is kept ON. Once the switch is turned OFF, the capacitor starts to discharge through the resistance R.

During the discharge process, the voltage v across the capacitor at any instant of time t is given by

Thus, the insulation resistance can be calculated as

$$R = \frac{t}{C \times \log_e\left(\frac{V}{v}\right)} \tag{4.24}$$

With known value of C and recorded values of t, V and v, the unknown resistance R can be estimated using (4.24).

The pattern of variation of voltage v with time is shown in Figure 4.19.

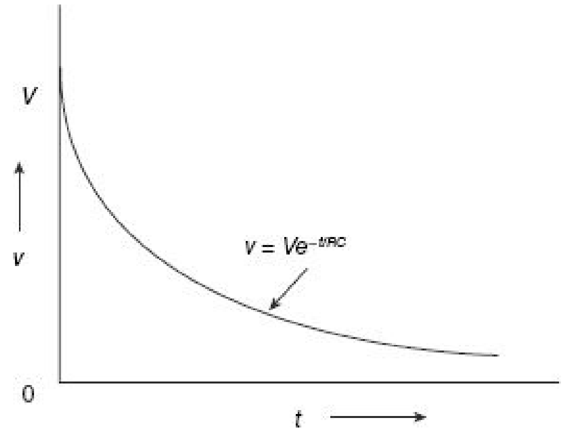


Figure 4.19 *Capacitor discharge pattern*

Great care must be taken to record the voltages V and v and also the time t very precisely, otherwise large errors may creep in to the calculation results.

This method, though simple in principle, require careful choice of the capacitor. The capacitor C itself must have sufficiently high value of its own leakage resistance, at least in the same range as the unknown resistance under measurement. The resistance of the voltmeter also needs to be very high to have more accurate results.

4.4.4 Megohmmeter, or Meggar, for High Resistance

Measurement

One of the most popular portable type insulation resistance measuring instruments is the megohmmeter or in short, meggar. The meggar is used very commonly for measurement of insulation resistance of electrical machines, insulators, bushings, etc. Internal diagram of a meggar is shown in Figure 4.20.

The traditional analog deflecting-type meggar is essentially a permanent magnet crossed-coil shunt type ohmmeter.

The instrument has a small permanent magnet dc generator developing 500 V dc (some other models also have 100 V, 250 V, 1000 or 2500 V generators). The generator is hand driven, through gear arrangements, and through a centrifugally controlled clutch switch which slips at a predefined speed so that a constant voltage can be developed. Some meggars also have rectified ac as power supply.

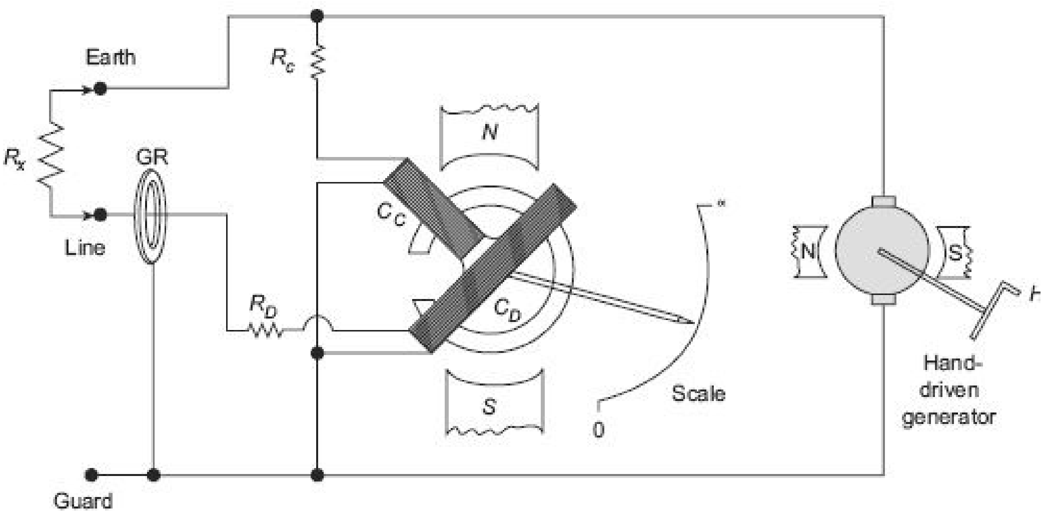
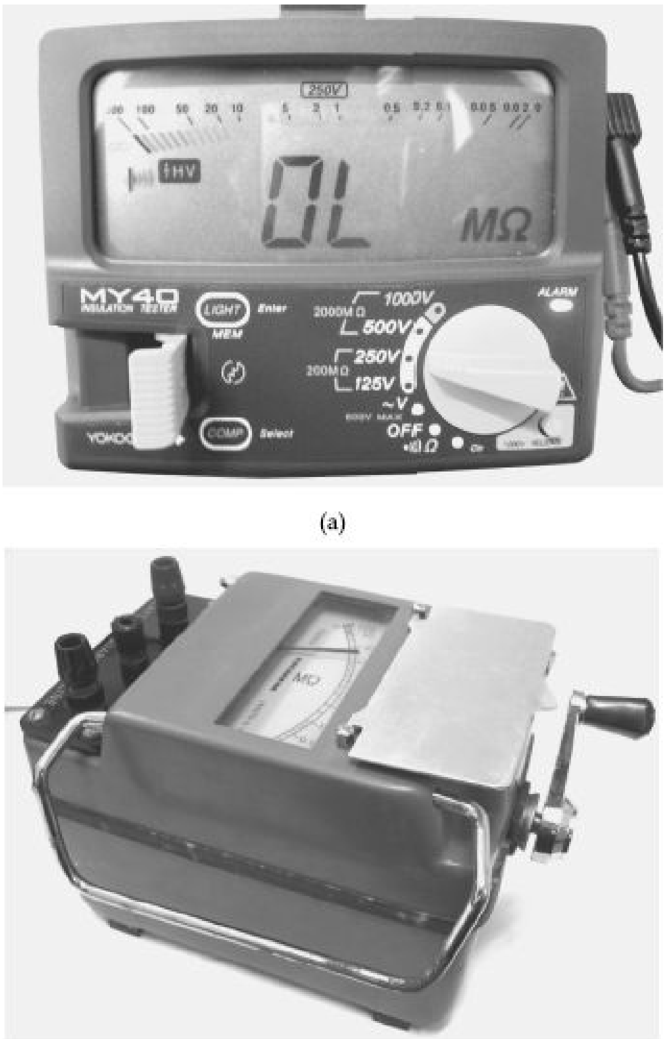


Figure 4.20 *Meggar for high resistance measurement*

The moving system in such instruments consists of two coils, the control coil C_C and the deflecting coil C_D. Both the coils are mounted rigidly on a shaft that carries the pointer as well. The two coils move in the air gap of a permanent magnet. The two coils are arranged with such numbers of turns, radii of action, and connected across the generator with such polarities that, for external magnetic fields of uniform intensity, the torque produced by the individual coils are in opposition thus giving an astatic combination. The deflecting coil is connected in series with the unknown resistance R_X under measurement, a fixed resistor R_D and then the generator. The current coil or the compensating coil, along with the fixed resistance R_C is connected directly across the generator. For any value of the unknown, the coils and the pointer take up a final steady position such that the torques of the two coils are equal and balanced against each other. For example, when the resistance R_X under measurement is removed, i.e., the test terminals are open-circuited, no current flows through the deflecting coil C_D, but maximum current will flow through the control coil C_C. The control coil C_C thus sets itself perpendicular to the magnetic axis with the pointer indicating '∞ Ω' as marked in the scale shown in Figure 4.20. As the value of R_X is brought down from open circuit condition, more and more current flows through the deflecting coil *CD*, and the pointer moves away from the '∞ Ω' mark clockwise (according to Figure 4.20) on the scale, and ultimately reaches the '0 Ω' mark when the two test terminals are short circuited.

The surface leakage problem is taken care of by the guard-wire arrangement. The guard ring (*GR* in Figure 4.20) and the guard wire diverts the surface leakage current from reaching the main moving system and interfering with its performance.

Photographs of some commercially available meggars are shown in Figure 4.21.



(a)

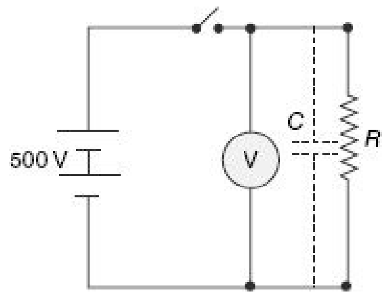
(b)

Figure 4.21 Commercial meggars: (a) Analog type (Courtesy, WACO) (b) Digital type (Courtesy, Yokogawa)

Example 4.7

A cable is tested for insulation resistance by loss of charge method. An electrostatic voltmeter is connected between the cable conductor and earth. This combination is found to form a capacitance of 800 pF between the conductor and earth. It is observed that after charging the cable with 500 V or sufficiently long time, when the voltage supply is withdrawn, the voltage drops down to 160 V in 1 minute. Calculate the insulation resistance of the cable.

Solution The arrangement can be schematically shown by the following figure.



While the cable is charged with 500 V for a long period of time, it is expected that the capacitance of the system is charged up to a steady voltage of 500 V before it is discharged.

Thus, the capacitor discharges from 500 V to 160 V in 1 minute.

During the discharge process, the voltage *v* across the capacitor at any instant of time *t* is given by

$$v = Ve^{-t/RC}$$

where *V* is the initial voltage in the capacitor *C*, connected across the unknown resistance *R*.

Thus, the insulation resistance is

$$R = \frac{t}{C \times \log_e\left(\frac{V}{v}\right)} = \frac{60}{800 \times 10^{-12} \times \log_e\left(\frac{500}{160}\right)} = 65 \times 10^9\ \Omega = 65{,}000\ \text{M}\Omega$$

4.5 LOCALISATION OF CABLE FAULTS

Underground cables during their operation can experience various fault conditions. Whereas routine standard tests are there to identify and locate faults in high-voltage cables, special procedures, as will be described in this section are required for localisation of cable faults in low distribution voltage level cables. Determination of exact location of fault sections in underground distribution cables is extremely important from the point of view of quick restoration of service without loss of time for repair.

The faults that are most likely to occur are *ground faults* where cable insulation may break down causing a current to flow from the core of the cable to the outer metal sheath or to the earth; or there may be *short-circuit faults* where a insulation failure between two cables, or between two cores of a multi-core cable results in flow of current between them.

Loop tests are popularly used in localisation of the aforesaid types of faults in low voltage cables. These tests can be carried out to localise a ground fault or a short-circuit fault, provided that an unfaulty cable runs along with the faulty cable. Such tests have the advantage that fault resistance does not affect the measurement sensitivity, that the fault resistance is not too high. Loop tests work on the simple principles of a Wheatstone bridge for measurement of unknown resistances.

4.5.1 Murray Loop Test

Connections for this test are shown in Figure 4.22. Figure 4.22(a) shows the connection diagram for localisation of ground faults and Figure 4.22(b) relates to localisation of short circuit faults. The general configuration of a Wheatstone bridge is given in Figure 4.23 for ready reference.

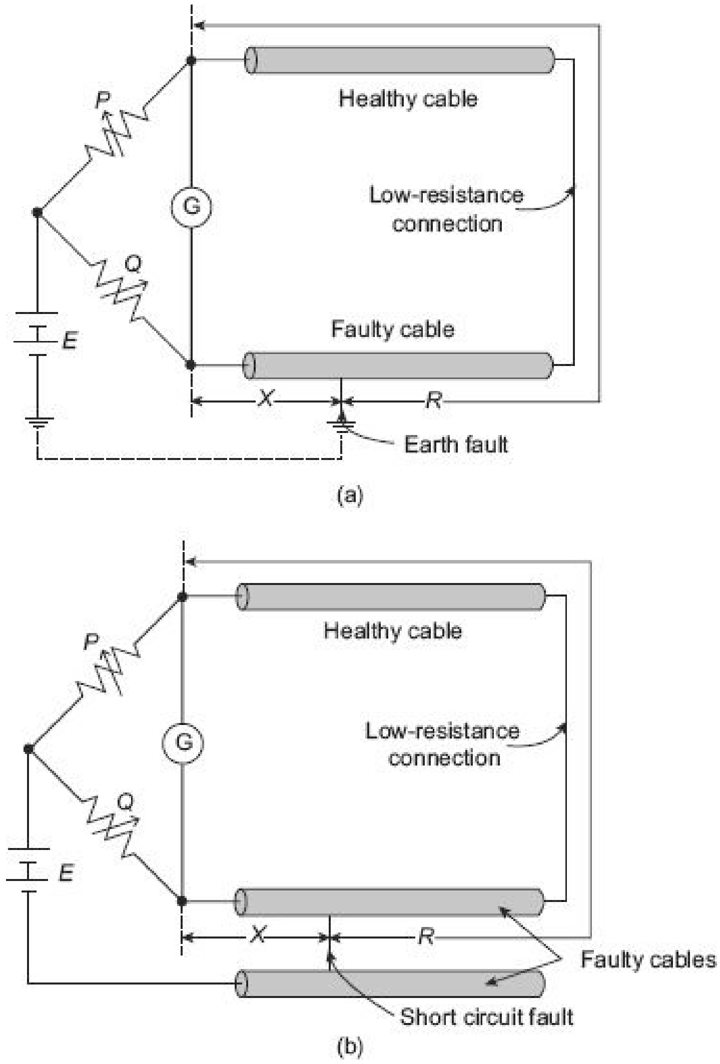


Figure 4.22 *Murray loop test for (a) earth fault, and (b) short-circuit fault localosation in cables*

The loop circuits formed by the cable conductors form a Wheatstone bridge circuit with the two externally controllable resistors *P* and *Q* and the cable resistance *X* and *R* as shown in Figure 4.22. The galvanometer *G* is used for balance detection. The ridge is balanced by adjustment of *P* and *Q* till the galvanometer indicates zero deflection.

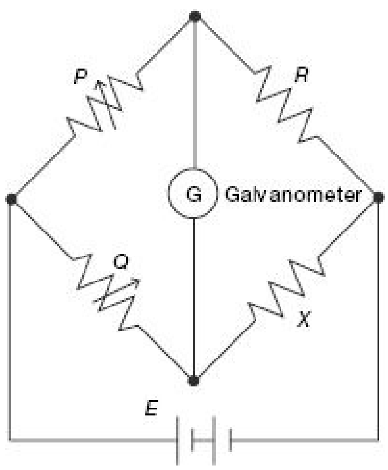


Figure 4.23 *Wheatstone bridge configuration relating to Figure 4.22*

Under balanced condition,

$$\frac{X}{R} = \frac{Q}{P}a$$

or,

$$\frac{X}{R+X} = \frac{Q}{Q+P}$$

$$X = \frac{Q}{Q+P} \times (R+X) \qquad (4.25)$$

Here, (R + *X)* is the total loop resistance formed by the good cable and the faulty cable. When the cables have the same cross section and same resistivity, their resistances are proportional to their lengths.

If L_X represents the distance of the fault point from the test end, and L is the total length of each cable under test, then we can write

1. The resistance X is proportional to the length L_X
2. The resistance (R + X) is proportional to the total length 2L

Equation (4.25) can now be expressed in terms of the lengths as

Thus, position of the fault can easily be located when the total length of the cables are known.

$$L_X = \frac{Q}{P+X} \times 2L \qquad (4.26)$$

4.5.2 Varley Loop Test

In this test, the total loop resistance involving the cables is determined experimentally to estimate the fault location, rather than relying upon the information of length of cables and their resistances per unit length. Connections diagrams for Varley loop test to detect ground fault location and short-circuit fault location in low voltage cables is shown in Figure 4.24 (a) and Figure 4.24 (b) respectively.

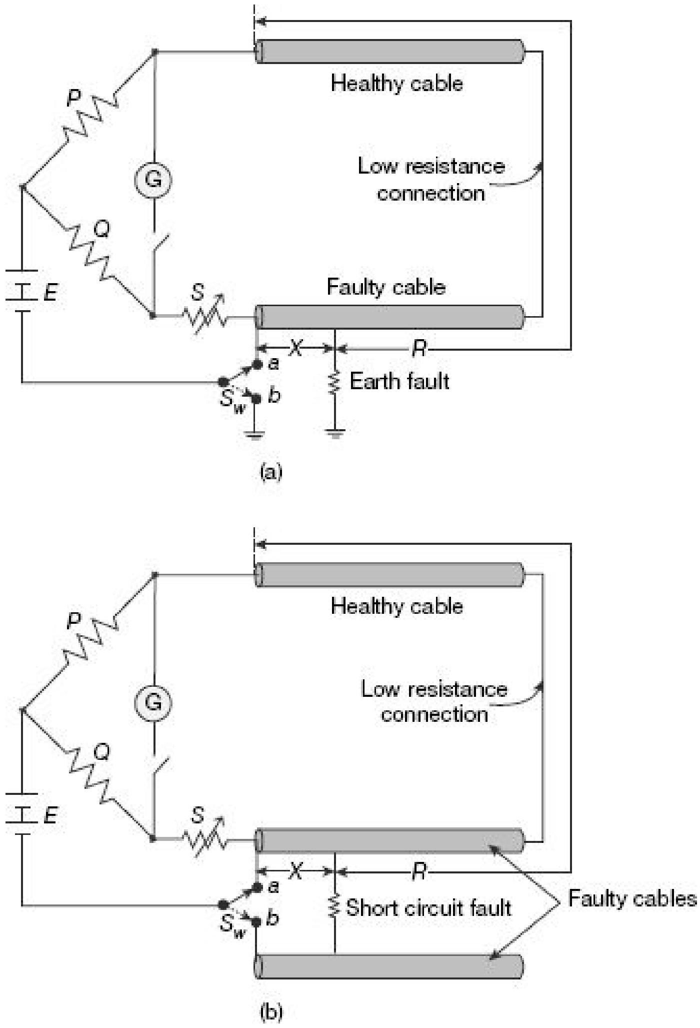


Figure 4.24 *Varley loop test for (a) earth fault, and (b) short-circuit fault localisation in cables*

The single pole double throw switch *Sw* is first connected to terminal 'a' and the resistance *S* is varied to obtain bridge balance.

Let, at this condition, the value of resistance $S = S_1$ when the bridge is balanced. Thus, from Wheatstone bridge principles, at balance condition with the switch at position *a*, we can write

$$\frac{(R+X)}{S_1} = \frac{P}{Q}$$

or, $$(R+X) = \frac{P}{Q} \times S_1 a \qquad (4.27)$$

The total loop resistance (R + *X)* can thus be experimentally determined using (4.27) by reading the values of *P*, *Q* and S_1 under bridge balanced condition with the switch at position *a*.

Now, the switch Sw is changed over to terminal b and the bridge is balanced again by varying S. Let, at this condition, the value of resistance $S = S_2$ when the bridge is balanced. Thus, once again, from Wheatstone bridge principles, at balance condition with the switch at position b, we can write

$$\frac{R}{X+S_2}=\frac{P}{Q}$$

or,

$$\frac{R+X+S_2}{X+S_2}=\frac{P+Q}{Q}$$

$$X=\frac{(R+X)Q-S_2P}{P+Q} \qquad (4.28)$$

Thus, *X* can be calculated from (4.28) from known values of P, Q, and S_2 and the value of total loop resistance (R + *X)* obtained from (4.27).

If L_X represents the distance of the fault point from the test end, and *L* is the total length of each cable under test, then we can write:

1. the resistance *X* is proportional to the length L_X
2. the resistance (R + *X)* is proportional to the total length 2L Then we can write

$$\frac{X}{R+X}=\frac{L_X}{2L}$$

$$L_X=\frac{X}{R+X}\times 2L \qquad (4.29)$$

Thus, the position of the fault can easily be located when the total length of the cables are known.

Both Murray and Varley loop tests are valid only when the cable cross sections are uniform throughout the loop and also between the healthy and faulty cables. Correction factors need also to be included to take care of temperature variation effects. Too many cable joints within length of the cables may also introduce errors in measurement.

Example 4.8

In a test for fault to earth by Murray loop test, the faulty cable has a length of 5.2 km.
The faulty cable is looped with a sound (healthy) cable of the same length and cross section. Resistances of the ratio arm of the measuring bridge circuit are 100 Ω and 41.2 Ω at balance. Calculate the distance of the fault point from the testing terminal.

Solution If *X* is the resistance of the cable from test end to the fault point, *R* is the resistance of cable loop from the fault point back over to the other end of the test point, and *P* and *Q* are the resistance of the ratio arm, then we can write

$$X=\frac{Q}{Q+P}\times(R+X)$$

If L_X be the distance of the fault point from the test end and *L* be the length of each cable. If resistivity of the cable core material *r* is and each of them has cross-sectional area A, then we can write

$$\therefore \quad X = \rho \frac{L_X}{A} \text{ and } (R+X) = \rho \frac{2L}{A}$$

$$\therefore \quad \rho \frac{L_X}{A} = \frac{Q}{P+Q} \times \rho \frac{2L}{A}$$

$$\text{or,} \quad L_X = \frac{Q}{P+Q} \times 2L$$

$$\text{or,} \quad L_X = \frac{41.2}{41.2+100} \times 2 \times 5.2$$

$$\text{or,} \quad L_X = 3.0.3 \text{ km}$$

Thus, the fault has occurred at a distance of 3.03 km from the test end.

Example 4.9

Varley Loop test is being used to locate short circuit fault. The faulty and sound cables are identical with resistances of 0.5 Ω per km. The ratio arms are set at 15 Ω and 40 Ω. Values of the variable resistance connected with the faulty cable are 20 Ω and 10 Ω at the two positions of the selector switch. Determine the length of each cable and fault distance from test end.

Solution Let S_1 be the resistance of the variable resistance at loop resistance measuring position of the selector switch and S_2 be the resistance of the variable resistor at fault location measuring position of the selector switch.

Hence, at the first position of the switch we can write:

$$\frac{(R+X)}{S_1} = \frac{P}{Q}$$

where X is the resistance of the cable from test end to the fault point, R is the resistance of cable loop from the fault point back over to the other end of the test point, and P and Q are the resistance of the ratio arm

Thus, total resistance of the loop is given as

$$(R+X) = \frac{P}{Q} \times S_1 = \frac{15}{40} \times 20 = 7.5 \,\Omega$$

Thus, resistance of each cable = 7.5/2 = 3.75 W
length of each cable = 3.75/0.5 = 7.5 km
At the second position of the switch, we have

$$\frac{R}{X+S_2} = \frac{P}{Q}$$

$$\text{or,} \quad \frac{R+X+S_2}{X+S_2} = \frac{P+Q}{Q}$$

$$\text{or,} \quad X = \frac{(R+X)Q - S_2 P}{P+Q} = \frac{7.5 \times 40 - 10 \times 15}{15+40} = 2.73 \,\Omega$$

. distance of the fault point from test end = 2.73/0.5 = 5.45 km

EXERCISE

Objective-type Questions

1. In a series-type ohmmeter
   (a) zero marking is on the left-hand side
   (b) zero marking is at the centre
   (c) zero marking is on the right-hand side
   (d) zero marking may be either on left or right-hand side
2. In series type ohmmeters, zero adjustment should be done by
   (a) changing the shunt resistance across the meter movement
   (b) changing the series resistance
   (c) changing the series and the shunt resistance
   (d) changing the battery voltage
3. Screw adjustments are preferred over shunt resistance adjustments for zero calibration in ohmmeters because
   (a) the former method is less costly
   (b) the former method does not disturb the scale calibration
   (c) the former method does not disturb the meter magnetic field
   (d) all of the above
4. The shape of scale in an analog series-type ohmmeter is
   (a) linearly spaced (b) cramped near the start
   (c) cramped near the end (d) directly proportional to the resistance
5. Shunt-type ohmmeters have on their scale
   (a) zero ohm marking on the right corresponding to zero current
   (b) zero ohm marking on the right corresponding to full scale current
   (c) infinite ohm marking on the right corresponding to zero current
   (d) infinite ohm marking on the right corresponding to full scale current
6. Shunt-type ohmmeters have a switch along with the battery to
   (a) disconnect the battery when not in use
   (b) prevent meter from getting damaged when measuring very low resistances
   (c) compensate for thermo-emf effects by reversing battery polarity
   (d) all of the above
7. The shape of scale in an analog shunt-type ohmmeter is
   (a) linearly spaced at lower scales (b) cramped near the start
   (c) linearly spaced at higher scales (d) uniform all throughout the scale
8. High resistances using the voltmeter–ammeter method should be measured with
   (a) voltmeter connected to the source side
   (b) ammeter connected to the source side
   (c) any of the two connections
   (d) readings are to be taken by interchanging ammeter and voltmeter positions
9. Low resistances using the voltmeter–ammeter method should be measured with
   (a) voltmeter connected to the source side
   (b) ammeter connected to the source side
   (c) any of the two connections

(d) readings are to be taken by interchanging ammeter and voltmeter positions

10. Accuracy of the substitution method for measurement of unknown resistance depends on
    (a) accuracy of the ammeter
    (b) accuracy of the standard resistance to which the unknown is compared
    (c) accuracy in taking the readings
    (d) all of the above

11. The null detector used in a Wheatstone bridge is basically a
    (a) sensitive voltmeter (b) sensitive ammeter
    (c) may be any of the above (d) none of (a) or (b)

12. Wheatstone bridge is not preferred for precision measurements because of errors due to
    (a) resistance of connecting leads (b) resistance of contacts
    (c) thermo-electric emf (d) all of the above

13. Error due to thermo-electric emf effects in a Wheatstone bridge can be eliminated by
    (a) taking the readings as quickly as possible
    (b) by avoiding junctions with dissimilar metals
    (c) by using a reversing switch to change battery polarity
    (d) all of the above

14. Low resistances are measured with four terminals to
    (a) eliminate effects of leads
    (b) enable the resistance value to be independent of the nature of contact at the current terminals
    (c) to facilitate connections to current and potential coils of the meters
    (d) all of the above

15. Kelvin's double bridge is called 'double' because
    (a) it has double the accuracy of a Wheatstone bridge
    (b) its maximum scale range is double that of a Wheatstone bridge
    (c) it can measure two unknown resistances simultaneously, i.e., double the capacity of a Wheatstone bridge
    (d) it has two additional ratio arms, i.e., double the number of ratio arms as compared to a Wheatstone bridge

16. Two sets of readings are taken in a Kelvin's double bridge with the battery polarity reversed in order to
    (a) eliminate the error due to contact resistance
    (b) eliminate the error due to thermo-electric effect
    (c) eliminate the error due to change in battery voltage
    (d) all of the above

17. Potentiometers, when used for measurement of unknown resistances, give more accurate results as compared to the voltmeter–ammeter method because
    (a) there is no error due to thermo-electric effect in potentiometers
    (b) the accuracy of voltage measurement is higher in potentiometers
    (c) personnel errors while reading a potentiometer is comparatively less
    (d) all of the above

18. 'Null detection method' is more accurate than 'deflection method' for measurement of unknown resistances because
    (a) the former does not include errors due to nonlinear scale of the meters
    (b) the former does not include errors due to change in battery voltage

(c) the former does to depend on meter sensitivity at balanced condition

(d) all of the above

19. Guard terminals are recommended for high resistance measurements to

    (a) bypass the leakage current

    (b) guard the resistance from effects of stray electro-magnetic fields

    (c) guard the resistance from effects of stray electro-static fields

    (d) none of the above

20. When measuring cable insulation using a dc source, the galvanometer used is initially short circuited to

    (a) discharge the stored charge in the cable

    (b) bypass the high initial charging current

    (c) prevent the galvanometer from getting damaged due to low resistance of the cable

    (d) all of the above

21. The loss of charge method is used for measurement of

    (a) high value capacitances

    (b) dissipation factor of capacitances

    (c) low value resistances

    (d) high value resistances

22. A meggar is used for measurement of

    (a) low value resistances

    (b) medium value resistances

    (c) high value, particularly insulation resistances

    (d) all of the above

23. Controlling torque in a meggar is provided by

    (a) control springs

    (b) balance weights

    (c) control coil

    (d) any one of the above

24. The advantage of Varley loop test over Murray loop test for cable fault localisation is

    (a) the former can be used for localising faults even without knowledge of cable resistance

    (b) the former can be used for localising both earth fault and short circuit faults

    (c) the former can experimentally determine the total loop resistance

    (d) all of the above

25. Possible sources of error in using loop test for cable fault localisation are

    (a) uneven cable resistance/km

    (b) temperature variations

    (c) unknown cable joint resistances

    (d) all of the above

Answers

1. (c)	2. (a)	3. (b)	4. (c)	5. (d)	6. (a)	7. (a)
8. (a)	9. (b)	10. (a)	11. (c)	12. (d)	13. (d)	14. (d)
15. (d)	16. (b)	17. (b)	18. (c)	19. (a)	20. (b)	21. (d)
22. (c)	23. (c)	24. (a)	25. (d)			

Short-answer Questions

1. Describe the operation of a series-type ohmmeter with the help of a schematic diagram. Comment on the scale markings and zero adjustment procedures in such an instrument.
2. Derive an expression for the meter current as a function of the full-scale deflection value in a series type ohmmeter to determine the shape of scale.
3. Describe the operation of a shunt-type ohmmeter with the help of a schematic diagram. Comment on the scale markings and zero adjustment procedures in such an instrument.
4. Derive an expression for the meter current as a function of the full-scale deflection value in a shunt type ohmmeter to determine the shape of scale.
5. List the sources of errors in a Wheatstone bridge that may affect its precision while measuring medium range resistances. Explain how these effects are eliminated/minimised?
6. What are the different problems encountered while measuring low resistances. Explain how a 4-terminal configuration can minimise these errors while measuring low resistances using a voltmeter–ammeter method.
7. Describe how low resistances can be measured with the help of a potentiometer.
8. Describe with suitable schematic diagram, how a high resistance can be effectively measured using Price's guard wire method.
9. Explain the principles of the loss of charge method for measurement of high resistances. Also comment on the compensations required to be made in the calculations to take care of circuit component nonidealities.
10. Draw and explain the operation of a meggar used for high resistance measurement.
11. Describe with suitable schematic diagram, the Murray Loop test for localising earth fault in low voltage cables.
12. Describe with suitable schematic diagram, the Varley loop test for localising earth fault in low voltage cables.

Long-answer Questions

1. (a) Discuss with suitable diagrams, the different ways of zero adjustment in a series-type ohmmeter.

   (b) Design a single range series-type ohmmeter using a PMMC ammeter that has internal resistance of 60 Ω and requires a current of 1.2 mA for full scale deflection. The internal battery has a voltage of 5 V. It is desired to read half scale at a resistance value of 3000 Ω. Calculate (a) the values of shunt resistance and current limiting series resistance, and (b) range of values of the shunt resistance to accommodate battery voltage variation in the range 4.7 to 5.2 V.
2. (a) Describe the operation of a shunt-type ohmmeter with the help of a schematic diagram. Comment son the scale markings and zero adjustment procedures in such an instrument?

   (b) A shunt-type ohmmeter uses a 2 mA basic d'Arsonval movement with an internal resistance of 50 Ω. The battery emf is 3 V. Calculate (a) value of the resistor in series with the battery to adjust the FSD, and (b) at what point (percentage) of full scale will 200 Ω be marked on the scale?
3. (a) Describe in brief, the use of voltmeter–ammeter method for measurement of unknown resistance.

   (b) A voltmeter of 500 Ω resistance and a milliammeter of 0.5 Ω resistance are used to measure two unknown resistances by voltmeter–ammeter method. If the voltmeter reads 50 V and milliammeter reads 50 mA in both the cases, calculate the percentage error in the values of measured resistances if (i) in the first case, the voltmeter is put across the resistance and the milliammeter connected in series with the supply, and (ii) in the second case, the voltmeter is connected in the supply, side and milliammeter connected directly in series with the resistance.
4. (a) Draw the circuit of a Wheatstone bridge for measurement of unknown resistances and derive the condition for balance.

   (b) Four arms of a Wheatstone bridge are as follows: AB = 150 W, BC = 15 W, CD = 6 W, DA = 60 Ω. A galvanometer with internal resistance of 25 Ω is connected between BD, while a battery of 20 V dc is connected between AC. Find the current through the galvanometer. Find the value of the resistance to be put on the arm DA so that the bridge is balanced.
5. (a) Explain the principle of working of a Kelvin's double bridge for measurement of unknown low resistances. Explain how the effects of contact resistance and resistance of leads are eliminated.

(b) A 4-terminal resistor was measured with the help of a Kelvin's double bridge having the following components: Standard resistor = 100.02 |iW, inner ratio arms = 100.022 Ω and 199 W, outer ratio arms = 100.025 Ω and 200.46 W, resistance of the link connecting the standard resistance and the unknown resistance = 300 mW. Calculate value of the unknown resistance.

6. Discuss the difficulties involved for measurement of high resistances. Explain the purpose of guarding a high resistance measurement circuits.

7. (a) Derive an expression for the unknown resistance measured using the loss of charge method.

(b) A cable is tested for insulation resistance by loss of charge method. An electrostatic voltmeter is connected between the cable conductor and earth. This combination is found to form a capacitance of 600 pF between the conductor and earth. It is observed that after charging the cable with 1000 V for sufficiently long time, when the voltage supply is withdrawn, the voltage drops down to 480 V in 1 minute. Calculate the insulation resistance of the cable.

8. (a) Describe with suitable schematic diagrams, the Murray loop test for localization of earth fault and short circuit fault in low voltage cables.

(b) n a test for fault to earth by Murray loop test, the faulty cable has a length of 6.8 km. The faulty cable is looped with a sound (healthy) cable of the same length and cross section. Resistances of the ratio arm of the measuring bridge circuit are 200 Ω and 444 Ω at balance. Calculate the distance of the fault point from the testing terminal.

9. (a) Describe with suitable schematic diagrams, the Varley loop test for localisation of earth fault and short-circuit fault in low voltage cables.

(b) Varley loop test is being used to locate short circuit fault. The faulty and sound cables are identical with resistances of 0.4 Ω per km. The ratio arms are set at 20 Ω and 50 Ω. Values of the variable resistance connected with the faulty cable are 30 Ω and 15 Ω at the two positions of the selector switch. Determine the length of each cable and fault distance from test end.

5 Potentiometers

5.1 INTRODUCTION

A potentiometer is an instrument which is used for measurement of potential difference across a known resistance or between two terminals of a circuit or network of known characteristics. A potentiometer is also used for comparing the emf of two cells. A potentiometer is extensively used in measurements where the precision required is higher than that can be obtained by ordinary deflecting instruments, or where it is required that no current be drawn from the source under test, or where the current must be limited to a small value.

Since a potentiometer measures voltage by comparing it with a standard cell, it can be also used to measure the current simply by measuring the voltage drop produced by the unknown current passing through a known standard resistance. By the potentiometer, power can also be calculated and if the time is also measured, energy can be determined by simply multiplying the power and time of measurement. Thus potentiometer is one of the most fundamental instruments of electrical measurement.

Some important characteristics of potentiometer are the following:

- A potentiometer measures the unknown voltage by comparing it with a known voltage source rather than by the actual deflection of the pointer. This ensures a high degree of accuracy.
- As a potentiometer measures using null or balance condition, hence no power is required for the measurement.
- Determination of voltage using potentiometer is quite independent of the source resistance.

5.2 A BASIC dc POTENTIOMETER

The circuit diagram of a basic dc potentiometer is shown in Figure 5.1.

Operation

First, the switch *S* is put in the 'operate' position and the galvanometer key *K* kept open, the battery supplies the working current through the rheostat and the slide wire. The working current through the slide wire may be varied by changing the rheostat setting. The method of measuring the unknown voltage, E_1, depends upon the finding a position for the sliding contact such that the galvanometer shows zero deflection, i.e., indicates null condition, when the galvanometer key K is closed. Zero galvanometer deflection means that the unknown voltage E_1 is equal to the voltage drop E_2, across position a–c of the slide wire. Thus, determination of the values of unknown voltage now becomes a matter of evaluating the voltage drop E_2 along the portion a–c of the slide wire.

When the switch S is placed at ‘ calibrate’ position, a standard or reference cell is connected to the circuit. This reference cell is used to standardize the potentiometer. The slide wire has a uniform cross-section and hence uniform resistance along its entire length. A calibrated scale in cm and fractions of cm, is placed along the slide wire so that the sliding Figure 5.1 *A basic potentiometer circuit* contact can be placed accurately at any desired position along the slide wire. Since the resistance of the slide wire is known accurately, the voltage drop along the slide wire can be controlled by adjusting the values of working current. The process of adjusting the working current so as to match the voltage drop across a portion of sliding wire against a standard reference source is known as ‘ standardisation’.

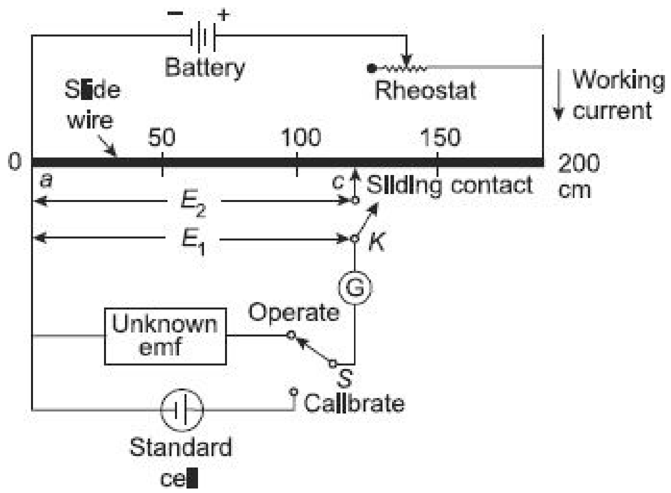


Figure 5.1 *A basic potentiometer circuit*

5.3 CROMPTON’S dc POTENTIOMETER

The general arrangement of a laboratory-type Crompton’s dc potentiometer is shown in Figure 5.2. It consists of a dial switch which has fifteen (or more) steps. Each steep has 10 Ω resistance. So the dial switch has total 150 Ω resistance. The working current of this potentiometer is 10 mA and therefore each step of dial switch corresponds to 0.1 volt. So the range of the dial switch is 1.5 volt.

The dial switch is connected in series with a circular slide wire. The circular slide wire has 10 Ω resistance. So the range of that slide wire is 0.1 volt. The slide wire calibrated with 200 scale divisions and since the total resistance of slide wire corresponds to a voltage drop of 0.1 volt, each division of the slide wire corresponds to $\frac{0.1}{200} = 0.0005$ volt. It is quite comfortable to interpolate readings up to $\frac{1}{5}$ of a scale division and therefore with this Crompton’s potentiometer it is possible to estimate the reading up to 0.0001 volt.

Procedure for Measurement of Unknown emf

- At first, the combination of the dial switch and the slide wire is set to the standard cell voltage. Let the standard sell voltage be 1.0175 volts, then the dial resistor is put in 1.0 volt and the slide wire at 0.0175 volts setting.
- The switch '*S*' is thrown to the calibrate position and the galvanometer switch '*K*' is pressed until the rheostat is adjusted for zero deflection on the galvanometer. The 10 kW protective resistance is kept in the circuit in the initial stages so as to protect the galvanometer from overload.
- After the null deflection on the galvanometer is approached the protective resistance is shorted so as to increase the sensitivity of the galvanometer. Final adjustment is made for the zero deflection with the help of the rheostat. This completes the standardisation process of the potentiometer.
- After completion of the standardisation, the switch '*S*' is thrown to the operate position thereby connecting the unknown emf into the potentiometer circuit. With the protective resistance in the circuit, the potentiometer is balanced by means of the main dial and the slide wire adjustment.
- As soon as the balanced is approached, the protective resistance is shorted and final adjustments are made to obtain true balance.
- After the final true balance is obtained, the value of the unknown emf is read off directly from the setting of the dial switch and the slide wire.
- The standardisation of the potentiometer is checked again by returning the switch '*S*' to the calibrate position. The dial setting is kept exactly the same as in the original standardisation process. If the new reading does not agree with the old one, a second measurement of unknown emf must be made. The standardisation again should be made after the measurement.

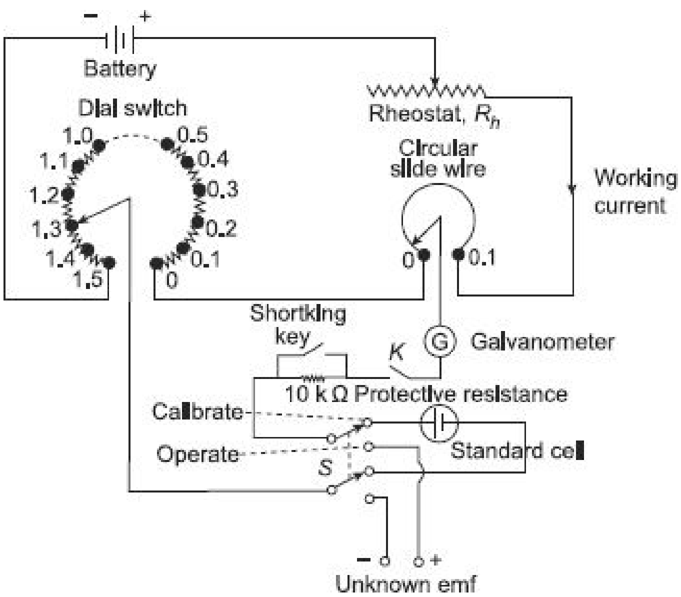


Figure 5.2 *General arrangement of Crompton's dc potentiometer*

A basic slide-wire potentiometer has a working battery

Example 5.1 *voltage of 3.0 volts with negligible resistance. The resistance of the slide-wire is 400 Ω and its length is 200 cm. A 200-cm scale is placed along the slide wire. The slide-wire has 1 mm scale divisions and it is possible to read up to 1/5 of a division. The instrument is standardised with 1.018 volt standard cell with with sliding contact at the 101.8 cm mark on scale.*

Calculate

(a) *Working current*

(b) *Resistance of series rheostat*

(c) *Measurement range*

(d) *Resolution of the instrument*

Solution

(a) Working current, I_m

Because the instrument is standardised with an emf of 1.018 volts with sliding contact at 101.8 cm, it is obvious that a length 101.8 cm represents a voltage of 1.018 volts.

$$\text{Resistance of 101.8 cm length of wire} = \frac{1.018}{200} \times 400 = 203.6\ \Omega$$

$$\therefore \quad \text{working current, } I_m = \frac{101.8}{203.6} = 0.005 \text{ A or 5 mA.}$$

(b) Resistance of series rheostat, R_h

Total resistance of battery circuit = Resistance of rheostat (R_h) + Resistance of slide wire.

$\therefore R_h$ = Total resistance – Resistance of slide wire

$$= \frac{3}{0.005} - 400 = 200\ \Omega$$

(c) Measurement range

The measurement range is the total voltage across the slide wire.

$\therefore$ range of voltage = 0.005 ×400 = 2.0 volt.

(d) Resolution of the instrument

A length of 200 cm represents 2.0 volt and therefore 1 mm represents a voltage ofê2 ê 1

$$\left(\frac{2}{200}\right) \times \left(\frac{1}{10}\right) = 1 \text{ mV}$$

Since it is possible to read $\frac{1}{5}$ of I mV, therefore, resolution of the instrument is $\frac{1}{5}$ × 1 = 0.2 mV

A single-range laboratory-type potentiometer has an 18-step dial switch where each step represents 0.1 volt. The

Example 5.2 *dial resistors are 10 Ω each. The slide wire of the potentiometer is circular and has 11 turns and a resistance of 1 Ω per turn. The slide wire has 100 divisions and interpolation can be done to one forth of a division. The working battery has a voltage of 6.0 volt. Calculate (a) the measuring range of the potentiometer, (b) the resolution, (c) working current, and (d) setting of the rheostat.*

SolutionDial resistor = 10 Weach

Each step = 0.1 v

$\therefore$ working current $= \frac{0.1}{10} = 10$ mA

(a) The measuring range of the potentiometer

Total resistance of measuring circuit,

R_m= Resistance of dial + resistance of slide wire

or, $R_m = 18 \times 10 + 11 = 191$

voltage range of the total instrument

$= R_m \times$working current

= 191 ×10 mA

= 1.91 V

(b) The resolution

The slide wire has a resistance of 11 Ω and therefore voltage drop across slide wire

= 11 × 10 mA = 0.11 volt

The slide wire has 11 turns, and therefore voltage drop across each turn

$= \frac{0.11}{11} = 0.01$ volt

Each turn is divided into 100 divisions and therefore each division represents a voltage drop of $\frac{0.01}{100} = 0.0001$.

Since each turn can be interpolated to of a division,

Resolution of instrument =×0 0001 =0 000025 volt =25 mV

(c) Working current, I_m

As previously mentioned, working current = 1 mA.

(d) Setting of rheostat

Total resistance across battery circuit $\frac{6}{0.01} = 600\ \Omega$

Total resistance of potentiometer circuit is 191 Ω

∴ resistance of series rheostat, R_h = 600 - 191 = 409 W.

5.4 APPLICATIONS OF dc POTENTIOMETERS

Practical uses of dc potentiometers are

- Measurement of current
- Measurement of high voltage
- Measurement of resistance
- Measurement of power
- Calibration of voltmeter
- Calibration of ammeter
- Calibration of wattmeter

5.4.1 Measurement of Current by Potentiometer

The circuit arrangement for measurement of current by a potentiometer is shown in Figure 5.3. The unknown current *I*, whose value is to be measured, is passed through a standard resistor *R* as shown. The standard resistor should be of such a value that voltage drop across it caused by flow of current to be measured, may not exceed the range of the potentiometer. Voltage drop across the standard resistor in volts divided by the value of *R* in ohms gives the value of unknown current in amperes.

$$\text{i.e., unknown current (in Amp) } I = \frac{\text{Voltage drop across } R \text{ (Volt)}}{\text{Value of } R \text{ (ohms)}}$$

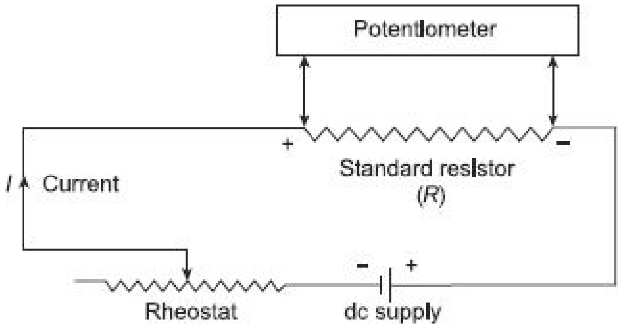


Figure 5.3 *Measurement of current with potentiometer*

Example 5.3

A simple slide wire potentiometer is used for measurement of current in a circuit. The voltage drop across a standard resistor of 0.1 Ω is balanced at 75 cm. find the magnitude of the current if the standard cell emf of 1.45 volt is balanced at 50 cm.

Solution For the same working current, if 50 cm corresponds to 1.45 volt. Then 75 cm of the slide wire corresponds to

$$= \frac{1.45}{50} \times 75 = 2.175 \text{ volt}$$

So, across the resistance 0.1 ? the voltage drop is 2.175 volt. Then the value of the current is

$$I = \frac{2.175}{0.1} = 21.75 \text{ A}$$

5.4.2 Measurement of High Voltage by Potentiometer

Special arrangements must be made to measure very high voltage by the potentiometer (say a hundreds of volts) as this high voltage is beyond the range of normal potentiometer. The voltage above the direct range of potentiometer (generally 1.8 volt) can be measured by using a volt–ratio box in conjunction with the potentiometer. The volt–ratio box consists of a simple resistance potential divider with various tapping on the input side. The arrangement is shown in Figure 5.4. Each input terminal is marked with the maximum voltage which can be applied and with the corresponding multiplying factor for the potential scale.

High emf to be measured is applied the suitable input terminal of volt–ratio box and leads to the potentiometer are taken from two tapping points intended for this purpose.

The potential difference across these two points is measured by the potentiometer. If the voltage measured by the potentiometer is v and k be the multiplying factor of the volt–ratio box, then the high voltage to be measured is $V = kv$ volt.

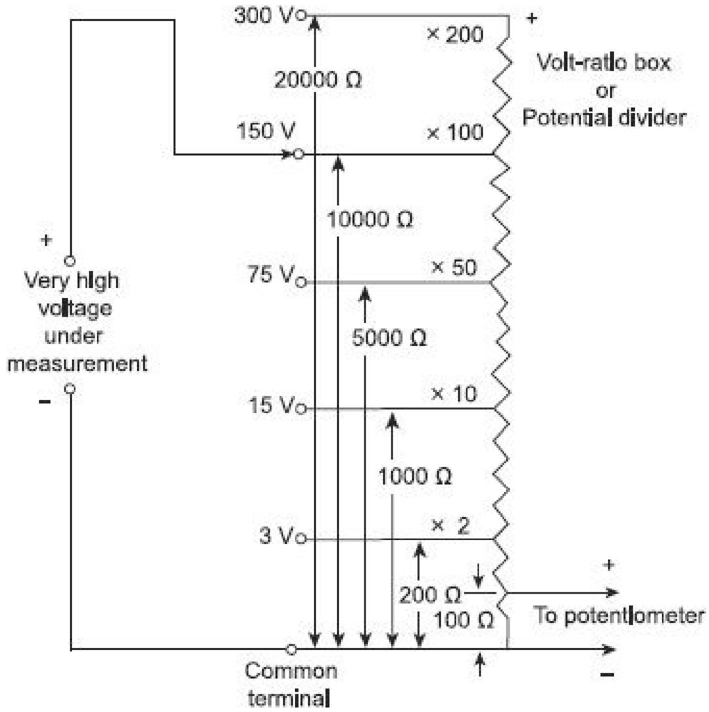


Figure 5.4 *Measurement of high voltage by potentiometer in conjunction with volt–ratio box*

5.4.3 Measurement of Resistance by Potentiometer

The connection diagram for measuring unknown resistance with the help of potentiometer is shown in Figure 5.5. The unknown resistance R, is connected in series with the known standard resistor S. The rheostat connected in the circuit controls the current flowing through the circuit. An ammeter is also connected in the circuit to indicate whether the value of the working current is within the limit of the potentiometer or not. Otherwise, the exact value of the working current need not be known.

When the two–pole double throw switch is put in position 1, the unknown resistance is connected to the potentiometer. Let the reading of the potentiometer in that position is V_R. Then

$$V_R = IR \tag{5.1}$$

Now the switch is thrown to position 2, this connects the standard resistor S to the potentiometer. If the reading of the potentiometer is that position is V_S then,

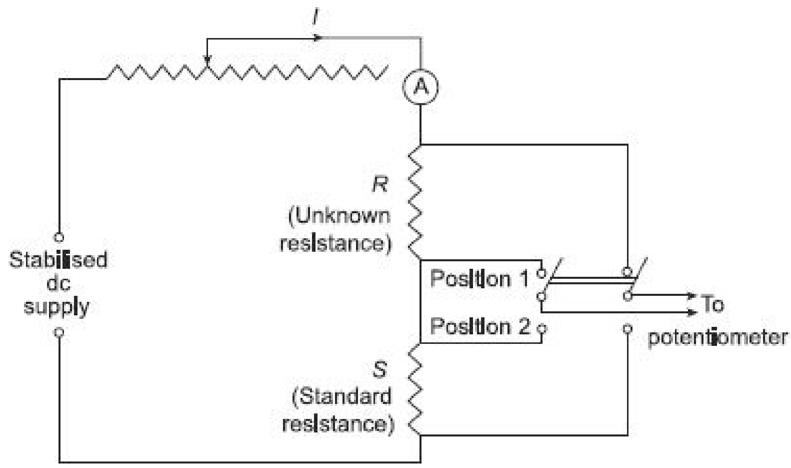


Figure 5.5 *Measurement of resistance by potentiometer*

$$V_S = IS \tag{5.2}$$

Dividing (5.1) by (5.2), we get

$$\frac{V_R}{V_S} = \frac{IR}{IS}$$

or $$R = \frac{V_R}{V_S} \times S$$

The value of R can be calculated accurately since the value of the standard resistor S is known. This method of measurement of resistance is used for low value of the resistor.

5.4.4 Measurement of Power by Potentiometer

In measurement of power by potentiometer the measurements are made one across the standard resistor S connected in series with the load and another across the volt–ratio box output terminals. The arrangement is shown in Figure 5.6.

The load current which is exactly equal to the current through the standard resistor S, as it is connected in series with the load, is calculated from the voltage drop across the standard resistor divided by the value of the standard resistor S.

Load current $I = \frac{V_S}{S}$

where V_S = voltage drop across standard resistor S as measured by the potentiometer.

Voltage drop across the load is found by the output terminal of the volt–ratio box. If V_R is the voltage drop across the output terminal of the volt–ratio box and V_L is the voltage drop across load then,

$$V_L = k \times V_R$$

where k is the multiplying factor of the volt–ratio box.

Then the power consumed, $P = V_L I = k \times V_R \times \frac{V_S}{S}$

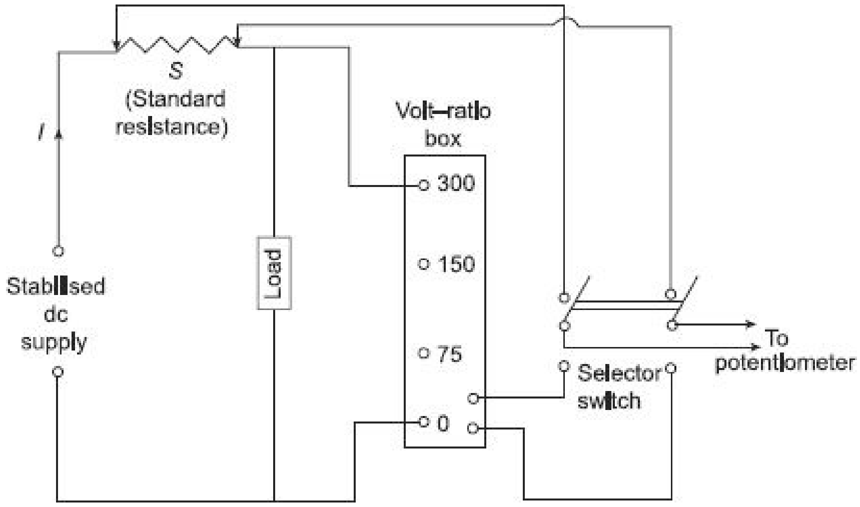


Figure 5.6 *Measurement of power by potentiometer*

5.4.5 Calibration of Voltmeter by Potentiometer

In case of calibration of voltmeter, the main requirement is that a suitable stable dc voltage supply is available, otherwise any change in the supply voltage will cause a change in the calibration process of the voltmeter.

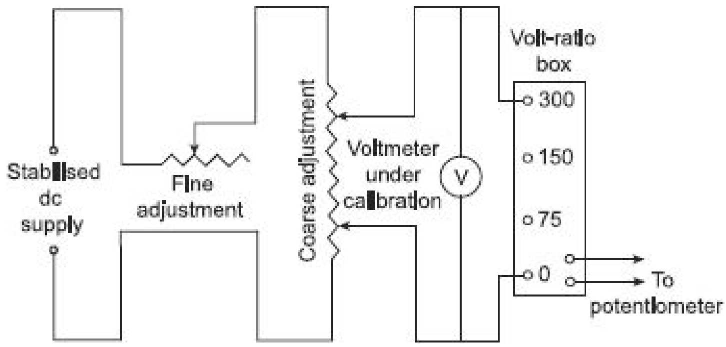


Figure 5.7 *Calibration of voltmeter by potentiometer*

The arrangement for calibrating a voltmeter by potentiometer is shown in Figure 5.7. The potential divider network consists of two rheostats. One for coarse and the other for fine control of calibrating voltage. With the help of these controls, it is possible to adjust the supply voltage so that the pointer coincides exactly with a major division of the voltmeter.

The voltage across the voltmeter is steeped down to a value suitable for the potentiometer with the help of the volt–ratio box. In order to get accurate measurements, it is necessary to measure voltages near the maximum range of the potentiometer, as far as possible.

The potentiometer measures the true value of the voltage. If the reading of the potentiometer does not match with the voltmeter reading, a positive or negative error is indicated. A calibration curve may be drawn with the help of the potentiometer and the voltmeter reading.

5.4.6 Calibration of Ammeter by Potentiometer

Figure 5.8 shows the circuit arrangement for calibration of an ammeter using potentiometer.

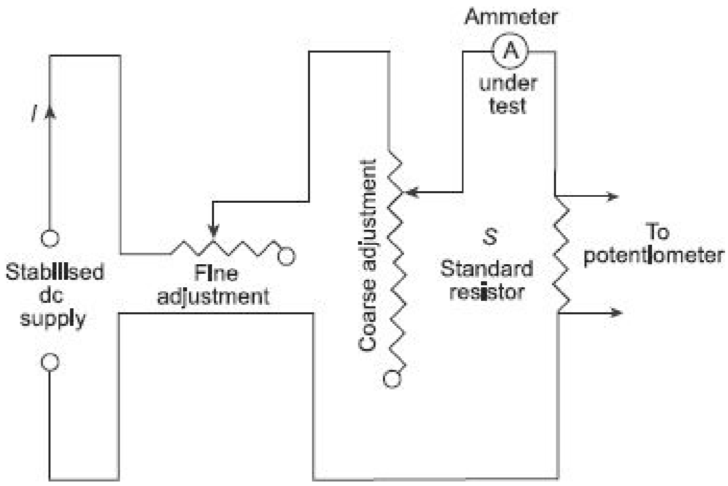


Figure 5.8s *Calibration of ammeter by potentiometer*

A standard resistor *S* of high current carrying capacity is placed in series with the ammeter under test. The voltage drop across *S* measured with the help of the potentiometer and then the current through *S* and hence the ammeter can be computed by dividing the voltage drop by the value of the standard resistor.

Current, $I = \frac{V_S}{S}$ is the voltage drop across the standard resistor *S*.

Now, compare the reading of the ammeter with the current found by calculation. If they do not match, a positive or negative error will be induced. A calibration curve may be drawn between the ammeter reading and the true value of the current as indicated by the potentiometer reading.

As the resistance of the standard resistor *S* is exactly known, the current through *S* is exactly calculated. This method of calibration of ammeter is very accurate.

5.4.7 Calibration of Wattmeter by Potentiometer

In this calibration process, the current coil of the wattmeter is supplied from low voltage supply and potential coil from the normal supply through potential divider. The voltage *V* across the potential coil of the wattmeter under calibration is measured directly by the potentiometer. The current through the current coil is measured by measuring the voltage drop across a standard resistor connected in series with the current coil divided by the value of the standard resistor.

The true power is then *VI*, where *V* is the voltage across the potential coil and *I* is the current through the current coil of the wattmeter. The wattmeter reading may be compared with this value, and a calibration curve may be drawn.

The arrangement for calibrating a wattmeter is shown in Figure 5.9.

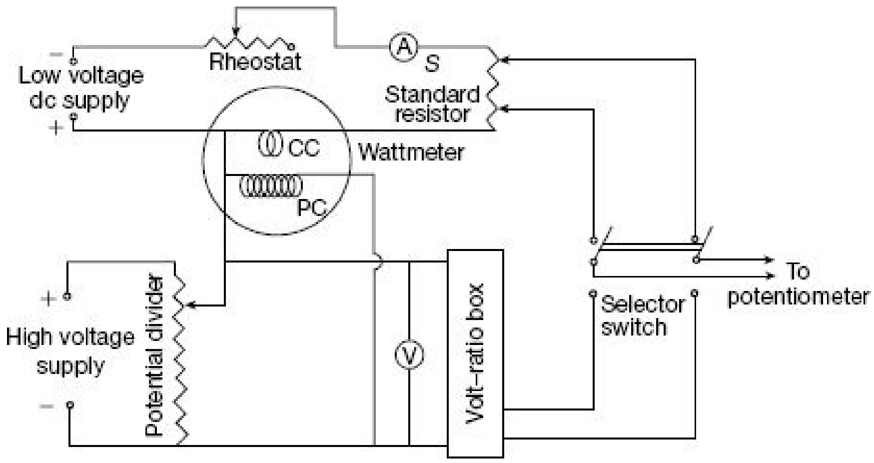


Figure 5.9 *Calibration of wattmeter by dc potentiometer*

Example 5.4

The emf of a standard cell used for standardisation is 1.0186 volt. If the balanced is achieved at a length of 55 cm, determine

(a) The emf of the cell which balances at 70 cm

(b) The current flowing through a standard resistance of 2 Ω if the potential difference across it balances at 60 cm

(c) The voltage of a supply main which is reduced by a volt–ratio box to one hundredth and balance is obtained at 85 cm

(d) The percentage error in a voltmeter reading 1.40 volt when balance is obtained at 80 cm

(e) The percentage error in ammeter reading 0.35 ampere when balance is obtained at 45 cm with the potential difference across a 2.5 Ω resistor in the ammeter circuit

Solution Emf of the standard cell = 1.0186 volts

The voltage drop per cm length of potentiometer wire,

$$v = \frac{1.0186}{55} = 0.01852$$

(a) The emf of a cell balanced at 70 cm,

$= v \cdot l = 0.01852 \times 70 = 1.2964$ volt

(b) The potential difference which is balanced at 60 cm

$= v \cdot l = 0.01852 \times 60 = 1.1112$ volt

Magnitude of the standard resistor, S =2 W

Therefore, current flowing through 2 Ω resistance$V$

$$= \frac{V}{S} = \frac{1.1112}{2} = 0.5556 \text{ A}$$

(c) The potential difference which balances at 85 cm,

$V = v \cdot l = 0.01852 \times 85 = 1.5742$ volt

voltage of supply main = V × ratio of volt–ratio box

1.5742 × 100 = 157.42 volt

(d) The potential difference which balances at 80 cm,

$V = v \cdot l = 0.01852 \times 80 = 1.4816$ volt

Voltmeter reading = 1.40 volt

percentage error in voltmeter reading

$$= \frac{1.4 - 1.4816}{1.4816} \times 100 = -5.507\%$$

(e) The potential difference which balances at 45 cm,

$V = v \cdot l = 0.01852 \times 45 = 0.8334$ volt

Current flowing through 2.5 Ω resistance$V$ 0.8334

$$I = \frac{V}{S} = \frac{0.8334}{2.5} = 0.33336 \text{ A}$$

percentage error in ammeter reading

$$= \frac{0.35 - 0.33336}{0.33336} \times 100 = 4.991\%$$

Example 5.5

The following readings were obtained during the measurement of a low resistance using a potentio meter:
Voltage drop across a 0.1 Ω standard resistance = 1.0437 V
Voltage drop across the low resistance under test = 0.4205 V
Calculate the value of unknown resistance, current and power lost in it.

Solution Given: S= 0.1 Ω; V_S= 1.0437 V; V_R= 0.4205 V

Resistance of unknown resistor, $R = \frac{V_R}{V_S} \times S = \frac{0.4205}{1.0437} \times 0.1 = 0.04\ \Omega$

Current through the resistor, $I = \frac{V_S}{S} = \frac{1.0437}{0.1} = 10.437$ A

Power loss, $PI^2 R = (10.437)^2 \times 0.04 = 4.357$ Wa

Example 5.6

A Crompton's potentiometer consists of a resistance dial having 15 steps of 10 Ω each and a series connected slide wire of 10 Ω which is divided into 100 divisions. If the working current of the potentio meter is 10 mA and each division of slide wire can be read accurately upto 1/5 th of its span, calculate the resolution of the potentiometer in volts.

Solution Total resistance of the potentiometer,

R = Resistance of the dial + Resistance of the slide wire

= 15 × 10 + 10 = 160 W

Working current, I = 10 mA = 0.01 A

Voltage range of the potentiometer = Working current × Total resistance of the potentiometer

= 0.01 × 160 = 1.6

Voltage drop across slide wire = Working current × Slide wire resistance

= 0.01 × 10 = 0.1 V

Since slide wire has 100 divisions, therefore, each division represents $\frac{0.1}{100}$ or 0.001 volt

As each division of slide wire can be read accurately up to $\frac{1}{5}$ potentiometer of its span, therefore, resolution of the potentiometer

$$= \frac{0.001}{5} = 0.0002 \text{ volt}$$

5.5 POTENTIOMETERS

An ac potentiometer is same as dc potentiometer by principle. Only the main difference between the ac and dc potentiometer is that, in case of dc potentiometer, only the magnitude of the unknown emf is compared with the standard cell emf, but in ac potentiometer, the magnitude as well as phase angle of the unknown voltage is compared to achieve balance.

This condition of ac potentiometer needs modification of the potentiometer as constructed for dc operation.

The following points need to be considered for the satisfactory operation of the ac potentiometer:

1. To avoid error in reading, the slide wire and the resistance coil of an ac potentiometer should be non-inductive.
2. The reading is affected by stray or external magnetic field, so in the time of measurement they must be eliminated or measured and corresponding correction factor should be introduced.
3. The sources of ac supply should be free from harmonics, because in presence of harmonics the balance may not be achieved.
4. The ac source should be as sinusoidal as possible.
5. The potentiometer circuit should be supplied from the same source as the voltage or current being measured.

5.6 CLASSIFICATION OF AC POTENTIOMETERS

There are two general types of ac potentiometers:

1. Polar Potentiometer

As the name indicates, in these potentiometers, the unknown emf is measured in polar form, i.e., in terms of its magnitude and relative phase. The magnitude is indicated by one scale and the phase with respect to some reference axis is indicated by another scale. There is provision for reading phase angles up to 360°.

The voltage is read in the form $V - \theta$.

Example: Drysdale polar potentiometer

2. Coordinate Potentiometer

Here, the unknown emf is measured in Cartesian form. Two components along and perpendicular to some standard axis are measured and indicated directly by two different scales known as in phase (V_1) and quadrature (V_2) scales (Figure 5.10). Provision is made in this instrument to read both positive and negative values of voltages so that all angles up to 360° are covered.

$$\text{Voltage } V = \sqrt{(V_1)^2 + (V_2)^2}; \ \theta = \tan^{-1}\left(\frac{V_2}{V_1}\right)$$

Example: Gall–Tinsley and Campbell–Larsen type potentiometer

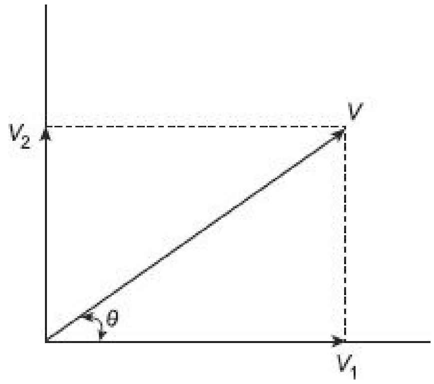


Figure 5.10 *Polar and coordinate representation of unknown emf*

5.6.1 Drysdale Polar Potentiometer

The different components of a Drysdale polar potentiometer is shown in Figure 5.11.

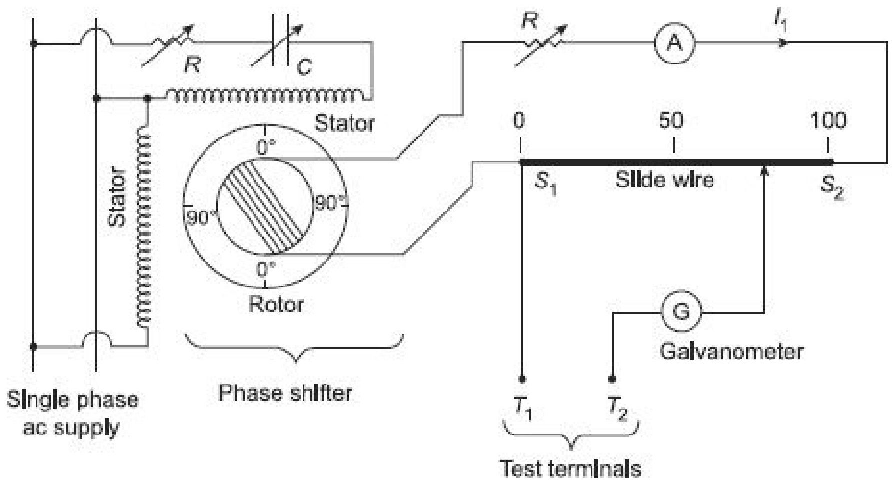


Figure 5.11 *Drysdale polar potentiometer*

The slide wire S_1–S_2 is supplied from a phase shifting circuit for ac measurement. The phase shifting circuit is so arranged that the magnitude of the voltage supplied by it remains constant while its phase can be varied through 360°. Consequently, slide wire current can be maintained constant in magnitude but varied in phase.

The phase shifting circuit consists of two stator coils connected in parallel supplied from the same source; their currents are made to differ by 90° by using very accurate phase shifting technique. The two windings produce rotating flux which induces a secondary emf in the rotor winding which is of constant magnitude but the phase of which can be varied by rotating the rotor in any position. The phase of the rotor emf is read from the circular dial attached in the potentiometer.

Before the ac measurement, the potentiometer is first calibrated by using dc supply for slide wire and standard cell for test terminals T_1 and T_2. The unknown alternating voltage to be measured is applied across test terminals and the balance is achieved by varying the slide wire contact and the position of the rotor. The ammeter connected in the slide wire circuit gives the magnitude of the unknown emf and the circular dial in the rotor circuit gives the phase angle of it.

5.6.2 Gall Coordinate Potentiometer

The Gall coordinate potentiometer consists of two separate potentiometer circuit in a single case. One of them is called the '*in-phase*' potentiometer and the other one is called the *quadrature* potentiometer. The slide-wire circuits of these two potentiometers are supplied with two currents having a phase difference of 90°. The value of the unknown voltage is obtained by balancing the voltages of *in-phase* and *quadrature* potentiometers slide wire simultaneously. If the measured values of *in-phase* and *quadrature* potentiometer slide-wires are V_1 and V_2 respectively then the magnitude of the unknown voltage is $V=\sqrt{V_1^2+V_2^2}$ $V = V_1^2 + V_2^2$ and the phase angle of the unknown voltage is given by $q = \tan^{-1}\frac{V_2}{V_1}$.

Figure 5.12 shows the schematic diagram of a Gall coordinate-type potentiometer. *W–X* and *Y–Z* are the sliding contacts of the *in-phase* and *quadrature* potentiometer respectively. *R* and *R′* are two rheostats to control the two slide-wire currents. The *in-phase* potentiometer slide-wire is supplied from a single-phase supply and the *quadrature* potentiometer slide-wire is supplied from a phase-splitting device to create a phase difference of 90° between the two slide-wire currents. T_1 and T_2 are two step-down transformers having an output voltage of 6 volts. These transformers also isolate the potentiometer from the high-voltage supply. *R* and *C* are the variable resistance and capacitance for phase-splitting purpose. *VG* is a vibration galvanometer which is tuned to the supply frequency and *K* is the galvanometer key. *A* is a dynamometer ammeter which is used to display the current in both the slide-wires so that they can be maintained at a standard value of 50 mA. SW_1 and SW_2 are two *sign- changing* switches which may be necessary to reverse the direction of unknown emf applied to the slide wires. SW_3 is a selector switch and it is used to apply the unknown voltage to the potentiometer.

Operation Before using the potentiometer for ac measurements, the current in the *in-*

phase potentiometer slide wire is first standardised using a standard dc cell of known value. The vibration galvanometer *VG* is replaced by a D'Arsonval galvanometer. Now the *in-phase* slide wire current is adjusted to the standard value of 50 mA by varying the rheostat *R*. This setting is left unchanged for ac calibration; the dc supply is replaced by ac and the D'Arsonval galvanometer by the vibration galvanometer.

The magnitude of the current in the quadrature potentiometer slide wire must be equal to the in- phase potentiometer slide wire current and the two currents should be exactly in quadrature. The switch SW_3 is placed to *test position* (as shown in Figure 5.12) so that the emf induced in the secondary winding of mutual inductance *M* is impressed across the *in-phase* potentiometer wire through the vibration galvanometer. Since the induced emf in the secondary of mutual inductance *M* will be equal to *2pfMi* volt in magnitude; where *f* is the supply frequency and will lag 90° behind the current in the quadrature slide-wire *i*, so the value of emf calculated from the relation *e*' =2*pf Mi* for a current *i* = 50 mA is set on *in-phase* potentiometer slide wire and *R*' are adjusted till exact balance point is obtained. At balance position, the current in the potentiometer wires will be exactly equal to 50 mA in magnitude and exactly in quadrature with each other. The polarity difference between the two circuits is corrected by changing switches SW_1 and SW_2.

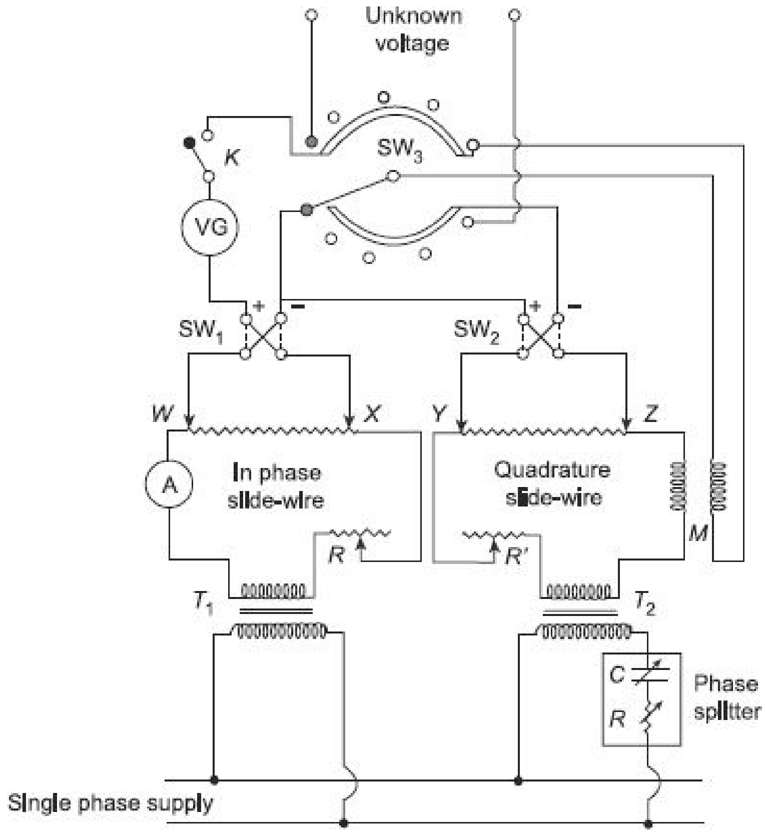


Figure 5.12 *Gall coordinate potentiometer*

Lastly, the unknown voltage is applied to the potentiometer by means of the switch SW_3 and balance is obtained on both the potentiometer slide-wire by adjusting the slide-wire setting. The reading of slide-wire *WX* gives the in-phase component (V_1) and slide wire *YZ* gives quadrature component (V_2) of the unknown voltage.

ADVANTAGES AND DISADVANTAGES OF ac

5.7 POTENTIOMETERS

Advantages

1. An ac potentiometer is a very versatile instrument. By using shunt and volt–ratio box, it can measure wide range of voltage, current and resistances.
2. As it is able to measure phase as well as magnitude of two signals, it is used to measure power, inductance and phase angle of a coil, etc.
3. The principle of ac potentiometer is also incorporated in certain special application like Arnold circuit for the measurement of CT (Current Transformer) errors.

Disadvantages

1. A small difference in reading of the dynamometer instrument either in dc or ac calibration brings on error in the alternating current to be set at standard value.
2. The normal value of the mutual inductance *M* is affected due to the introduction of mutual inductances of various potentiometer parts and so a slight difference is observed in the magnitude of the current of quadrature wire with compared to that in the in–phase potentiometer wire.
3. Inaccuracy in the measured value of frequency will also result in the quadrature potentiometer wire current to differ from that of in–phase potentiometer wire.
4. The presence of mutual inductances in the various parts of the potentiometer and the inter capacitance, the potential gradient of the wires is affected.
5. Since the standardisation is done on the basis of rms value and balance is obtained dependent upon the fundamental frequency only, therefore, the presence of harmonics in the input signal introduces operating problem and the vibration galvanometer tuned to the fundamental frequency may not show full null position at all.

5.8 APPLICATIONS OF ac POTENTIOMETER

The major applications of the ac potentiometers are

1. Measurement of self-inductance
2. Calibration of voltmeter
3. Calibration of ammeter
4. Calibration of wattmeter

5.8.1 Measurement of Self–inductance

The circuit diagram for measurement of self inductance of a coil by ac potentiometer is shown in Figure 5.13(a). A standard non-inductive resistor is connected in series with the coil under test and two potential differences V_1 and V_2 are measured in magnitude and phase by the potentiometer.

The vector diagram is shown in Figure 5.13(b). Refer to this figure.

Voltage drop across standard resistor $R_S, V2 = IR_S$

where, I= current flowing through the circuit, and

R_S= resistance of the standard non-inductive resistor

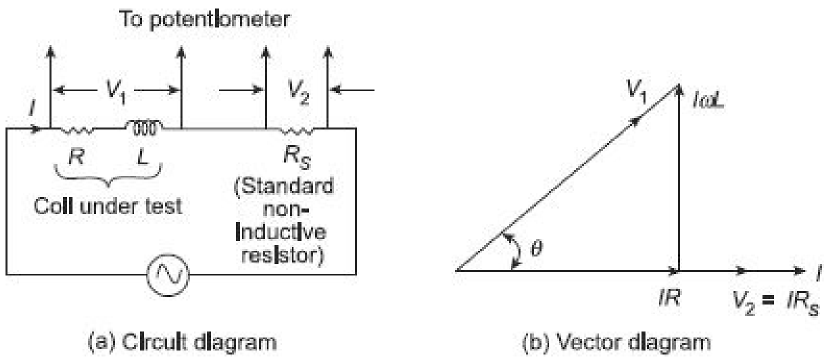


Figure 5.13 *Measurement of self-inductance by ac potentiometer*

or, $$I = \frac{V_2}{R_S}$$

Voltage drop across inductive coil = V_1

Phase angle between voltage across and current through the coil = θ

Voltage drop due to resistance of coil, $IR = V_1 \cos\theta$

$$\therefore \text{ resistance of the coil, } R = \frac{V_1 \cos\theta}{I} = \frac{V_1 \cos\theta}{\frac{V_2}{R_S}} = \frac{R_S V_1 \cos\theta}{V_2}$$

Voltage drop due to inductance of coil, $I\omega L = V_1 \sin\theta$

$$\therefore \text{ inductance of the coil, } L = \frac{V_1 \sin\theta}{I\omega} = \frac{V_1 \sin\theta}{\omega\left(\frac{V_2}{R_S}\right)} = \frac{R_S V_1 \sin\theta}{V_2 \omega}$$

5.8.2 Calibration of Ammeter

The method of calibration of an ac ammeter is similar to dc potentiometer method for dc ammeter (refer to Section 5.4.6) i.e., ac ammeter under calibration is connected in series with a non inductive variable resistance for varying the current, and a non-inductive standard resistor and voltage drop across standard resistor is measured on ac potentiometer. However, the standardising of the ac potentiometer involves the use of a suitable transfer instrument.

5.8.3 Calibration of Voltmeter

The method of calibration of an ac voltmeter by using ac potentiometer is similar to that adopted for calibration of dc voltmeter by using dc potentiometer (refer Section 5.4.5).

5.8.4 Calibration of Wattmeter

The circuit diagram for calibration of wattmeter by ac potentiometer is shown in Figure 5.14. The calibration process is same that adopted in case of calibration of wattmeter by dc potentiometer (refer Section 5.4.7).

The current coil of the wattmeter is supplied through a stepdown transformer and the potential coil from the secondary of a variable transformer whose primary is supplied from the rotor of a phase shifting transformer.

The voltage V across the potential coil of the wattmeter and the current I through the current coil of wattmeter are measured by the potentiometer, introducing a volt–ratio box and a standard resistor as shown in Figure 5.14.

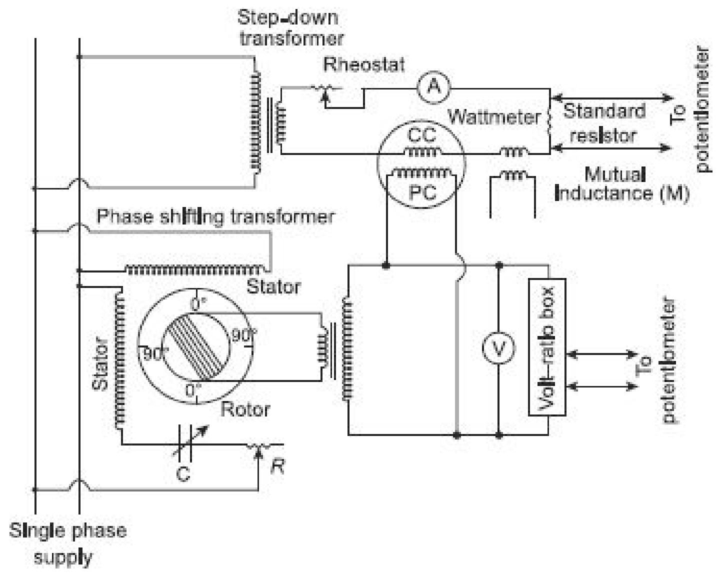


Figure 5.14 *Calibration of wattmeter by ac potentiometer*

The power factor cos *q* is varied by rotating the phase shift rotor, the phase angle between voltage and current, F being given by the reading on the dial of the phase shifter. The power is then *VI* cos F and the wattmeter reading may be compared with this reading. A calibration curve may be drawn if necessary. A small mutual inductance *M* is included to ensure accuracy of measurement of zero power factor.

Example 5.7

The following readings were obtained during measurement of inductance of a coil on an ac potentiometer:Voltage drop across 0.1 Ω standard resistor connected in series with the coil= 0.613 -12°6'Voltage across the test coil through a 100:1 volt–ratio box = 0.781 -50°48'Frequency 50 Hz. Determine the value of the inductance of the coil.

Solution

Current through the coil $=\bar{I}=\dfrac{0.613\angle 12°6'}{0.1}=6.13\angle 12°6'\ \text{A}$

Voltage across the coil $\bar{V}=100\times 0.781\angle 50°48'=78.1\angle 50°48'\ \text{V}$

∴impedance of the coil $\bar{Z}=\dfrac{\bar{V}}{\bar{I}}=\dfrac{78.1\angle 50°48'}{6.13\angle 12°6'}=12.74\angle 38°42'\ \Omega\alpha$

Resistance of the coil $R=12.74\cos 38°42'=9.94\ \Omega$

Reactance of the coil $X=12.74\sin 38°42'=7.96\ \Omega$

$$\therefore \text{inductance of the coil } L=\frac{X}{2\pi f}=\frac{7.96}{2\times\pi\times 50}=0.0253\ \text{H}$$

The power factor cos *q* is varied by rotating the phase shift rotor, the phase angle between voltage and current, F being given by the reading on the dial of the phase shifter. The power is then *VI* cos F and the wattmeter reading may be compared with this reading.

A calibration curve may be drawn if necessary. A small mutual inductance M is included to ensure accuracy of measurement of zero power factor.

Example 5.7

The following results were obtained for determination of impedance of a coil by using a coordinate type potentiometer: Voltage across the 1.0 Ωresistor in series with the coil = +0.2404 V in phase dial and 0.0935 V on quadrature dial Voltage across 10:1 potential divider used with the coil = +0.3409 V on in phase dial and +0.2343 V on quadrature dial Calculate the resistance and reactance of the coil.

Solution

Current through the coil

$$\bar{I} = \frac{(+0.2404 - j0.0935)}{1.0} = (+0.2404 - j0.0935)\ \text{A}$$

Voltage across the coil

$$\bar{V} = 10(0.3409 + j0.2343) = (3.409 + j2.343)\ \text{V}$$

∴ impedance of the coil

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{3.409 + j2.343}{0.2404 - j0.0935} = \frac{4.136 \angle 34.5^\circ}{0.258 \angle -21.2^\circ} = 16.03 \angle 55.7^\circ$$

$$= (9.03 + j13.24)\ \Omega$$

∴ resistance of the coil $R = 9.03\ \Omega$

Reactance of the coil $L = 13.24\ \Omega$

Example 5.9

The following results were obtained during the measurement of power by a polar potentiometer:Voltage across 0.2 Ωstandard resistor in series with the load = 1.52 -35°Voltage across 200:1 potential divider across the line = 1.43 -53°Calculate the current, voltage, power and power factor of the load.

Solution

(a) Current through the load, $\bar{I} = \frac{1.52 \angle 35^\circ}{0.2} = 7.6 \angle 35^\circ\ \text{A}$

Magnitude of the current I= 7.6 A

(b) Voltage across the load V=200 ×(1.43 <53°)=286 <53°V

Magnitude of the voltage V= 286 V

(c) Phase angle of the load = 53° <35° = 18°

∴power factor of the load cos φ = cos 18° = 0.951 (lagging)

(d) Power consumed by the load, P = Icos φ=286 ×× 0.951

EXERCISE

Objective-type Questions

1. The transfer instrument which is used for standardisation of a polar-type ac potentiometer is

   (a) an electrostatic instrument

   (b) a dynamometer instrument

   (c) a moving coil instrument

   (d) a thermal instrument

2. A dc potentiometer is designed to measure up to about 2 volts with a slide wire of 800 mm. A standard cell of emf 1.18 volt obtains balance at 600 mm. a test cell is seen to obtained balance at 680 mm. The emf of the test cell is

   (a) 1.50 volts

   (b) 1.00 volts

   (c) 1.34 volts

   (d) 1.70 volts

3. For measuring an ac voltage by an ac potentiometer, it is desirable that the supply for the potentiometer is taken from

   (a) a battery

   (b) the same source as the unknown voltage

   (c) a source other than the source of unknown voltage

   (d) any of the above

4. The calibration of a voltmeter can be carried out by using

   (a) an ammeter

   (b) a function generator

   (c) a frequency meter

   (d) a potentiometer

5. A slide wire potentiometer has 10 wires of 1 m each. With the help of a standard voltage source of 1.018 volt it is standardise by keeping the jockey at 101.8 cm. If the resistance of the potentiometer wire is 1000 Ω then the value of the working current is

   (a) 1 mA

   (b) 0.5 mA

   (c) 0.1 A

   (d) 10 mA

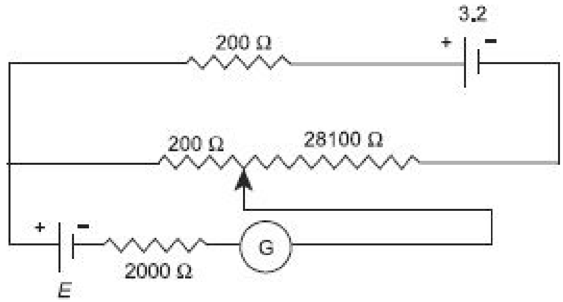
6. In the potentiometer circuit, the value of the unknown voltage *E* under balance condition will be

   (a) 3 V

   (b) 200 mV

   (c) 2.8 V

   (d) 3.2 V



7. The potentiometer is standardised for making it
   (a) precise
   (b) accurate
   (c) accurate and precise
   (d) accurate and direct reading
8. Consider the following statements.A dc potentiometer is the best means available for the measurement of dc voltage because
   1. The precision in measurement is independent of the type of detector used
   2. It is based on null balance technique
   3. It is possible to standardize before a measurement is undertaken
   4. It is possible to measure dc voltages ranging in value from mV to hundreds of volts Of these statements,
   (a) 2 and 3 are correct
   (b) 1 and 4 are correct
   (c) 2 and 4 are correct
   (d) 3 and 4 are correct
9. In a dc potentiometer measurements, a second reading is often taken after reversing the polarities of dc supply and the unknown voltage, and the average of the two readings is taken. This is with a view to eliminate the effects of
   (a) ripples in the dc supply
   (b) stray magnetic field
   (c) stray thermal emfs
   (d) erroneous standardisation

Answers

1. (d)	2. (c)	3. (b)	4. (d)	5. (d)	6. (b)	7. (d)
8. (c)	9. (c)					

Short-answer Questions

1. Explain why a potentiometer does not load the voltage source whose voltage is being measured.
2. Describe the procedure of standardisation of a dc potentiometer.
3. What is a *volt ratio* box? Explain its principle with a suitable block diagram.
4. Explain the reasons why dc potentiometers cannot be used for ac measurement directly.
5. Explain the procedure for measurement of self-reactance of a coil with the help of ac potentiometer.
6. What is the difference between a slide-wire potentiometer and direct reading potentiometer?
7. How can a dc potentiometer be used for calibration of voltmeter?
8. Explain with suitable diagram how a dc potentiometer can be used for calibration of an ammeter.
9. Explain with a suitable diagram how a dc potentiometer can be used for calibration of wattmeter.
10. What are the different forms of ac potentiometers and bring out the differences between them.

Long-answer Questions

1. (a) Explain the working principle of a Crompton dc potentiometer with a suitable diagram.

   (b) The emf of a standard cell is measured with a potentiometer which gives a reading of 1.01892 volts. When a 1 MΩ resistor is connected across the standard cell terminals, the potentiometer reading drops to 1.01874 volts. Calculate the internal resistance of the cell.

   [Ans: 176.6 Ω]

2. (a) Name the different types of dc potentiometers and explain one of them.

   (b) A slidewire potentiometer is used to measure the voltage between the two points of a certain dc circuit. The potentiometer reading is 1.0 volt. Across the same two points when a 10000 Ω/V voltmeter is connected, the reading on the voltmeter is 0.5 volt of its 5–volt range. Calculate the input resistance between two points.

   [Ans: 50000 Ω]

3. (a) Write down the procedure of standardisation of a dc potentiometer. How can it be used for calibration of ammeters and voltmeters?

   (b) A slidewire potentiometer has a battery of 4 volts and negligible internal resistance. The resistance of slide wire is 100 Ω and its length is 200 cm. A standard cell of 1.018 volts is used for standardising the potentiometer and the rheostat is adjusted so that balance is obtained when the sliding contact is at 101.8 cm.

   (i) Find the working current of the slidewire and the rheostat setting. [Ans: 20 mA, 100 Ω]

   (ii) If the slidewire has divisions marked in mm and each division can be interpolated to one fifth, calculate the resolution of the potentiometer. [Ans: 0.2 mV]

4. (a) What are the problems associated with ac potentiometers? Describe the working of any one ac potentiometer.

   (b) Power is being measured with an ac potentiometer. The voltage across a 0.1 Ω standard resistance connected in series with the load is (0.35 – *j*0.10) volt. The voltage across 300:1 potential divider connected to the supply is (0.8 + *j*0.15) volt. Determine the power consumed by the load and the power factor.

   [Ans: 801 W, 0.8945]

5. (a) Describe the construction and working of an ac coordinate–type potentiometer.

   (b) Measurements for the determination of the impedance of a coil are made on a coordinate type potentiometer. The result are: Voltage across 1 Ω standard resistance in series with the coil = +0.952 V on inphase dial and –0.340 V on quadrature dial; voltage across 10:1 potential divider connected to the terminals of the coil = +1.35 V on inphase dial and +1.28 V on quadrature dial.

   Calculate the resistance and reactance of the coil.

   [Ans: R = 8.32 Ω, × = 16.41 Ω]

6. Describe briefly the applications of ac potentiometers.

7. Write short notes on the following (any three):

   (a) Simple dc potentiometer and its uses

   (b) Calibration of low range ammeter

   (c) Measurement of high voltage by dc potentiometer

   (d) Polar potentiometer

   (e) Coordinate–type potentiometer

   (f) Comparison between ac and dc potentiometer

6 AC Bridges

6.1 INTRODUCTION

Alternating current bridges are most popular, convenient and accurate instruments for measurement of unknown inductance, capacitance and some other related quantities. In its simplest form, ac bridges can be thought of to be derived from the conventional dc Wheatstone bridge. An ac bridge, in its basic form, consists of four arms, an alternating power supply, and a balance detector.

6.2 SOURCES AND DETECTORS IN ac BRIDGES

For measurements at low frequencies, bridge power supply can be obtained from the power line itself. Higher frequency requirements for power supplies are normally met by electronic oscillators. Electronic oscillators have highly stable, accurate yet adjustable frequencies. Their output waveforms are very close to sinusoidal and output power level sufficient for most bridge measurements.

When working at a single frequency, a tuned detector is preferred, since it gives maximum sensitivity at the selected frequency and discrimination against harmonic frequencies. *Vibration galvanometers* are most commonly used as tuned detectors in the power frequency and low audio-frequency ranges. Though vibration galvanometers can be designed to work as detectors over the frequency range of 5 Hz to 1000 Hz, they have highest sensitivity when operated for frequencies below 200 Hz.

Head phones or *audio amplifiers* are popularly used as balance detectors in ac bridges at frequencies of 250 Hz and above, up to 3 to 4 kHz.

Transistor amplifier with *frequency tuning* facilities can be very effectively used as balance detectors with ac bridges. With proper tuning, these can be used to operate at a selective band of frequencies with high sensitivity. Such detectors can be designed to operate over a frequency range of 10 Hz to 100 kHz.

6.3 GENERAL BALANCE EQUATION FOR FOUR-ARM BRIDGE

An ac bridge in its general form is shown in Figure 6.1, with the four arms being represented by four unspecified impedances Z_1, Z_2, Z_3 and Z_4.

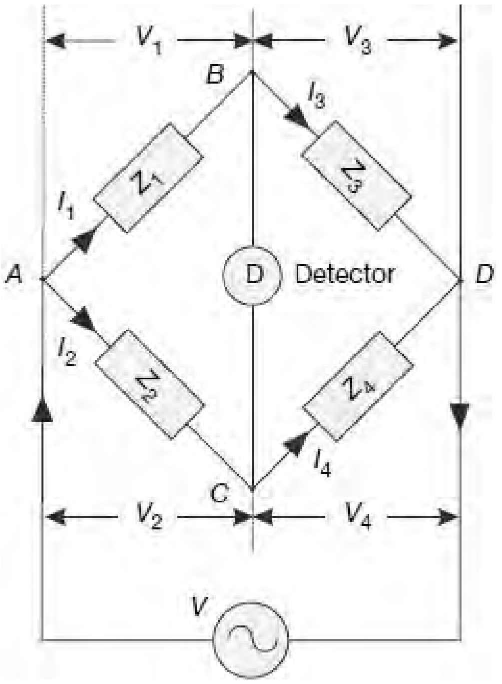


Figure 6.1 *General 4-arm bridge confi guration*

Balance in the bridge is secured by adjusting one or more of the bridge arms. Balance is indicated by zero response of the detector. At balance, no current flows through the detector, i.e., there is no potential difference across the detector, or in other words, the potentials at points *B* and *C* are the same. This will be achieved if the voltage drop from *A* to *B* equals the voltage drop from *A* to *C*, both in magnitude and phase.

Thus, we can write in terms of complex quantities:

$$\bar{V}_1 = \bar{V}_2$$

or, $$\bar{I}_1 \bar{Z}_1 = \bar{I}_2 \bar{Z}_2 \quad (6.1)$$

Also at balance, since no current flows through the detector,

$$\bar{I}_1 = \bar{I}_3 = \frac{\bar{V}}{\bar{Z}_1 + \bar{Z}_3} \quad (6.2)$$

and $$\bar{I}_2 = \bar{I}_4 = \frac{\bar{V}}{\bar{Z}_2 + \bar{Z}_4} \quad (6.3)$$

Combining Eqs (6.2) and (6.3) into Eq. (6.1), we have

$$\frac{\bar{V}}{\bar{Z}_1 + \bar{Z}_3} \bar{Z}_1 = \frac{\bar{V}}{\bar{Z}_2 + \bar{Z}_4} \bar{Z}_2$$

or, $$\bar{Z}_1 \bar{Z}_2 + \bar{Z}_1 \bar{Z}_4 = \bar{Z}_2 \bar{Z}_1 + \bar{Z}_2 \bar{Z}_3$$

or, $$\bar{Z}_1 \bar{Z}_4 = \bar{Z}_2 \bar{Z}_3 \quad (6.4)$$

or, $$\frac{\bar{Z}_1}{\bar{Z}_3} = \frac{\bar{Z}_2}{\bar{Z}_4} \quad (6.5)$$

When using admittances in place of impedances, Eq. (6.4) can be re-oriented as

$$\bar{Y}_1 \bar{Y}_4 = \bar{Y}_2 \bar{Y}_3 \quad (6.6)$$

Equations (6.4) and (6.6) represent the basic balance equations of an ac bridge. Whereas (6.4) is convenient for use in bridge configurations having series elements, (6.6) is more useful when bridge configurations have parallel elements.

Equation (6.4) indicates that under balanced condition, the product of impedances of one pair of *opposite* arms must be equal to the product of impedances of the other pair of *opposite* arms, with the impedances expressed as complex numbers. This will mean, both magnitude and phase angles of the complex numbers must be taken into account.

Re-writing the expressions in polar form, impedances can be expressed as $\overline{Z} = Z\angle\theta$ where *Z* represents the magnitude and *θ* represents the phase angle of the complex impedance.

If similar forms are written for all impedances and substituted in (6.4), we obtain:

$$Z_1\angle\theta_1 \times Z_4\angle\theta_4 = Z_2\angle\theta_2 \times Z_3\angle\theta_3$$

Thus, for balance we have,

$$Z_1 Z_4 \angle(\theta_1 + \theta_4) = Z_2 Z_3 \angle(\theta_2 + \theta_3) \qquad (6.7)$$

Equation (6.7) shows that two requirements must be met for satisfying balance condition in a bridge.

The first condition is that the magnitude of the impedances must meet the relationship; $Z_1 Z_4 = Z_2 Z_3$ (6.8)

The second condition is that the phase angles of the impedances must meet the relationship; $\angle(\theta_1 + \theta_4) = \angle(\theta_2 + \theta_3)$ (6.9)

Example 6.1

In the AC bridge circuit shown in Figure 6.1, the supply voltage is 20 V at 500 Hz. Arm AB is 0.25 mμ pure capacitance; arm BD is 400 Ω pure resistance and arm AC has a 120 Ω resistance in parallel with a 0.15 mμ capacitor. Find resistance and inductance or capacitance of the arm CD considering it as a series circuit.

Solution Impedance of the arm *AB* is

$$Z_1 = \frac{1}{2\pi f C_1} = \frac{1}{2\pi \times 500 \times 0.25 \times 10^{-6}} = 1273\ \Omega$$

Since it is purely capacitive, in complex notation, $\overline{Z}_1 = 1273\angle -90°\ \Omega$

Impedance of arm *BD* is $Z_3 = 400\ \Omega$

Since it is purely resistive, in complex notation, $\overline{Z}_3 = 400\angle 0°\ \Omega$

Impedance of arm AC containing 120 Ω resistance in parallel with a 0.15 µF capacitor is

$$\overline{Z}_2 = \frac{R_2}{1+j2\pi fC_2R_2} = \frac{120}{1+j(2\pi\times500\times0.15\times10^{-6}\times120)}$$
$$= 119.8\angle-3.2^\circ\ \Omega$$

For balance, $\overline{Z}_1\overline{Z}_4 = \overline{Z}_2\overline{Z}_3$

∴ impedance of arm *CD* required for balance is $\overline{Z}_4 = \frac{\overline{Z}_2\overline{Z}_3}{\overline{Z}_1}$

or, $\overline{Z}_4 = \frac{119.88\times400}{1273}\angle(-3.2^\circ+0^\circ+90^\circ) = 37.65\angle86.8^\circ$

The positive angle of impedance indicates that the branch consists of a series combination of resistance and inductance.

Resistance of the unknown branch $R_4 = 37.65\times\cos(86.8^\circ) = 2.1\ \Omega$

Inductive reactance of the unknown branch

$$X_4 = 37.65\times\sin(86.8^\circ) = 37.59\ \Omega$$

Inductance of the unknown branch $L_4 = \frac{37.59}{2\pi\times500}H = 11.97$ mH

6.4 MEASUREMENT OF SELF-INDUCTANCE

6.4.1 Maxwell's Inductance Bridge

This bridge is used to measure the value of an unknown inductance by comparing it with a variable standard self-inductance. The bridge configuration and phasor diagram under balanced condition are shown in Figure 6.2.

The unknown inductor L_1 of resistance R_1 in the branch *AB* is compared with the standard known inductor L_2 of resistance R_2 on arm *AC*. The inductor L_2 is of the same order as the unknown inductor L_1. The resistances R_1, R_2, etc., include, of course the resistances of contacts and leads in various arms. Branch *BD* and *CD* contain known no-inductive resistors R_3 and R_4 respectively.

The bridge is balanced by varying L_2 and one of the resistors R_3 or R_4. Alternatively, R_3 and R_4 can be kept constant, and the resistance of one of the other two arms can be varied by connecting an additional resistor.

Under balanced condition, no current flows through the detector. Under such condition, currents in the arms *AB* and *BD* are equal (I_1). Similarly, currents in the arms *AC* and *CD* are equal (I_2). Under balanced condition, since nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drop across arms *AB* and *AC* are equal ($V_1 = V_2$).

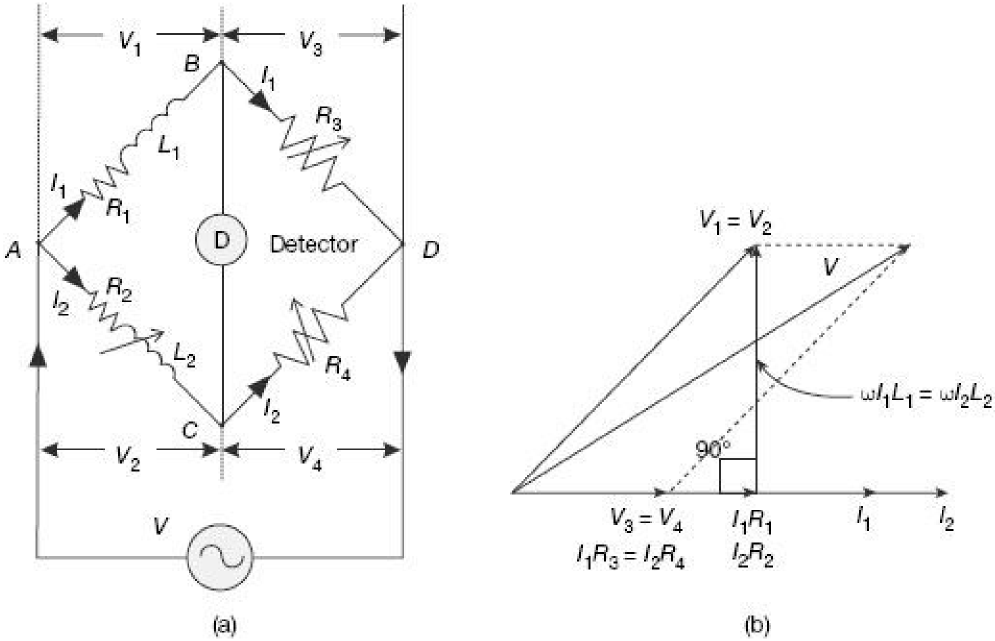


Figure 6.2 *Maxwell's inductance bridge under balanced condition: (a) Configuration (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.2 (b), V_3 and V_4 being equal, they are overlapping. Arms *BD* and *CD* being purely resistive, currents through these arms will be in the same phase with the voltage drops across these two respective branches. Thus, currents I_1 and I_2 will be collinear with the phasors V_3 and V_4. The same current I_1 flows through branch *AB* as well, thus the voltage drop I_1R_1 remains in the same phase as I_1. Voltage drop wI_1L_1 in the inductor L_1 will be 90° out of phase with I_1R_1 as shown in Figure 6.2(b). Phasor summation of these two voltage drops I_1R_1 and wI_1L_1 will give the voltage drop V_1 across the arm *AB*. At balance condition, since voltage across the two branches *AB* and *AC* are equal, thus the two voltage drops V_1 and V_2 are equal and are in the same phase. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2 + j\omega L_2}{R_4}$$

or, $$R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

Equating real and imaginary parts, we have

$$R_1R_4 = R_2R_3$$

or, $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

and also, $$j\omega L_1R_4 = j\omega L_2R_3$$

or, $$\frac{L_1}{L_2} = \frac{R_3}{R_4}$$

Thus, $$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_1}{L_2}$$

Unknown quantities can hence be calculated as

$$L_1 = L_2 \times \frac{R_3}{R_4} \text{ and } R_1 = R_2 \times \frac{R_3}{R_4} \tag{6.10}$$

Care must be taken that the inductors L_1 and L_2 must be placed at a distance from each other to avoid effects of mutual inductance.

The final expression (6.10) shows that values of L1 and R1 do not depend on the supply frequency. Thus, this bridge confi guration is immune to frequency variations and even harmonic distortions in the power supply.

6.4.2 Maxwell's Inductance–Capacitance Bridge

In this bridge, the unknown inductance is measured by comparison with a standard variable capacitance. It is much easier to obtain standard values of variable capacitors with acceptable degree of accuracy. This is however, not the case with finding accurate and stable standard value variable inductor as is required in the basic Maxwell's bridge described in Section 6.4.1.

Configuration of a Maxwell's inductance–capacitance bridge and the associated phasor diagram at balanced state are shown in Figure 6.3.

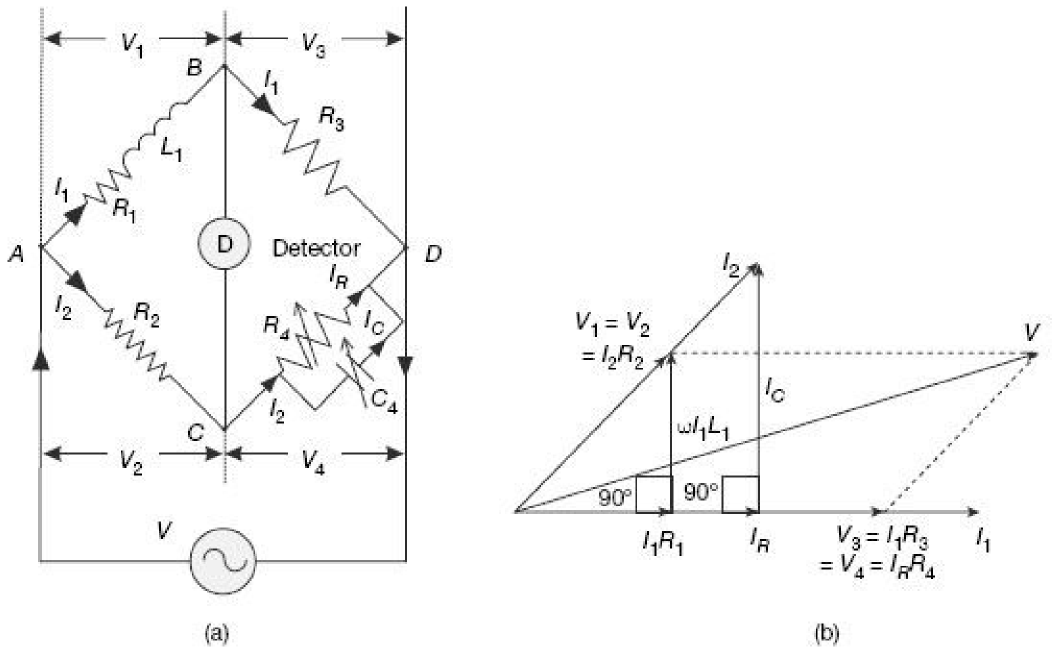


Figure 6.3 Maxwell's inductance–capacitance bridge under balanced condition: (a) Confi guration (b) Phasor diagram

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known variable capacitor C_4 on arm *CD*. The other resistances R_2, R_3, and R_4 are known as non– inductive resistors.

The bridge is preferably balanced by varying C_4 and R_4, giving independent adjustment settings.

Under balanced condition, no current flows through the detector. Under such condition, currents in the arms *AB* and *BD* are equal (I_1). Similarly, currents in the arms *AC* and *CD* are equal (I_2). Under balanced condition, since nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drops across arms *AB* and *AC* are equal ($V_1 = V_2$).

As shown in the phasor diagram of Figure 6.3 (b), V_3 and V_4 being equal, they are overlapping both in magnitude and phase. The arm *BD* being purely resistive, current I_1

through this arm will be in the same phase with the voltage drop V_3 across it. Similarly, the voltage drop V_4 across the arm CD, current I_R through the resistance R_4 in the same branch, and the resulting resistive voltage drop $I_R R_4$ are all in the same phase [horizontal line in Figure 6.3(b)]. The resistive current I_R when added with the quadrature capacitive current I_C, results in the main current I_2 flowing in the arm CD. This current I_2 while flowing through the resistance R_2 in the arm AC, produces a voltage drop $V_2 = I_2R_2$, that is in same phase as I_2. Under balanced condition, voltage drops across arms AB and AC are equal, i.e., $V_1 = V_2$. This voltage drop across the arm AB is actually the phasor summation of voltage drop I_1R_1 across the resistance R_1 and the quadrature voltage drop wI_1L_1 across the unknown inductor L_1. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage V.

At balance, $$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2}{\left(\dfrac{R_4}{1 + j\omega C_4 R_4}\right)}$$

or, $$R_1R_4 + j\omega L_1 R_4 = R_2R_3 + j\omega C_4 R_2 R_3 R_4$$

Equating real and imaginary parts, we have

$$R_1R_4 = R_2R_3$$

or, $$R_1 = R_2 \times \frac{R_3}{R_4}$$

and also, $$j\omega L_1 R_4 = j\omega C_4 R_2 R_3 R_4$$

or, $$L_1 = C_4 R_2 R_3$$

Thus, the unknown quantities are

$$L_1 = C_4R_2R_3 \text{ and } R_1 = R_2 \times \frac{R_3}{R_4} \tag{6.11}$$

Once again, the final expression (6.11) shows that values of L_1 and R_1 do not depend on the supply frequency. Thus, this bridge confi guration is immune to frequency variations and even harmonic distortions in the power supply.

It is interesting to note that both in the Maxwell's Inductance Bridge and Inductance-Capacitance Bridge, the unknown Inductor L_1 was always associated with a resistance R_1. This series resistance has been included to represent losses that take place in an inductor coil. An ideal inductor will be lossless irrespective of the amount of current flowing through it. However, any real inductor will have some non-zero resistance associated with it due to resistance of the metal wire used to form the inductor winding. This series resistance causes heat generation due to power loss. In such cases, the Quality Factor or the Q-Factor of such a lossy inductor is used to indicate how closely the real inductor comes to behave as an ideal inductor. The Q-factor of an inductor is defined as the ratio of its inductive reactance to its resistance at a given frequency. Q-factor is a measure of the efficiency of the inductor. The higher the value of Q-factor, the closer it approaches the behavior of an ideal, loss less inductor. An ideal inductor would have an infinite Q at all frequencies.

The Q-factor of an inductor is given by the formula $Q = \frac{\omega L}{R}$, where R is its internal resistance R (series resistance) and wL is its inductive reactance at the frequency ω.

Q-factor of an inductor can be increased by either increasing its inductance value (by using a good ferromagnetic core) or by reducing its winding resistance (by using good quality conductor material, in special cases may be super conductors as well).

In the Maxwell's Inductance-Capacitance Bridge, Q-factor of the inductor under measurement can be found at balance condition to be $Q = \frac{\omega L_1}{R_1}$ or,

$$Q = \frac{\omega C_4 R_2 R_3}{R_2 \times \frac{R_3}{R_4}} = \omega C_4 R_4 \tag{6.12}$$

The above relation (6.12) for the inductor Q factor indicate that this bridge is not suitable for measurement of inductor values with high Q factors, since in that case, the required value of R_4 for achieving balance becomes impractically high.

Advantages of Maxwell's Bridge

1. The balance equations (6.11) are independent of each other, thus the two variables C_4 and R_4 can be varied independently.
2. Final balance equations are independent of frequency.
3. The unknown quantities can be denoted by simple expressions involving known quantities.
4. Balance equation is independent of losses associated with the inductor.
5. A wide range of inductance at power and audio frequencies can be measured.

Disadvantages of Maxwell's Bridge

1. The bridge, for its operation, requires a standard variable capacitor, which can be very expensive if high accuracies are asked for. In such a case, fixed value capacitors are used and balance is achieved by varying R_4 and R_2.
2. This bridge is limited to measurement of low Q inductors ($1 < Q < 10$).
3. Maxwell's bridge is also unsuited for coils with very low value of Q (e.g., $Q < 1$). Such low Q inductors can be found in inductive resistors and RF coils. Maxwell's bridge finds difficult and laborious to obtain balance while measuring such low Q inductors.

6.4.3 Hay's Bridge

Hay's bridge is a modification of Maxwell's bridge. This method of measurement is particularly suited for high Q inductors.

Configuration of Hay's bridge and the associated phasor diagram under balanced state are shown in Figure 6.4.

The unknown inductor L_1 of effective resistance R_1 in the branch AB is compared with the standard known variable capacitor C_4 on arm *CD*. This bridge uses a resistance R_4 in series with the standard capacitor C_4 (unlike in Maxwell's bride where R_4 was in parallel with C_4). The other resistances R_2 and R_3 are known no-inductive resistors.

The bridge is balanced by varying C_4 and R_4.

Under balanced condition, since no current flows through the detector, nodes *B* and *D* are at the same potential, voltage drops across arm *BD* and *CD* are equal ($V_3 = V_4$); similarly, voltage drops across arms *AB* and *AC* are equal ($V_1 = V_2$).

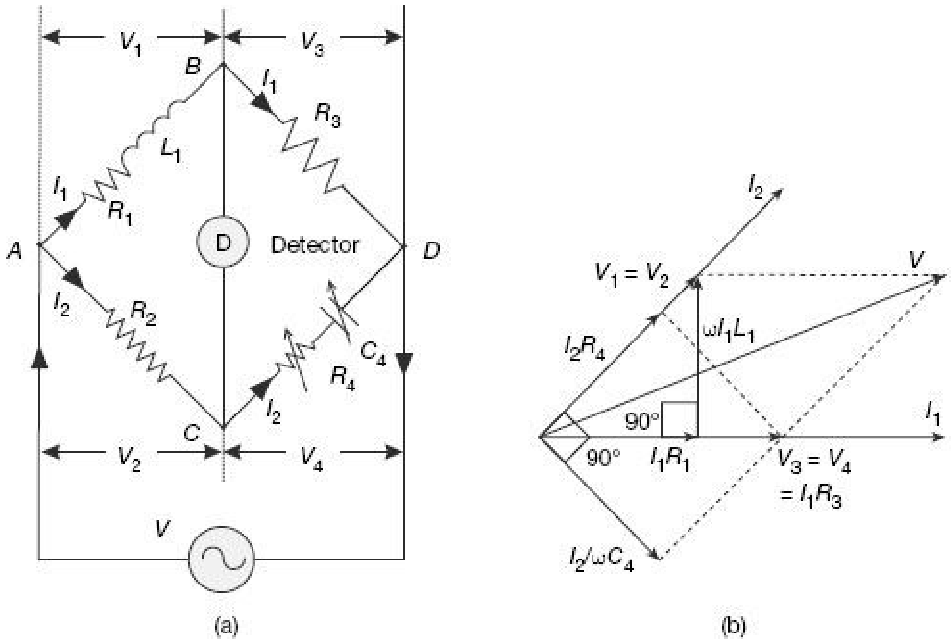


Figure 6.4 *Hay's bridge under balanced condition: (a) Confi guration, (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.4 (b), V_3 and V_4 being equal, they are overlapping both in magnitude and phase and are draw on along the horizontal axis. The arm *BD* being purely resistive, current I_1 through this arm will be in the same phase with the voltage drop $V_3 = I_1R_3$ across it. The same current I_1, while passing through the resistance R_1 in the arm *AB*, produces a voltage drop I_1R_1 that is once again, in the same phase as *I* 1. Total voltage drop *V* 1 across the arm *AB* is obtained by adding the two quadrature phasors *I* 1*R* 1 and *wI* 1*L* 1 representing resistive and inductive voltage drops in the same branch *AB*. Since under balance condition, voltage drops across arms *AB* and *AC* are equal, i.e., (*V* 1 = *V* 2), the two voltages *V* 1 and *V* 2 are overlapping both in magnitude and phase. The branch *AC* being purely resistive, the branch current *I* 2 and branch voltage *V* 2 will be in the same phase as shown in the phasor diagram of Figure 6.4 (b). The same current *I* 2 fl ows through the arm *CD* and produces a voltage drop *I* 2*R* 4 across the resistance *R* 4. This resistive voltage drop *I* 2*R* 4, obviously is in the same phase as *I* 2. The capacitive voltage drop *I* 2/*wC* 4 in the capacitance *C* 4 present in the same arm *AC* will however, lag the current *I* 2 by 90°. Phasor summation of these two series voltage drops across *R* 4 and *C* 4 will give the total voltage drop *V* 4 across the arm *CD*. Finally, phasor summation of *V* 1 and *V* 3 (or *V* 2 and *V* 4) results in the supply voltage *V*.

At balance, $$\frac{R_1 + j\omega L_1}{R_3} = \frac{R_2}{\left(R_4 - \frac{j}{\omega C_4}\right)}$$

or, $$R_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 - \frac{jR_1}{\omega C_4} = R_2 R_3$$

Equating real and imaginary parts, we have

Equating real and imaginary parts, we have

$$R_1 R_4 + \frac{L_1}{C_4} = R_2 R_3 \qquad (6.13)$$

and $$\omega L_1 R_4 = \frac{R_1}{\omega C_4} \qquad (6.14)$$

Solving Eqs (6.13) and (6.14) we have the unknown quantities as

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega^2 R_4^2 C_4^2} \qquad (6.15)$$

and

$$R_1 = \frac{R_2 R_3 R_4 \omega^2 C_4^2}{1 + \omega^2 R_4^2 C_4^2} \qquad (6.16)$$

Q factor of the inductor in this case can be calculated at balance condition as

$$Q = \frac{\omega L_1}{R_1} = \frac{1}{\omega C_4 R_4} \qquad (6.17)$$

Hay's bridge is more suitable for measurement of unknown inductors having Q factor more than 10. In those cases, bridge balance can be attained by varying R_2 only, without losing much accuracy.

From (6.15) and (6.17), the unknown inductance value can be written as

$$L_1 = \frac{R_2 R_3 C_4}{1 + (1/Q)^2} \qquad (6.18)$$

For inductors with Q > 10, the quantity $(1/Q)^2$ will be less than 1/100, and thus can be neglected from the denominator of (6.18). In such a case, the inductor value can be simplifi ed to L1 = R2R3C4, which essentially is the same as obtained in Maxwell's bridge.

6.4.4 Anderson's Bridge

This method is a modification of Maxwell's inductance–capacitance bridge, in which value of the unknown inductor is expressed in terms of a standard known capacitor. This method is applicable for precise measurement of inductances over a wide range of values.

Figure 6.5 shows Anderson's bridge configuration and corresponding phasor diagram under balanced condition.

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known capacitor *C* on arm *ED*. The bridge is balanced by varying *r*.

Under balanced condition, since no current flows through the detector, nodes *B* and *E* are at the same potential.

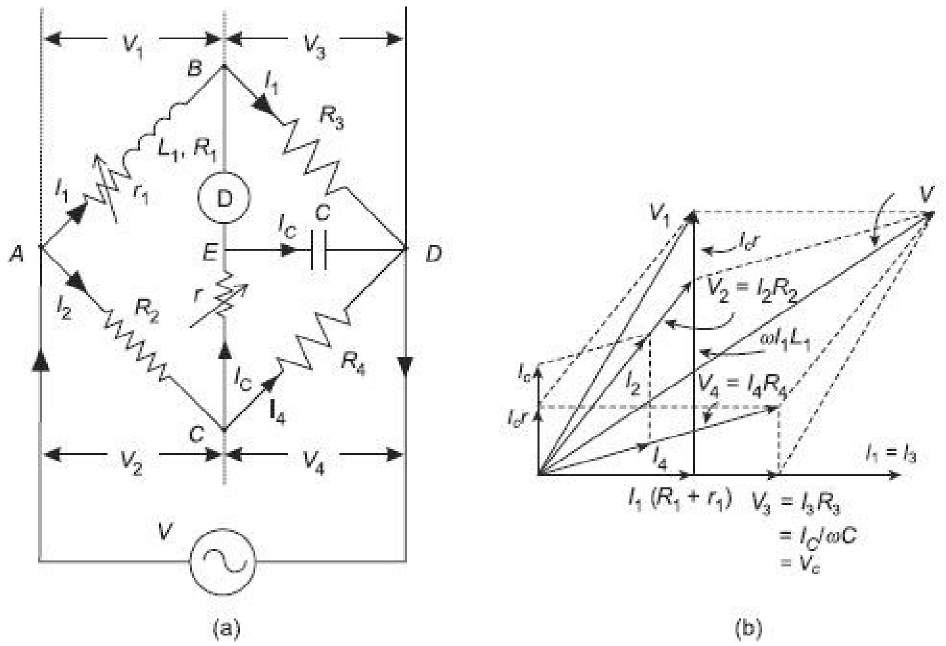


Figure 6.5 *Anderson's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

As shown in the phasor diagram of Figure 6.5 (b), I_1 and $V_3 = I_1R_3$ are in the same phase along the horizontal axis. Since under balance condition, voltage drops across arms *BD* and *ED* are equal, $V_3 = I_1R_3 = I_C/wC$ and all the three phasors are in the same phase. The same current I_1, when flowing through the arm *AB* produces a voltage drop $I_1(R_1 + r_1)$ which is once again, in phase with I_1. Since under balanced condition, no current flows through the detector, the same current I_C flows through the resistance *r* in arm *CE* and then through the capacitor *C* in the arm *ED*. Phasor summation of the voltage drops I_Cr in arm the *CE* and $I_C\,/wC$ in the arm *ED* will be equal to the voltage drop V_4 across the arm *CD*. V_4 being the voltage drop in the resistance R_4 on the arm *CD*, the current I_4 and V_4 will be in the same phase. As can be seen from the Anderson's bridge circuit, and also plotted in the phasor diagram, phasor summation of the currents I_4 in the arm *CD* and the current I_C in the arm *CE* will give rise to the current I_2 in the arm *AC*. This current I_2, while passing through the resistance R_2 will give rise to a voltage drop $V_2 = I_2R_2$ across the arm *AC* that is in phase with the current I_2. Since, under balance, potentials at nodes *B* and *E* are the same, voltage drops between nodes *A* -*B* and between *A* -*C* -*E* will be equal. Thus, phasor summation of the voltage drop $V_2 = I_2R_2$ in the arm *AC* I_C *r* in arm the *CE* will build up to the voltage V_1 across the arm *AB*. The voltage V_1 can also be obtained by adding the resistive voltage drop $I_1(R_1 + r_1)$ with the quadrature inductive voltage drop wI_1L_1 in the arm *AB*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $I_2 = I_C + I_4$

and, $$V_{BD} = V_{ED},\ \text{or}\ I_1R_3 = I_C \times \frac{1}{j\omega C}$$

$$\therefore \quad I_C = j\omega I_1R_3C \tag{6.19}$$

The other balance equations are:

$$V_{AB} = V_{AC} + V_{CE},\ \text{or}\ I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + I_Cr \tag{6.20}$$

and, $$V_{CD} = V_{CE} + V_{ED}, \text{or } I_C\left(r + \frac{1}{j\omega C}\right) = (I_2 - I_C)R_4 \quad (6.21)$$

Putting the value of I_C from Eq. (6.19) in Eq. (6.20), we have:*Ir (+R +jwL) =IR +jwI RCr*

$$I_1(r_1 + R_1 + j\omega L_1) = I_2R_2 + j\omega I_1R_3Cr$$

or, $$I_1(r_1 + R_1 + j\omega L_1 - j\omega R_3Cr) = I_2R_2 \quad (6.22)$$

Then, putting the value of I_C from Eq. (6.19) in Eq. (6.21), we have:Ê1 ˆ

$$j\omega I_1R_3C\left(r + \frac{1}{j\omega C}\right) = (I_2 - j\omega I_1R_3C)R_4$$

or, $$I_1(j\omega R_3Cr + R_3 + j\omega R_3CR_4) = I_2R_4 \quad (6.23)$$

From Eqs (6.22) and (6.23), we obtain:

From Eqs (6.22) and (6.23), we obtain:

$$I_1(r_1 + R_1 + j\omega L_1 - j\omega R_3Cr) = I_1(j\omega R_3Cr + R_3 + j\omega R_3CR_4)\frac{R_2}{R_4}$$

Equating real and imaginary parts, we get

$$R_1 = \frac{R_2R_3}{R_4} - r_1 \quad (6.24)$$

and, $$L_1 = C\frac{R_3}{R_4}\left[r(R_2 + R_4) + R_2R_4\right] \quad (6.25)$$

The advantage of Anderson's bridge over Maxwell's bride is that in this case a fixed value capacitor is used thereby greatly reducing the cost. This however, is at the expense of connection complexities and balance equations becoming tedious.

6.4.5 Owen's Bridge

This bridge is used for measurement of unknown inductance in terms of known value capacitance.

Figure 6.6 shows the Owen's bridge configuration and corresponding phasor diagram under balanced condition.

The unknown inductor L_1 of effective resistance R_1 in the branch *AB* is compared with the standard known capacitor C_2 on arm *AC*. The bridge is balanced by varying R_2 and C_2 independently.

Under balanced condition, since no current flows through the detector, nodes *B* and *C* are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.5 (b), I_1, $V_3 = I_1R_3$ and $V_4 = I_2/wC_4$ are all in the same phase along the horizontal axis. The resistive voltage drop I_1R_1 in the arm *AB* is also in the same phase

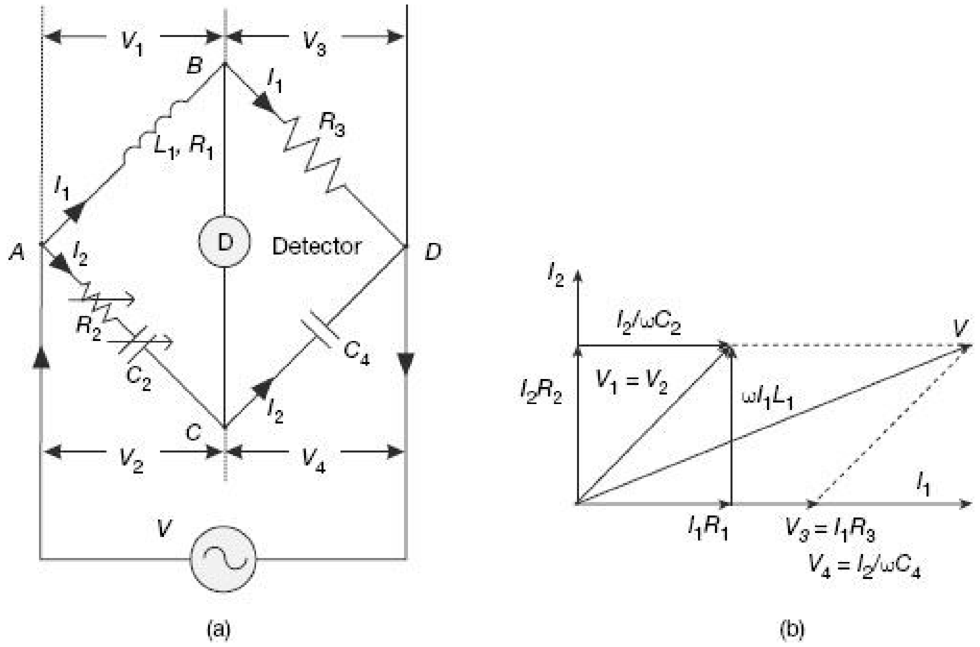


Figure 6.6 *Owen's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

with I_1. The inductive voltage drop wI_1L_1 when added in quadrature with the resistive voltage drop I_1R_1 gives the total voltage drop V_1 across the arm *AB*. Under balance condition, voltage drops across arms *AB* and *AC* being equal, the voltages V_1 and V_2 coincide with each other as shown in the phasor diagram of Figure 6.6 (b). The voltage V_2 is once again summation of two mutually quadrature voltage drops I_2R_2 (resistive) and I_2/wC_2 (capacitive) in the arm *AC*. It is to be noted here that the current I_2 leads the voltage V_4 by 90° due to presence of the capacitor C_4. This makes I_2 and hence I_2R_2 to be vertical, as shown in the phasor diagram. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

$$\text{At balance,}\quad \frac{(R_1 + j\omega L_1)}{R_3} = \frac{\left(R_2 + \dfrac{1}{j\omega C_2}\right)}{\dfrac{1}{j\omega C_4}}$$

Simplifying and separating real and imaginary parts, the unknown quantities can be found out as

$$R_1 = R_3\frac{C_4}{C_2} \tag{6.26}$$

and

$$L_1 = R_2R_3C_4 \tag{6.27}$$

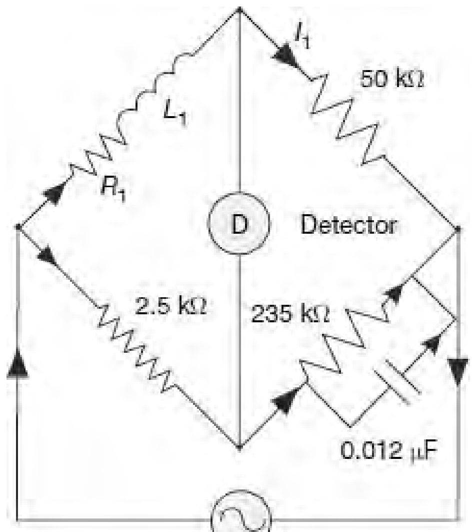
It is thus possible to have two independent variables C_2 and R_2 for obtaining balance in Owen's bridge. The balance equations are also quite simple. This however, does come with additional cost for the variable capacitor

Example 6.2

A Maxwell's inductance–capacitance bridge is used to measure a unknown inductive impedance. The bridge constants at bridge balance are: Pure resistance arms = 2.5 *kΩ and 50 kΩ. In between these two resistors, the third arm has a capacitor of value 0.012 μF in series with a resistor of value 235 kΩ. Find the series equivalent of the unknown impedance.*

Solution Referring to the diagram of a Maxwell's inductance–capacitance bridge:

Using the balance equation,



$$L_1 = C_4 R_2 R_3 = 0.012 \times 10^{-6} \times 2.5 \times 10^3 \times 50 \times 10^3 = 1.5 \text{ H}$$

$$\text{and } R_1 = R_2 \times \frac{R_3}{R_4} = 2.5 \times 10^3 \times \frac{50 \times 10^3}{235 \times 10^3} = 0.53 \text{ k}\Omega$$

Example 6.3 *The four arms of a bridge are connected as follows:*

Arm AB: A choke coil L_1 with an equivalent series resistance r_1

Arm BC: A noninductive resistance R_3

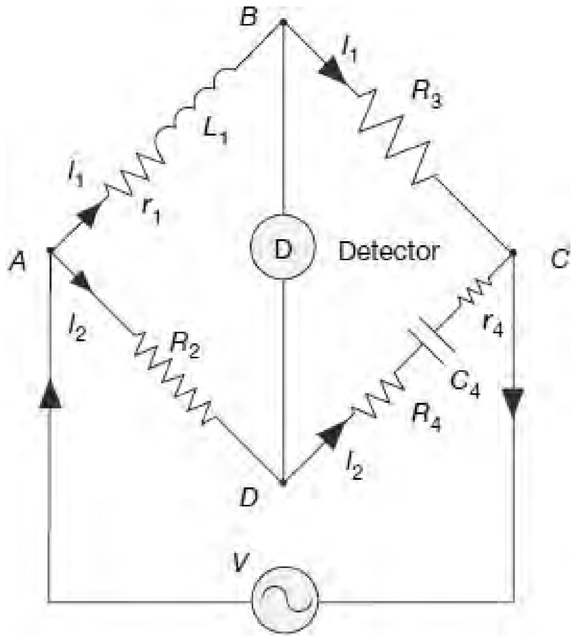
Arm CD: A mica capacitor C_4 in series a noninductive resistance R_4

Arm DA: A noninductive resistance R_2

When the bridge is supplied from a source of 450 Hz is given between terminals A and C and the detector is connected between nodes B and D, balance is obtained the following conditions: R_2 = 2400 Ω, R_3 = 600 Ω, C_4 = 0.3 μF and R_4 = 55.4 Ω. Series resistance of the capacitor is 0.5 Ω. Calculate the resistance and inductance of the choke coil.

Solution The bridge configuration is shown below:

Given that at balance,



$R_2 = 2400\ \Omega$, $R_3 = 600\ \Omega$, $C_4 = 0.3\ \mu F$, $R_4 = 55.4\ \Omega$ and $r_4 = 0.5\ \Omega$.

At balance, $\dfrac{r_1 + j\omega L_1}{R_3} = \dfrac{R_2}{r_4 + R_4 - \dfrac{j}{\omega C_4}}$

or,

$$r_1 r_4 + r_1 R_4 + \frac{L_1}{C_4} + j\omega L_1 R_4 + j\omega L_1 r_4 - \frac{j r_1}{\omega C_4} = R_2 R_3$$

Equating real and imaginary parts, we have

$$r_1 r_4 + r_1 R_4 \frac{L_1}{C_4} = R_2 R_3 \qquad \text{(i)}$$

and $\omega L_1 R_4 + \omega L_1 r_4 = \dfrac{r_1}{\omega C_4}$ (ii)

Solving (i) and (ii), we have the unknown quantities as

$$L_1 = \frac{R_2 R_3 C_4}{1 + \omega (R_4 + r_4)^2 C_4^{\ 2}} = \frac{2400 \times 600 \times 0.3 \times 10^{-6}}{1 + (2\pi \times 450 \times 55.9 \times 0.3 \times 10^{-6})^2} = 0.43\ \text{H}$$

and

$$r_1 = \frac{R_2 R_3 (R_4 + r_4)\omega^2 C_4^{\ 2}}{1 + \omega (R_4 + r_4)^2 C_4^{\ 2}} = \frac{2400 \times 600 \times 55.9 \times (2\pi \times 450)^2 \times (0.3 \times 10^{-6})^2}{1 + (2\pi \times 450 \times 55.9 \times 0.3 \times 10^{-6})^2} = 57.8\ \Omega$$

6.5 MEASUREMENT OF CAPACITANCE

Bridges are used to make precise measurements of unknown capacitances and associated losses in terms of some known external capacitances and resistances. An ideal capacitor is formed by placing a piece of dielectric material between two conducting plates or electrodes. In practical cases, this dielectric material will have some power losses in it due to dielectric's conduction electrons and also due to dipole relaxation phenomena. Thus, whereas an ideal capacitor will not have any losses, a real capacitor will have some losses associated with its operation. The potential energy across a capacitor is thus dissipated in all real capacitors as heat loss inside its dielectric material. This loss is equivalently represented by a series resistance, called the equivalent series resistance (ESR). In a good capacitor, the ESR is very small, whereas in a poor capacitor the ESR is large. *C* real *C* ideal *ESR*

A real, lossy capacitor can thus be equivalently represented by an ideal loss less capacitor in series *CR* with its equivalent series resistance (ESR) shown Figure 6.7 *Equivalent series resistance (ESR)* in Figure 6.7.

Figure 6.7 *Equivalent series resistance (ESR)*

The quantifying parameters often used to describe performance of a capacitor are ESR, its dissipation factor (DF), Quality Factor (*Q*-factor) and Loss Tangent (tan *d*). Not only that these parameters describe operation of the capacitor in radio frequency (RF) applications, but ESR and DF are also particularly important for capacitors operating in power supplies where a large dissipation factor will result in large amount of power being wasted in the capacitor. Capacitors with high values of ESR will need to dissipate large amount of heat. Proper circuit design needs to be practiced so as to take care of such possibilities of heat generation.

Dissipation factor due to the non-ideal capacitor is defined as the ratio of the resistive power loss in the ESR to the reactive power oscillating in the capacitor, or

$$DF = \frac{i^2 R}{i^2 X_C} = \frac{R}{1/\omega C} = \omega CR$$

When representing the electrical circuit parameters as phasors, a capacitor's dissipation factor is equal to the tangent of the angle between the capacitor's impedance phasor and the negative reactive axis, as shown in the *impedance triangle diagram* of Figure 6.8 This gives rise to the parameter known as the loss tangent *d* where Figure. 6.8 *Impedance*

$$\tan\delta = \frac{R}{X_C} = \frac{R}{1/\omega C} = \omega CR$$

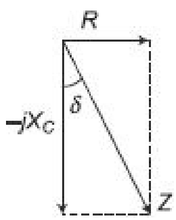


Figure. 6.8 *Impedance triangle diagram*

Loss tangent of a real capacitor can also be defined in the *voltage triangle diagram* of Figure 6.9 as the ratio of voltage drop across the *ESR* to the voltage drop across the capacitor only, i.e. tangent of the angle between the capacitor voltage only and the total voltage drop across the combination of capacitor and *ESR*.

$$\tan\delta = \frac{V_R}{V_C} = \frac{iR}{iX_C} = \frac{R}{1/\omega C} = \omega CR$$

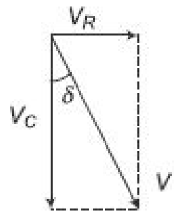


Figure. 6.9 *Voltage triangle diagram*

Though the expressions for dissipation factor (*DF*) and loss tangent (tan *δ*) are the same, normally the dissipation factor is used at lower frequencies, whereas the loss tangent is more applicable for high frequency applications. A good capacitor will normally have low values of dissipation factor (*DF*) and loss tangent (tan *δ*).

In addition to *ESR*, *DF* and loss tangent, the other parameter used to quantify performance of a real capacitor is its Quality Factor or *Q*-Factor. Essentially for a capacitor it is the ratio of the energy stored to that dissipated per cycle.

$$Q = \frac{i^2 X_C}{i^2 R} = \frac{X_C}{R}$$

It can thus be deduced that the *Q* can be expressed as the ratio of the capacitive reactance to the *ESR* at the frequency of interest.

$$Q = \frac{X_C}{R} = \frac{1}{\omega CR} = \frac{1}{DF} = \frac{1}{\tan\delta}$$

A high quality capacitor (high *Q*-factor) will thus have low values of dissipation factor (*DF*) and loss tangent (tan *δ*), i.e. less losses.

The most commonly used bridges for capacitance measurement are De Sauty's bridge and Schering Bridge.

6.5.1 De Sauty's Bridge

This is the simplest method of finding out the value of a unknown capacitor in terms of a known standard capacitor. Configuration and phasor diagram of a De Sauty's bridge under balanced condition is shown in Figure 6.10.

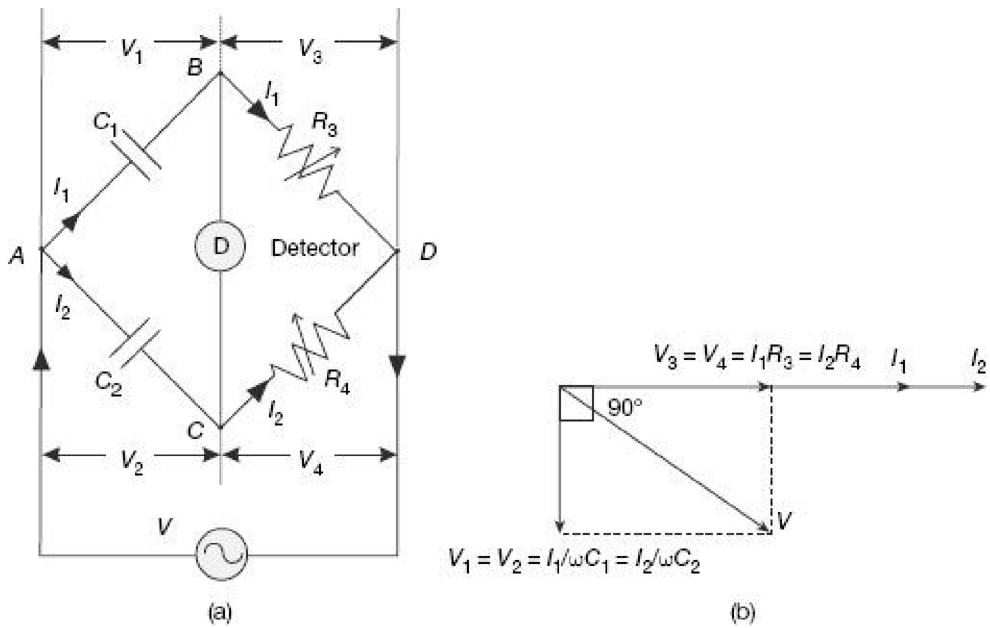


Figure 6.10 *De Sauty's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

The unknown capacitor C_1 in the branch AB is compared with the standard known standard capacitor C_2 on arm AC. The bridge can be balanced by varying either of the non-inductive resistors R_3 or R_4.

Under balanced condition, since no current flows through the detector, nodes B and C are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.7 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_2 in the arm CD are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. Capacitive voltage drop $V_1 = I_1/wC_1$ in the arm AB lags behind I_1 by 90°. Similarly, the other capacitive voltage drop $V_2 = I_2/wC_2$ in the arm AC lags behind I_2 by 90°. Under balanced condition, these two voltage drops V_1 and V_2 being equal in magnitude and phase, they overlap each other along the vertical axis as shown in Figure 6.7 (b). Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage V.

$$\text{At balance,} \quad \frac{\left(\frac{1}{j\omega C_1}\right)}{R_3} = \frac{\left(\frac{1}{j\omega C_2}\right)}{R_4}$$

$$\text{or,} \quad C_1 = C_2 \frac{R_4}{R_3} \tag{6.28}$$

The advantage of De Sauty's bridge is its simplicity. However, this advantage may be nullifi ed by impurities creeping in the measurement if the capacitors are not free from dielectric losses. This method is thus best suited for loss-less air capacitors.

In order to make measurement in capacitors having inherent dielectric losses, the *modified De Sauty's bridge* as suggested by Grover, can be used. This bridge is also called the series resistance-capacitance bridge. Configuration of such a bridge and its corresponding phasor diagram under balanced condition is shown in Figure 6.11.

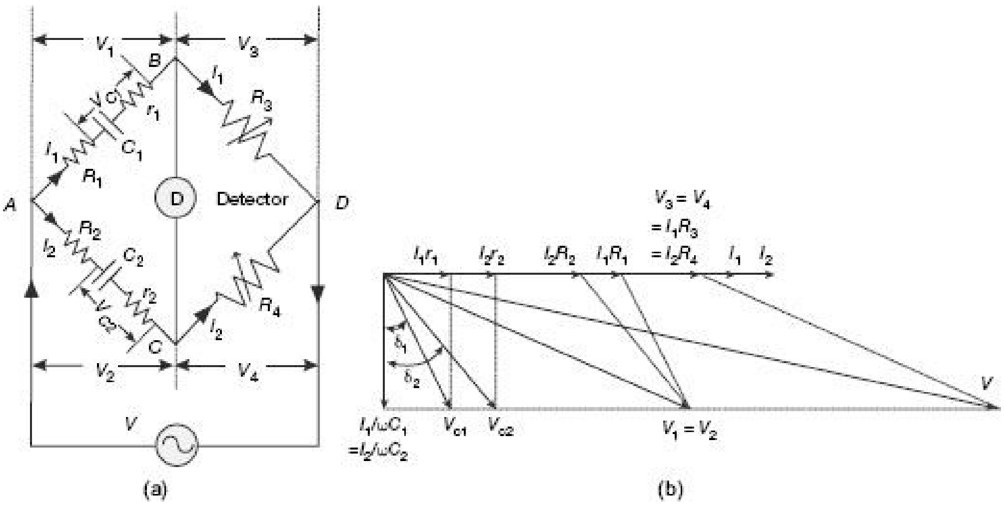


Figure 6.11 *Modified De Sauty's bridge under balanced condition: (a) Configuration, and (b) Phasor diagram*

The unknown capacitor C_1 with internal resistance r_1 representing losses in the branch AB is compared with the standard known standard capacitor C_2 along with its internal resistance r_2 on arm AC. Resistors R_1 and R_2 are connected externally in series with C_1

and C_2 respectively. The bridge can be balanced by varying either of the non-inductive resistors R_3 or R_4.

Under balanced condition, since no current flows through the detector, nodes *B* and *C* are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.11 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm *BD* and I_2 in the arm *CD* are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. The other resistive drops, namely, I_1R_1 in the arm *AB* and I_2R_2 in the arm *AC* are also along the same horizontal line. Finally, resistive drops inside the capacitors, namely, I_1r_1 and I_2r_2 are once again, in the same phase, along the horizontal line. Capacitive voltage drops I_1/wC_1 lags behind I_1r_1 by 90°. Similarly, the other capacitive voltage drop I_2/wC_2 lags behind I_2r_2 by 90°. Phasor summation of the resistive drop I_1r_1 and the quadrature capacitive drop I_1/wC_1 produces the total voltage drop V_{C1} across the series combination of capacitor C_1 and its internal resistance r_1. Similarly, phasor summation of the resistive drop I_2r_2 and the quadrature capacitive drop I_2/wC_2 produces the total voltage drop V_{C2} across the series combination of capacitor C_2 and its internal resistance r_2. d_1 and d_2 represent loss angles for capacitors C_1 and C_2 respectively. Phasor summation of I_1R_1 and V_{C1} gives the total voltage drop V_1 across the branch *AB*. Similarly, phasor summation of I_2R_2 and V_{C2} gives the total voltage drop V_2 across the branch *AC*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{\left(R_1 + r_1 + \frac{1}{j\omega C_1}\right)}{R_3} = \frac{\left(R_2 + r_2 + \frac{1}{j\omega C_2}\right)}{R_4}$$

or, $$R_1R_4 + r_1R_4 - \frac{jR_4}{\omega C_1} = R_2R_3 + r_2R_3 - \frac{jR_3}{\omega C_2}$$

Equating real and imaginary parts, we have

$$\frac{C_1}{C_2} = \frac{(R_2 + r_2)}{(R_1 + r_1)} = \frac{R_4}{R_3} \qquad (6.29)$$

$$\therefore \qquad C_1 = C_2\frac{R_4}{R_3} \qquad (6.30)$$

The modified De Sauty's bridge can also be used to estimate dissipation factor for the unknown capacitor as described below:

Dissipation factor for the capacitors are defined as

$$D_1 = \tan\delta_1 = \frac{I_1r_1}{\frac{I_1}{\omega C_1}} = \omega C_1r_1 \text{ and } D_2 = \tan\delta_2 = \frac{I_2r_2}{\frac{I_2}{\omega C_2}} = \omega C_2r_2 \qquad (6.31)$$

From Eq. (6.29), we have

$$\frac{C_1}{C_2} = \frac{(R_2 + r_2)}{(R_1 + r_1)}$$

or, $$C_2 r_2 - C_1 r_1 = C_1 R_1 - C_2 R_2$$

or, $$\omega C_2 r_2 - \omega C_1 r_1 = \omega C_1 R_1 - C_2 R_2$$

Using Eq. (6.31), we getor, *D* -*D* =*w* (*CR* -*CR*)

or, $$D_2 - D_1 = \omega (C_1 R_1 - C_2 R_2)$$

Substituting the value of C_1 from Eq. (6.30), we have

$$D_2 - D_1 = \omega C_2 \left(\frac{R_1 R_4}{R_3} - R_2 \right) \quad (6.32)$$

Thus, dissipation factor for one capacitor can be estimated if dissipation factor of the other capacitor is known.

6.5.2 Schering Bridge

Schering bridges are most popularly used these days in industries for measurement of capacitance, dissipation factor, and loss angles. Figure 6.12 illustrates the confi guration of a Schering bridge and corresponding phasor diagram under balanced condition.

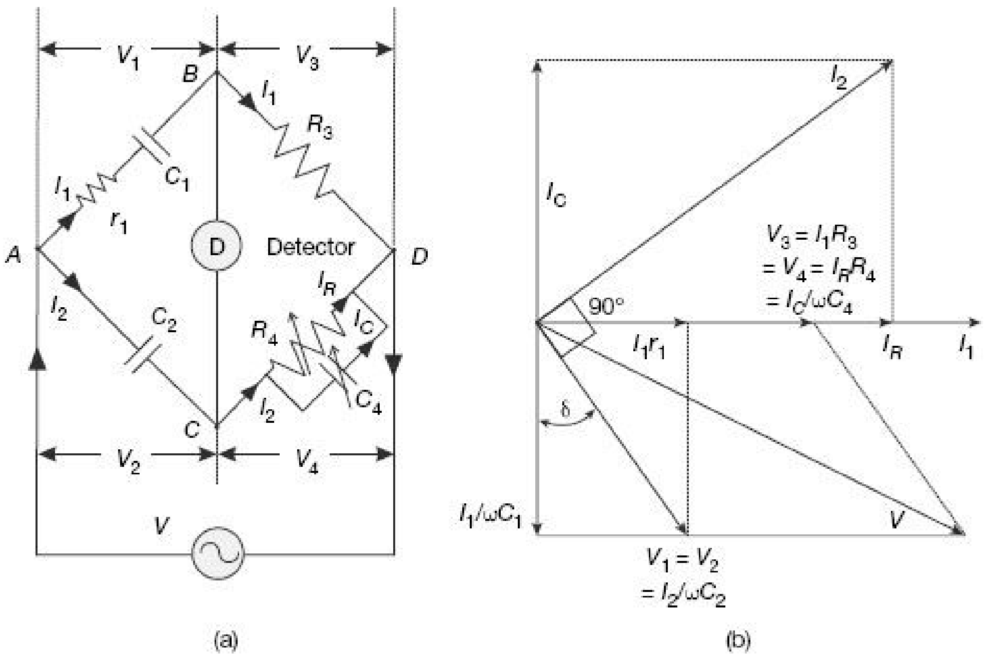


Figure 6.12 *Schering bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

The unknown capacitor C_1 along with its internal resistance r_1 (representing loss) placed on the arm *AB* is compared with the standard loss-less capacitor C_2 placed on the arm *AC*. This capacitor C_2 is either an air or a gas capacitor to make it loss free. R_3 is a non-inductive resistance placed on arm *BD*. The bridge is balanced by varying the capacitor C_4 and the non-inductive resistor R_4 parallel with C_4, placed on arm *CD*.

Under balanced condition, since no current flows through the detector, nodes *B* and *C* are at the same potential, i.e., $V_1 = V_2$ and $V_3 = V_4$.

As shown in the phasor diagram of Figure 6.12 (b), $V_3 = I_1 R_3$ and $V_4 = I_R R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_R flowing through R_4 are also in the same phase with I_1R_3 and I_RR_4 along the horizontal line. The other resistive drop namely, I_1R_1 in the arm AB is also along the same horizontal line. The resistive current I_R through R_4 and the quadrature capacitive current I_C through C_4 will add up to the total current I_2 in the branch CD (and also in A C under balanced condition). Across the arm AB, the resistive drop I_1r_1 and the quadrature capacitive drop I_1/wC_1 will add up to the total voltage drop V_1 across the arm. At balance, voltage drop V_1 across arm AB will be same as the voltage drop $V_2 = I_2/wC_2$ across the arm AC. It can be confirmed from the phasor diagram in Figure 6.12(b) that the current I_2 has quadrature phase relationship with the capacitive voltage drop I_2/wC_2 in the arm AC. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage V.

At balance, $$\frac{\left(r_1 + \frac{1}{j\omega C_1}\right)}{R_3} = \frac{\left(\frac{1}{j\omega C_2}\right)}{\left(\frac{R_4}{1 + j\omega C_4 R_4}\right)}$$

or, $$R_4\left(r_1 + \frac{1}{j\omega C_1}\right) = \left(\frac{R_3}{j\omega C_2}\right)(1 + j\omega C_4 R_4)$$

or, $$R_4 r_1 - \frac{jR_4}{\omega C_1} = \frac{R_3 R_4 C_4}{C_2} - \frac{jR_3}{\omega C_2}$$

Equating real and imaginary parts, we have the unknown quantities:

$$r_1 = \frac{R_3 C_4}{C_2} \tag{6.33}$$

and

$$C_1 = C_2 \frac{R_4}{R_3} \tag{6.34}$$

Dissipation Factor

$$D_1 = \tan\ \delta_1 = \frac{I_1 r_1}{\frac{I_1}{\omega C_1}} = \omega C_1 r_1 = \omega \times C_2 \frac{R_4}{R_3} \times \frac{R_3 C_4}{C_2} = \omega R_4 C_4 \tag{6.35}$$

Thus, using Schering bridge, dissipation factor can be obtained in terms of the bridge parameters at balance condition.

6.6 MEASUREMENT OF FREQUENCY

6.6.1 Wien's Bridge

Wien's bridge is primarily used for determination of an unknown frequency. However, it can be used for various other applications including capacitance measurement, in harmonic distortion analysers, where it is used as notch filter, and also in audio and HF oscillators.

Configuration of a Wien's bridge for determination of unknown frequency and corresponding phasor diagram under balanced condition is shown in Figure 6.13.

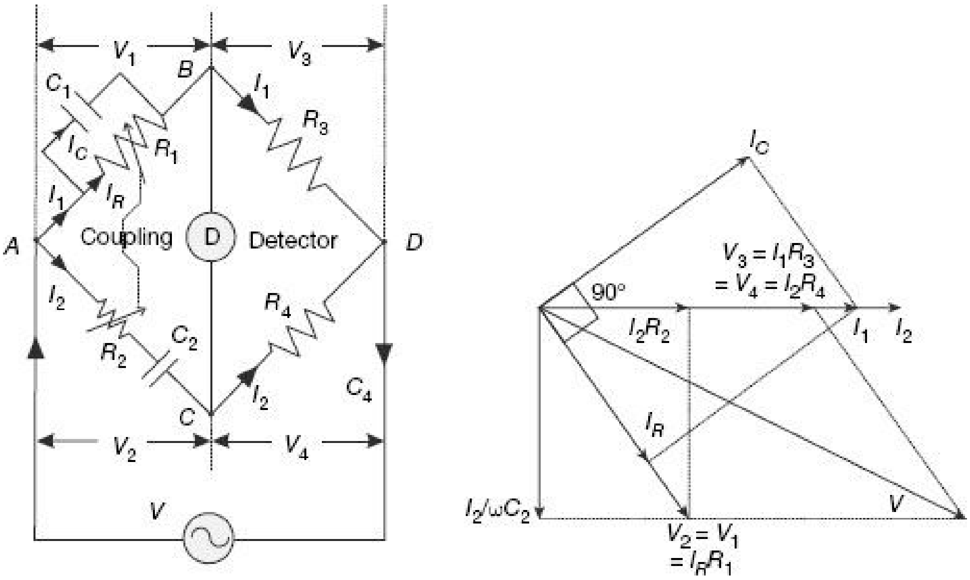


Figure 6.13 *Wien's bridge under balanced condition: (a) Confi guration (b) Phasor diagram*

Under balanced condition, since no current fl ows through the detector, nodes B and C are at the same potential, i.e., $V_1=V_2$ and $V_3=V_4$.

As shown in the phasor diagram of Figure 6.13 (b), $V_3 = I_1R_3$ and $V_4 = I_2R_4$ being equal both in magnitude and phase, they overlap. Current I_1 in the arm BD and I_2 flowing through R4 are also in the same phase with I_1R_3 and I_2R_4 along the horizontal line. The other resistive drop, namely, I_2R_2 in the arm *AC* is also along the same horizontal line. The resistive voltage drop I_RR_2 across R_2 and the quadrature capacitive drop I_2/wC_2 across C_2 will add up to the total voltage drop V_2 in the arm *AC*. Under balanced condition, voltage drops across arms *AB* and *AC* are equal, thus $V_1 = V_2$ both in magnitude and phase. The voltage V_1 will be in the same phase as the voltage drop I_RR_1 across the resistance R_1 in the same arm *AB*. The resistive current I_R will thus be in the same phase as the voltage $V_1 = I_RR_1$. Phasor addition of the resistive current I_R and the quadrature capacitive current I_C, which flows through the parallel R_1C_1 branch, will add up to the total current I_1 in the arm *AB*. Finally, phasor summation of V_1 and V_3 (or V_2 and V_4) results in the supply voltage *V*.

At balance, $$\frac{\left(\dfrac{R_1}{1+j\omega C_1R_1}\right)}{R_3} = \frac{\left(R_2 - \dfrac{j}{\omega C_2}\right)}{R_4}$$

or, $$\frac{R_1R_4}{1+j\omega C_1R_1} = \frac{\omega C_2R_2R_3 - jR_3}{\omega C_2}$$

or, $$\omega C_2R_1R_4 = \omega C_2R_2R_3 - jR_3 + j\omega^2C_1C_2R_1R_2R_3 + \omega C_1R_1R_3$$

or, $$\omega(C_2R_1R_4) = \omega(C_2R_2R_3 + C_1R_1R_3) - j(R_3 - \omega^2C_1C_2R_1R_2R_3)$$

Equating real and imaginary parts, we get

$$C_2R_1R_4 = C_2R_2R_3 + C_1R_1R_3$$

$$\text{or,} \quad \frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \tag{6.36}$$

and

$$\omega^2 C_1 C_2 R_1 R_2 R_3 = R_3$$

$$\text{or,} \quad \omega = \sqrt{\frac{1}{C_1 C_2 R_1 R_2}}$$

$$\text{or,} \quad f = \frac{1}{2\pi\sqrt{C_1 C_2 R_1 R_2}} \tag{6.37}$$

In most bridges, the parameters are so chosen that,

$$R_1 = R_2 = R \text{ and } C_1 = C_2 = C$$

Then, from Eq. (6.37), we get

$$f = \frac{1}{2\pi RC} \tag{6.38}$$

Sliders for the resistors R_1 and R_2 are mechanically coupled to satisfy the criteria $R_1 = R_2$.

Wien's bridge is frequency sensitive. Thus, unless the supply voltage is purely sinusoidal, achieving balance may be troublesome, since harmonics may disturb balance condition. Use of filters with the null detector in such cases may solve the problem.

Example 6.4 *The four arms of a bridge are connected as follows:*

Arm AB: A capacitor C_1 with an equivalent series resistance r_1

Arm BC: A noninductive resistance R_3

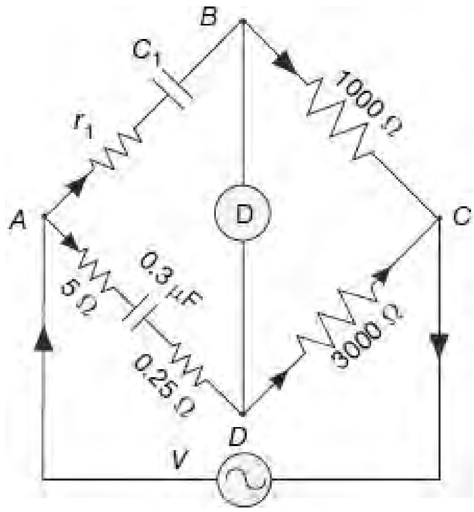
Arm CD: A noninductive resistance R_4

Arm DA: A capacitor C_2 with an equivalent series resistance r_2 in series with a resistance R_2

A supply of 500 Hz is given between terminals A and C and the detector is connected between nodes B and D. At balance, R_2 = 5 Ω, R_3 = 1000 Ω, R_4 = 3000 Ω, C_2 = 0.3 µF and r_2 = 0.25 Ω. Calculate the values of C_1 and r_1, and also dissipation factor of the capacitor.

Solution The configuration can be shown as

$$\text{At balance,} \quad \frac{r_1 + \dfrac{1}{j\omega C_1}}{1000} = \frac{5.25 + \dfrac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}}{3000}$$



or, $$3r_1 + \frac{3}{j\omega C_1} = 5.25 + \frac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}$$

Equating real and imaginary terms, we get

$$r_1 = \frac{5.25}{3} = 1.75\ \Omega$$

and, $$\frac{3}{j2\pi \times 500 \times C_1} = \frac{1}{j2\pi \times 500 \times 0.3 \times 10^{-6}}$$

or, $$C_1 = 3 \times 0.3 \times 10^{-6} = 0.9\ \mu\text{F}$$

Example 6.5 *The four arms of a bridge supplied from a sinusoidal source are configured as follows:*

Arm AB: A resistance of 100 Ω in parallel with a capacitance of 0.5 μF

Arm BC: A 200 Ω noninductive resistance

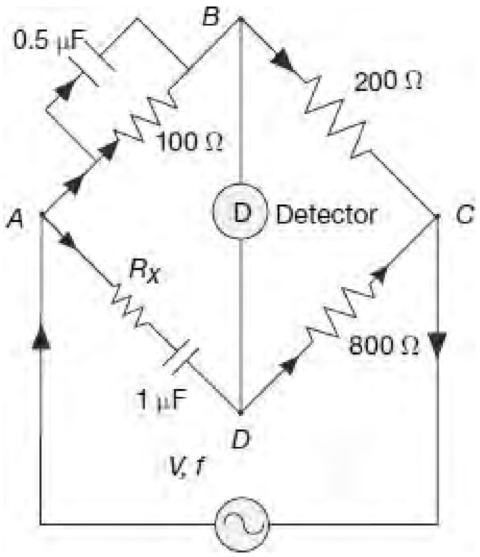
Arm CD: A 800 Ω noninductive resistance

Arm DA: A resistance R_x in series with a 1 μF capacitance

Determine the value of R_x and the frequency at which the bridge will balance.

Supply is given between terminals A and C and the detector is connected between nodes B and D.

Solution The configuration can be shown as



The configuration shows that it is a Wien's bridge. Thus, following Eq. (6.36), the balance equation can be written as

$$\frac{R_4}{R_3} = \frac{R_x}{R_1} + \frac{C_1}{C_2}$$

Thus,
$$R_x = R_1 \times \left(\frac{R_4}{R_3} - \frac{C_1}{C_2} \right) = 100 \times \left(\frac{800}{200} - \frac{0.5 \times 10^{-6}}{1 \times 10^{-6}} \right) = 350\ \Omega$$

The frequency at which bridge is balanced is given by Eq. (6.37):

$$f = \frac{1}{2\pi\sqrt{0.5 \times 10^{-6} \times 1 \times 10^{-6} \times 100 \times 350}} = 1203\ \text{Hz}$$

6.7 WAGNER EARTHING DEVICE

A serious problem encountered in sensitive ac bridge circuits is that due to stray capacitances. Stray capacitances may be formed in an ac bridge between various junction points within the bridge configuration and nearest ground (earthed) object. These stray capacitors affect bridge balance in severe ways since these capacitors carry leakage current when the bridge is operated with ac, especially at high frequencies. Formation of such stray capacitors in a simple ac bridge circuit is schematically shown in Figure 6.14.

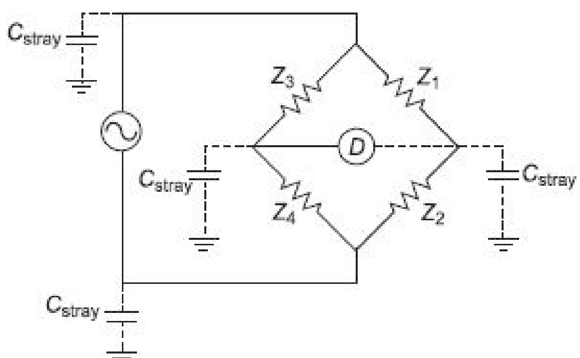


Figure 6.14 *Formation of stray capacitors in an ac bridge*

One possible way of reducing this effect is to keep the detector at ground potential, so there will be no ac voltage between it and the ground, and thus no current through Figure 6.14

Formation of stray capacitors in an ac bridge the stray capacitances can leak out. However, directly connecting the null detector to ground is not an option, since it would create a direct current path for other stray currents. Instead, a special voltage-divider circuit, called a *Wagner ground* or *Wagner earth*, may be used to maintain the null detector at ground potential without having to make a direct connection between the detector and ground.

The Wagner earth circuit is nothing more than a voltage divider as shown in Figure 6.15. There are two additional (auxiliary) arms Z_A and Z_B in the bridge configuration with a ground connection at their junction E. The switch S is used to connect one end of the detector alternately to the ground point e and the bridge connection point d. The two impedances Z_A and Z_B must be made of such components (R, L, or C) so that they are capable of forming a balanced bridge with the existing bride arm pairs Z_1–Z_2 or Z_3–Z_4. Stray capacitances formed between bridge junctions and the earthing point E are shown as C_1, C_2, C_3 and C_4.

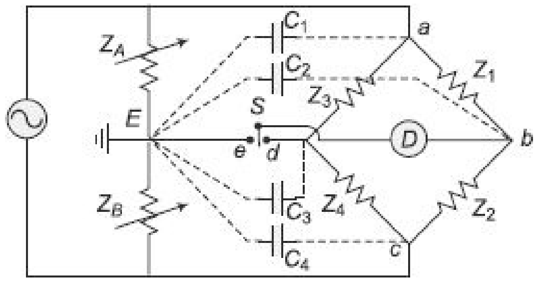


Figure 6.15 *Wagner earthing device*

The bridge is first balanced with the arms Z_1–Z_2 and Z_3–Z_4 with the switch at the position d. The switch is next thrown to position e and balance is once again attained between arms Z_1–Z_2 and Z_5–Z_6. The process is repeated till both bridge configurations become balanced. At this point, potential at the points b, d, e are the same and are all at earth potential. Thus, the Wagner earthing divider forces the null detector to be at ground potential, without a direct connection between the detector and ground. Under these conditions, no current can flow through the stray capacitors C_2 and C_3 since their terminals are both at earth potential. The other two stray capacitors C_1 and C_4 become part of (shunt to) the Wagner arms Z_A and Z_B, and thus get eliminated from the original bridge network.

The Wagner earthing method gives satisfactory results from the point of view of eliminating stray capacitance charging effects in ac bridges, but the entire balancing process is time consuming at times.

EXERCISE

Objective-type Questions

1. A bridge circuit works at a frequency of 2 kHz. Which of the following can be used as null detector in such a bridge?
   (a) Vibration galvanometers and tunable amplifiers
   (b) Headphones and tunable amplifiers

(c) Vibration galvanometers and headphones

(d) All of the above

2. Under balanced condition of a bridge for measuring unknown impedance, if the detector is suddenly taken out

   (a) measured value of the impedance will be lower

   (b) measured value of the impedance will be higher

   (c) measured value of the impedance will not change

   (d) the impedance can not be measured

3. Harmonic distortions in power supply does not affect the performance of Maxwell's bridge since

   (a) filters are used to remove harmonics

   (b) final expression for unknown inductance contain only fundamental frequency

   (c) mechanical resonance frequency of null detectors are beyond the range of harmonic frequencies

   (d) final expression for unknown inductance is independent of frequency

4. Maxwell's bridge can be used for measurement of inductance with

   (a) high Q factors

   (b) very low Q factors

   (c) medium Q factors

   (d) wide range of Q factor variations

5. The advantage of Hay's bridge over Maxwell's inductance–capacitance bridge is that

   (a) its final balance equations are independent of frequency

   (b) it reduces cost by not making capacitor or inductor as the variable parameters

   (c) it can be used measuring low Q inductors

   (d) it can be used measuring high Q inductors

6. The advantage of Anderson's bridge over Maxwell's bridge is that

   (a) its final balance equations are independent of inductor losses

   (b) it reduces cost by not making capacitor or inductor as the variable parameters

   (c) number of bridge components required are less

   (d) attaining balance condition is easier and less time consuming

7. The main advantage of Owen's bridge for measurement of unknown inductance is that

   (a) it has two independent elements R and C for achieving balance

   (b) it can be used for measurement of very high Q coils

   (c) it is very inexpensive

   (d) it can be used for measurement of unknown capacitance as well

8. DeSauty's bridge is used for measurement of

   (a) high Q inductances

   (b) low Q inductances

   (c) loss less capacitors

   (d) capacitors with dielectric losses

9. Schering bridge can be used for measurement of

   (a) capacitance and dissipation factor

   (b) dissipation factor only

   (c) inductance with inherent loss

   (d) capacitor but not dissipation factor

10. Frequency can be measured using
    (a) Anderson's bridge
    (b) Maxwell's bridge
    (c) De Sauty's bridge
    (d) Wien's bridge

Answers

1. (b)	2. (c)	3. (d)	4. (c)	5. (d)	6. (b)	7. (a)
8. (c)	9. (a)	10. (d)				

Short-answer Questions

1. Derive the general equations for balance in ac bridges. Show that both magnitude and phase conditions need to be satisfied for balancing an ac bridge.
2. Derive the expression for balance in Maxwell's inductance bridge. Draw the phasor diagram under balanced condition.
3. Show that the final balance expressions are independent of supply frequency in a Maxwell's bridge. What is the advantage in having balance equations independent of frequency?
4. Discuss the advantages and disadvantages of Maxwell's bridge for measurement of unknown inductance.
5. Explain why Maxwell's inductance–capacitance bridge is suitable for measurement of inductors having quality factor in the range 1 to 10.
6. Explain with the help of phasor diagram, how unknown inductance can be measured using Owen's bridge.
7. Explain how Wien's bridge can be used for measurement of unknown frequencies. Derive the expression for frequency in terms of bridge parameters.

Long-answer Questions

1. (a) Explain with the help of a phasor diagram, how unknown inductance can be measured using Maxwell's inductance–capacitance bridge.

   (b) The following data relate to a basic ac bridge:

   $\bar{Z}_1 = 50\ \Omega\angle 80^\circ \quad \bar{Z}_2 = 125\ \Omega \quad \bar{Z}_3 = 200\ \Omega\angle 30^\circ \quad \bar{Z}_4 = \text{unknown}$

   Determine the unknown arm parameters. [10 + 5]
2. Describe the working of Hay's bridge for measurement of inductance. Derive the equations for balance and draw the phasor diagram under balanced condition. Explain how this bridge is suitable for measurement of high *Q* chokes?
3. Derive equations for balance for an Anderson's bridge. Draw its phasor diagram under balance. What are its advantages and disadvantages?
4. Describe how unknown capacitors can be measured using De Sauty's bridge. What are the limitations of this bridge and how they can be overcome by using a modified De Sauty's bridge? Draw relevant phasor diagrams
5. Describe the working of a Schering bridge for measurement of capacitance and dissipation factor. Derive relevant equations and draw phasor diagram under balanced condition.
6. In an Anderson's bridge for measurement of inductance, the arm *AB* consists of an unknown impedance with *L* and *R*, the arm *BC* contains a variable resistor, fixed resistances of 500 Ω each in arms *CD* and *DA*, a known variable resistance in the arm *DE*, and a capacitor of fixed capacitance 2 μF in the arm *CE*. The ac supply of 200 Hz is connected across *A* and *C*, and the detector is connected between *B* and *E*. If balance is obtained with a resistance of 300 Ω in the arm *DE* and a resistance of 600 Ω in the arm *BC*, calculate values of unknown impedance *L* and *R*. Derive the relevant equations for balance and draw the phasor diagram.
7. The four arms of a Maxwell's inductance–capacitance bridge at balance are Arm *AB* : A choke coil L1 with an equivalent series resistance R_1 Arm *BC* : A non-inductive resistance of 800 ΩArm *CD* : A mica capacitor of 0.3 μF in parallel with a noninductive resistance of 800 ΩArm *DA* : A non-inductive resistance 800 ΩSupply is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*. Derive the equations for balance

of the bridge and hence determine values of L_1 and R_1. Draw the phasor diagram of the bridge under balanced condition.

8. The four arms of a Hay's bridge used for measurement of unknown inductance is configured as follows: Arm *AB* : A choke coil of unknown impedance Arm *BC* : A non-inductive resistance of 1200 ΩArm *CD* : A non-inductive resistance of 900 Ω in series with a standard capacitor of 0.4 μF Arm *DA* : A noninductive resistance 18000 ΩIf a supply of 300 V at 50 Hz is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*, determine the inductance and inherent resistance of the unknown choke coil. Derive the conditions for balance and draw the phasor diagram under balanced condition.

9. A capacitor busing forms the arm *AB* of a Schering bridge and a standard capacitor of 400 μF capacitance and negligible loss, form the arm *AD*. Arm *BC* consists of a non-inductive resistance of 200 Ω. When the detector connected between nodes *B* and *D* shows no deflection, the arm *CD* has a resistance of 82.4 Ωin parallel with a capacitance of 0.124 μF. The supply frequency is 50 Hz. Calculate the capacitance and dielectric loss angle of the capacitor. Derive the equations for balance and draw the relevant phasor diagram at balanced state.

10. (a) An ac bridge is configured as follows:

    Arm *AB* : A resistance of 600 Ω in parallel with a capacitance of 0.3 μF

    Arm *BC* : An unknown non-inductive resistance

    Arm *CD* : A noninductive resistance of 1000 Ω

    Arm *DA* : A resistance of 400 Ω in series with a capacitance of 0.1 μF

    If a supply is given between terminals *A* and *C* and the detector is connected between nodes *B* and *D*, find the resistance required in the arm *BC* and also the supply frequency for the bridge to be balanced.

    (b) Explain how Wien's bridge can be used for measurement of unknown frequency. Draw the phasor diagram under balanced condition and derive the expression for balance.

7 Power Measurement

7.1 INTRODUCTION

Measurement of electric power is as essential in industry as in commercial or even domestic applications. Prior estimation and subsequent measurements of instantaneous and peak power demands of any installation are mandatory for design, operation and maintenance of the electric power supply network feeding it. Whereas an under-estimation of power demand may lead to blowing out of power supply side accessories, on the other hand, over-estimation can end up with over-design and additional cost of installation. Knowledge about accurate estimation, calculation and measurement of electric power is thus of primary concern for designers of new installations. In this chapter, the most popular power measurement methods and instruments in dc and ac circuits are illustrated.

7.2 POWER MEASUREMENT IN dc CIRCUITS

Electric power (P) consumed by a load (R) supplied from a dc power supply (V_S) is the product of the voltage across the load (V_R) and the current flowing through the load (I_R):

$$P = V_R \times I_R \tag{7.1}$$

Thus, power measurement in a dc circuit can be carried out using a voltmeter (V) and an ammeter (A) using any one of the arrangements shown in Figure 7.1.

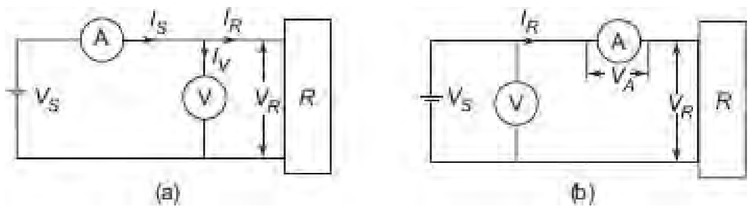


Figure 7.1 *Two arrangements for power measurement in dc circuits*

One thing should be kept in mind while using any of the two measuring arrangements shown in Figure 7.1; that both the voltmeter and the ammeter requires power for their own operations. In the arrangement of Figure 7.1(a), the voltmeter is connected between the load and the ammeter. The ammeter thus, in this case measures the current flowing into the voltmeter, in addition to the current flowing into the load.

$$\text{Current through the voltmeter} = I_V = V_R/R_V \tag{7.2}$$

where, R_V is the internal resistance of the voltmeter.

$$\begin{aligned}\text{Power consumed by the load} &= V_R \times I_R = V_R \times (I_S - I_V)\\ &= V_R \times I_S - V_R \times I_V\\ &= V_R \times I_S - V_R^2/R_V \qquad (7.3)\\ &= \text{Power indicated by instruments} - \text{Power loss in voltmeter}\end{aligned}$$

Thus, Power indicated = Power consumed + Power loss in voltmeter

In the arrangement of Figure 7.1(b), the voltmeter measures the voltage drop across the ammeter in addition to that dropping across the load.

$$\text{Voltage drop across ammeter} = V_A = I_R \times R_A \quad (7.4)$$

where, R_A is the internal resistance of the ammeter.

$$\begin{aligned}\text{Power consumed by the load } &= V_R \times I_R = (V_S - V_A) \times I_R \\ &= V_S \times I_R - V_A \times I_R \\ &= V_S \times I_R - I_R^2 \times R_A \quad (7.5) \\ &= \text{power indicated by instruments} - \text{Power loss in ammeter}\end{aligned}$$

Thus, Power indicated = Power consumed + Power loss in Ammeter

Thus, both arrangements indicate the additional power absorbed by the instruments in addition to indicating the true power consumed by the load only. The corresponding measurement errors are generally referred to as insertion errors.

Ideally, in theory, if we consider voltmeters to have infi nite internal impedance and ammeters to have zero internal impedance, then from (7.3) and (7.5) one can observe that the power consumed by the respective instruments go down to zero. Thus, in ideal cases, both the two arrangements can give correct indication of the power consumed by the load. Under practical conditions, the value of power loss in instruments is quite small, if not totally zero, as compared with the load power, and therefore, the error introduced on this account is small.

Example 7.1

Two incandescent lamps with 80 Ω and 120 Ω resistances are connected in series with a 200 V dc source. Find the errors in measurement of power in the 80 Ω lamp using a voltmeter with internal resistance of 100 kΩ and an ammeter with internal resistance of 0.1 mΩ, when (a) the voltmeter is connected nearer to the lamp than the ammeter, and (b) when the ammeter is connected nearer to the lamp than the voltmeter

Solution Assuming both the instruments to be ideal, i.e., the voltmeter with infinite internal impedance and ammeter with zero internal impedance, the current through the series circuit should be

$$= 200/(80 + 120) = 1 \text{ A}$$

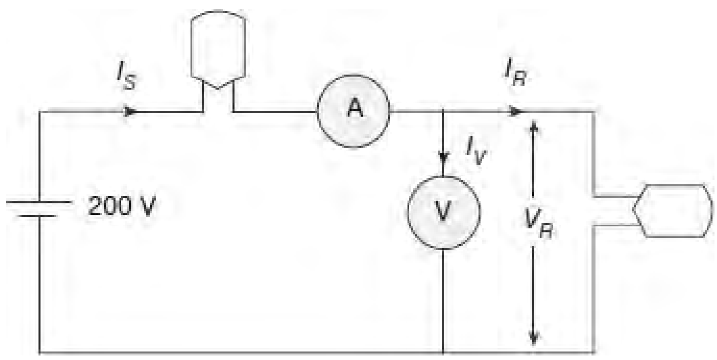


Figure 7.2 *Actual connections for Example 7.1(a)*

Hence, true power consumed by the 80 Ω lamp would have been

$= 1^2 \times 80 = 80\ \Omega$

However, considering the internal resistance of the ammeter and voltmeter, the equivalent circuit will look like Figure 7.3.

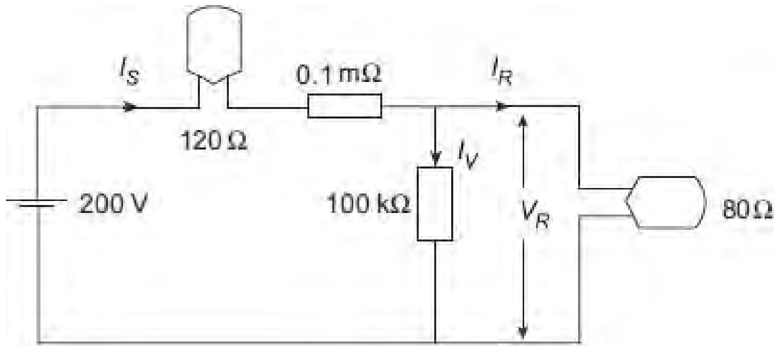


Figure 7.3 *Equivalent circuit for Example 7.1(a)*

Supply current (ammeter reading)

I_S = Supply voltage/Equivalent resistance of the circuit

$$= \frac{\text{Supply voltage}}{(\text{Series of Lamp 1 and ammeter}) + (\text{Parallel of Lamp 2 and voltmeter})}$$

$$= 200 \Big/ \left((120 + 0.1\times10^{-3}) + \frac{(100\times10^3\times80)}{(100\times10^3+80)} \right)$$

$= 1.0003$ A

Actual current through the 80 Ω lamp is

$$I_R = 1.0003 \times \frac{100\times10^3}{100\times10^3+80}\ \text{A}$$

$= 0.9995$ A

Voltage across the 80 Ω lamp (voltmeter reading) is

$V_R = I_R \times 80$

$= 79.962$ V

Thus, actual power consumed by the 80 Ω lamp is

$V_R \times I_R = 79.962 \times 0.9995 = 79.922$ W

Power consumption as indicated by the two meters

= Voltmeter reading × Ammeter reading

$= 79.962 \times 1.0003 = 79.986$ W

(b) In this case, the actual circuit and its equivalent will look like Figure 7.4.

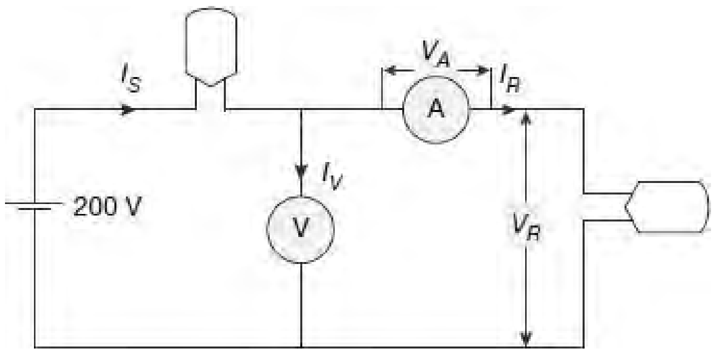


Figure 7.4 *Actual connection for Example 7.1(b)*

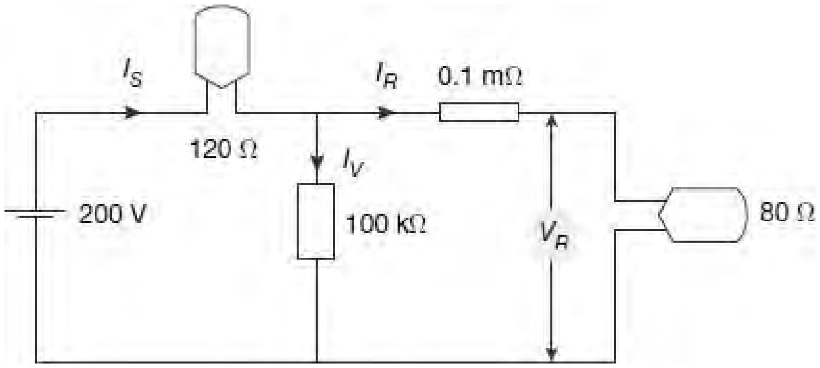


Figure 7.5 *Equivalent circuit for Example 7.1(a)*

Supply current

$$I_S = \text{Supply voltage/Equivalent resistance of the circuit}$$

$$= \frac{\text{Supply voltage}}{\text{Series of Lamp 1 and [Parallel of voltmeter and (Series of ammeter and Lamp 2)]}}$$

$$= 200\Big/\left(120 + \frac{[100\times10^3 \times (80+0.1\times10^{-3})]}{100\times10^3 + (80+0.1\times10^{-3})}\right)$$

$$= 1.003 \text{ A}$$

Current through the 80 Ω lamp (ammeter reading) is

$$I_R = 1.0003 \times \frac{100\times10^3}{100\times10^3 + (80+0.1\times10^{-3})}$$

$$= 0.9995 \text{ A}$$

Voltage across the 80 Ω lamp

$$= I_R \times 80 = 79.96 \text{ V}$$

Voltmeter reading

$$= I_V \times 100 \times 10^3$$

$$= (I_S - I_R) \times 100 \times 10^3 = 80 \text{ V}$$

Thus, actual power consumed by the 80 Ω lamp is

$$V_R \times I_R = 79.96 \times 0.9995 = 79.92 \text{ W}$$

Power consumption as indicated by the two meters

$$= \text{Voltmeter reading} \times \text{Ammeter reading}$$

$$= 80 \times 0.9995 = 79.96 \text{ W}$$

Thus, we can have the following analysis:

Case	*Power Consumption by 80 W lamp (W)*			*% Error from ideal*
	Ideal Power	*Actual Power*	*Meter Indication*	
a	80	79.922	79.986	0.0175
b	80	79.92	79.96	0.05

Power in dc circuits can also be measured by wattmeter. Wattmeter can give direct indication of power and there is no need to multiply two readings as in the case when ammeter and voltmeter is used.

The type of wattmeter most commonly used for such power measurement is the *dynamometer*. It is built by (1) two fixed coils, connected in series and positioned coaxially with space between them, and (2) a moving coil, placed between the fixed coils and fitted with a pointer. Such a construction for a dynamometer-type wattmeter is shown in Figure 7.6.

It can be shown that the torque produced in the dynamometer is proportional to the product of the current flowing through the fixed coils times that through the moving coil.

The fixed coils, generally referred to as current coils, carry the load current while the moving coil, generally referred to as voltage coil, carries a current that is proportional, via the multiplier resistor R_V, to the voltage across the load resistor R. As a consequence, the deflection of the moving coil is proportional to the power consumed by the load.

A typical connection of such a wattmeter for power measurement in dc circuit is shown in Figure 7.7.

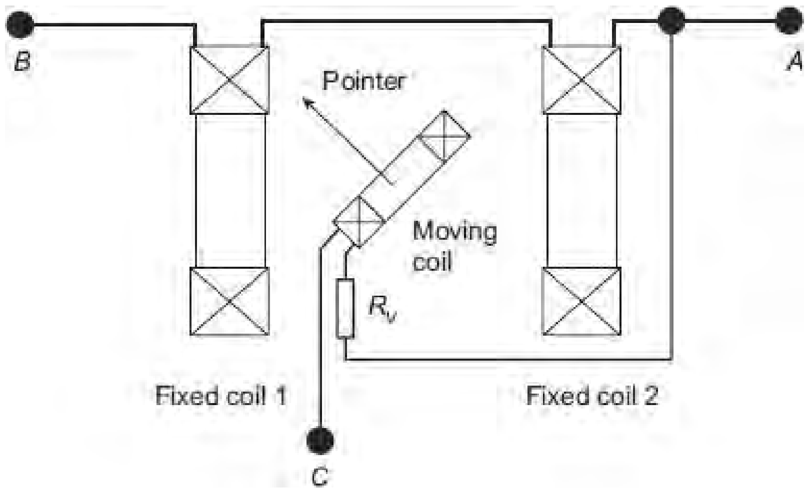


Figure 7.6 *Basic construcion of dynamometer-type wattmeter*

In such a connection of the wattmeter, the insertion error, as in the pervious case with ammeter and voltmeter, still exists. Relative β positioning of the current coil and the voltage coil with respect to load, introduce similar V_S errors in measurement of actual power. In particular, by connecting the voltage coil between A and C (Figure 7.7), the current coils carry the surplus current flowing through the voltage coil. On the other hand, by connecting the moving coil between B and C, this current error can be avoided, but now the voltage coil measures the surplus voltage drop across the current coils.

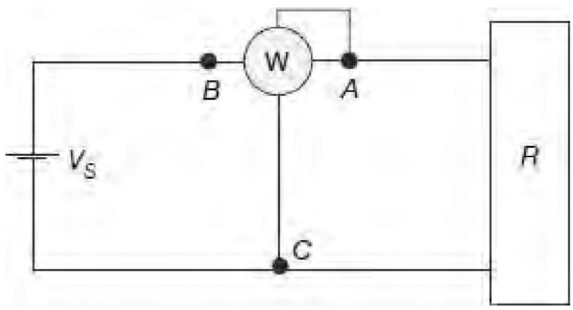


Figure 7.7 *Connection of dynamometer-type wattmeter for power measurement in dc circuit*

7.3 POWER MEASUREMENT IN ac CIRCUITS

In alternating current circuits, the instantaneous power varies continuously as the voltage and current varies while going through a cycle. In such a case, the power at any instant is given by

$$p(t) = v(t) \times i(t) \tag{7.6}$$

where, $p(t)$, $v(t)$, and $i(t)$ are values of instantaneous power, voltage, and current respectively.

Thus, if both voltage and current can be assumed to be sinusoidal, with the current lagging the voltage by phase-angle φ, then

$$v(t) = V_m \sin \omega t$$

and $$i(t) = I_m \sin (\omega t - \varphi)$$

where, V_m and I_m are peak values of voltage and current respectively, and *w* is the angular frequency.

The instantaneous power *p*is therefore given by

$$p(t) = V_m I_m \sin \omega t \sin (\omega t - \varphi) \tag{7.7}$$

or, $$p(t) = \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]$$

Average value of power over a complete cycle in such a case will be

$$\begin{aligned} P &= \frac{1}{2T}\int_0^{2T} p(t)\,dt = \frac{1}{2T}\int_0^{2T} \frac{V_m I_m}{2}[\cos\varphi - \cos(2\omega t - \varphi)]\,dt \\ &= \frac{V_m I_m}{2T}\int_0^{2T}\left[\cos\varphi - \cos\left(\frac{4\pi}{T}t - \varphi\right)\right]dt \\ &= \frac{V_m I_m}{2T}\left[\cos\varphi\, t\Big|_0^T - \frac{T}{4\pi}\sin\left(\frac{4\pi}{T}t - \varphi\right)\Bigg|_0^T\right] \\ &= \frac{V_m I_m}{4T}[\cos\varphi\, T - 0] \\ &= \frac{V_m I_m}{2}\cos\varphi \\ &= \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi \\ &= V\, I \cos\varphi \end{aligned} \tag{7.8}$$

where, *V*and *I* are rms values of voltage and current respectively and cos *j* is power factor of the load.

Involvement of the power-factor term cos *j* in the expression for power in ac circuit indicates that ac power cannot be measured simply by connecting a pair of ammeter and voltmeter. A wattmeter, with in-built facility for taking in to account the power factor, can only be used for measurement of power in ac circuits.

Figure 7.8 plots the waveforms of instantaneous power *p*(*t*), voltage *v*(*t*), and current *i*(*t*).

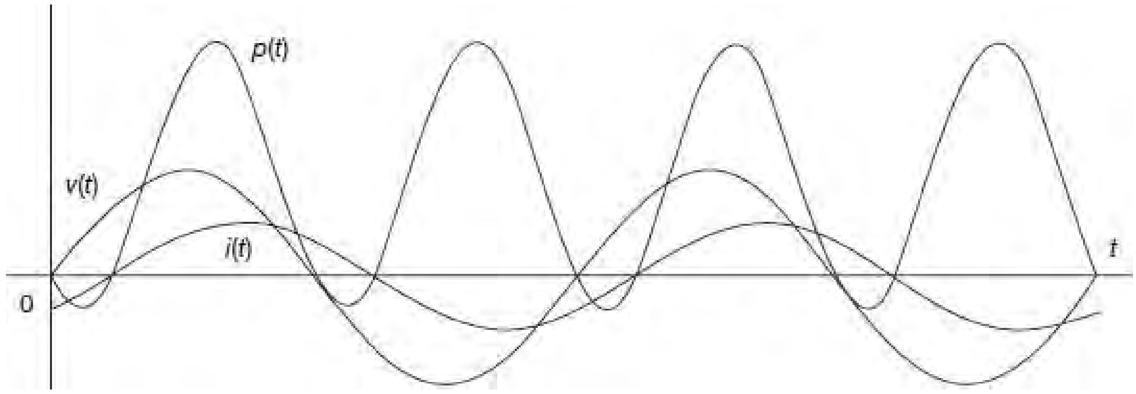


Figure 7.8 *Plot of the waveforms of instantaneous power voltage and current in ac circuit*

Readers may find interesting to note in Figure 7.8 that though voltage and current waveforms have zero average value over a complete cycle, the instantaneous power has offset above zero having non-zero average value.

7.4 ELECTRODYNAMOMETER-TYPE WATTMETER

An electrodynamometer-type wattmeter is similar in design and construction with the analog electrodynamometer-type ammeter and voltmeter described in Chapter 2.

7.4.1 Construction of Electrodynamometer-type Wattmeter

Schematic diagram displaying the basic constructional features of a electrodynamometer-type wattmeter is shown in Figure 7.9.

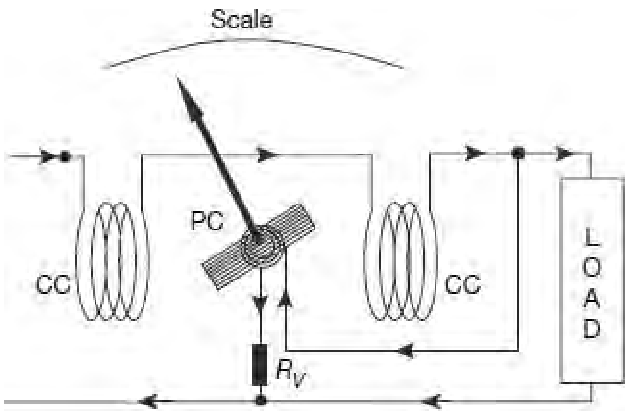


Figure 7.9 *Schematic of electrodynamometer-type wattmeter*

Internal view of such an arrangement is shown in the photograph of Figure 7.10.

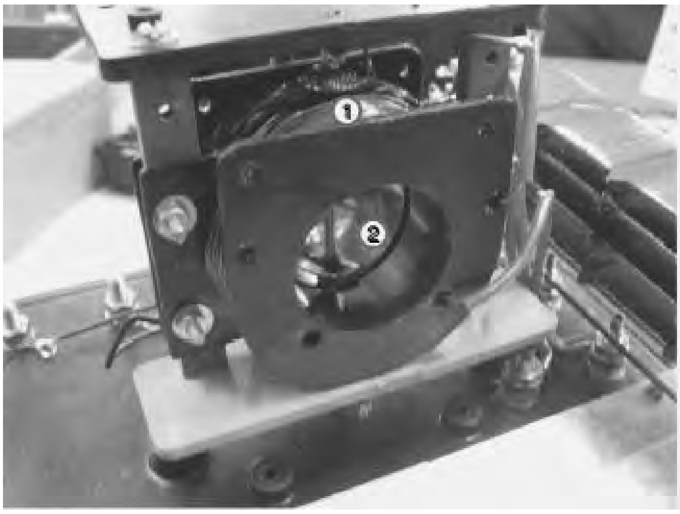


Figure 7.10 *nternal photograph of electrodynamometer-type wattmeter:(1) Fixed (current) coil (2) Moving (potential) coil*

1. Fixed Coil System

Such an instrument has two coils connected in different ways to the same circuit of which power is to be measured. The *fixed coils* or the *field coils* are connected in series with the load so as to carry the same current as the load. The fixed coils are hence, termed as the *Current Coils (CC)* of the wattmeter. The main magnetic field is produced by these fixed coils. This coil is divided in two sections so as to provide more uniform magnetic field near the centre and to allow placement of the instrument moving shaft.

Fixed coils are usually wound with thick wires for carrying the main load current through them. Windings of the fixed coil is normally made of stranded conductors running together but, insulated from each other. All the strands are brought out to an external commutating terminator so that a number of current ranges of the instrument may be obtained by grouping them all in series, all in parallel, or in a series–parallel combination. Such stranding of the fixed coils also reduces Eddy-current loss in the conductors. Still higher current or voltage ranges, however, can be accommodated only through the use of instrument transformers.

Fixed coils are mounted rigidly with the coil supporting structures to prevent any small movement whatsoever and resulting field distortions. Mounting supports are made of ceramic, and not metal, so as not to disturb the magnetic field distribution.

2. Moving Coil System

The **moving coil** that is connected across the load carries a current proportional to the voltage. Since the moving coil carries a current proportional to the voltage, it is called the *voltage coil* or the *pressure coil* or simply *PC* of the wattmeter. The moving coil is entirely embraced by the pair of fixed coils. A high value **non-inductive resistance** is connected in series with the voltage coil to restrict the current through it to a small value, and also to ensure that voltage coil current remains as far as possible in phase with the load voltage.

The moving coil, made of fine wires, is wound either as a self-sustaining air-cored coil, or else wound on a nonmetallic former. A metallic former, otherwise would induce Eddy-currents in them under influence of the alternating field.

3. Movement and Restoring System

The moving, or voltage coil along with the pointer is mounted on an aluminum spindle in case jewel bearings are used to support the spindle. For higher sensitivity requirements, the moving coil may be suspended from a torsion head by a metallic suspension which serves as a lead to the coil. In other constructions, the coil may be suspended by a silk fibre together with a spiral spring which gives the required torsion. The phosphor-bronze springs are also used to lead current into and out of the moving coil. In any case, the torsion head with suspension, or the spring, also serves the purpose of providing the restoring torque to bring the pointer back to its initial position once measurement is over.

> The moving, or voltage coil current must be limited to much low values keeping in mind the design requirements of the movement system. Current is lead to and out of the moving coil through two spiral springs. Current value in the moving coil is thus to be limited to values that can be safely carried by the springs without appreciable heating being caused.

4. Damping System

Damping in such instruments may be provided by small aluminum vanes attached at the bottom of the spindle. These vanes are made to move inside enclosed air chambers, thereby creating the damping torque. In other cases, the moving coil itself can be stitched on a thin sheet of mica, which acts as the damping vane while movements. Eddy-current damping, however, cannot be used with these instruments. This is due to the fact that any metallic element to be used for Eddy-current damping will interfere and distort the otherwise weak operating magnetic field. Moreover, introduction of any external permanent magnet for the purpose of Eddy-current damping will severely hamper the operating magnetic field.

5. Shielding System

The operating field produced by the fixed coils, is comparatively lower in electrodynamometer-type instruments as compared to other type of instruments. In some cases, even the earth's magnetic field can pollute the measurement readings. It is thus essential to shield the electrodynamometer-type instruments from effects of external magnetic fields. Enclosures of such instruments are thus made of alloys with high permeability to restrict penetration of external stray magnetic fields into the instrument.

7.4.2 Operation of Electrodynamometer-type Wattmeter

The schematic operational circuit of an electrodynamometer-type wattmeter being used for measurement of power in a circuit is shown in Figure 7.11.

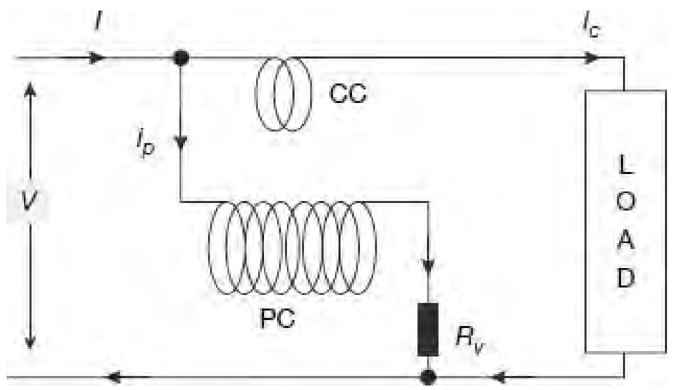


Figure 7.11 *Operational circuit of electrodynamometer-type wattmeter*

V = voltage to be measured (rms)

I = current to be measured (rms)

i_P = voltage (pressure) coil instantaneous current

i_C = current coil instantaneous current

R_V = external resistance connected with pressure coil

R_P = resistance of pressure coil circuit (PC resistance + R_V)

M = mutual inductance between current coil and pressure coil

θ = angle of deflection of the moving system

ω = angular frequency of supply in radians per second

φ = phase-angle lag of current I with respect to voltage V

As described in Chapter 2, the instantaneous torque of the electrodynamometer wattmeter shown in Figure 7.11 is given by

$$T_i = i_P i_C \frac{dM}{d\theta} \tag{7.9}$$

Instantaneous value of voltage across the pressure-coil circuit is

$$v_P = \sqrt{2} \times V \sin \omega t$$

If the pressure coil resistance can be assumed to be very high, the whole pressure coil can be assumed to be behaving like a resistance only. The current i_P in the pressure coil thus, can be assumed to in phase with the voltage v_P, and its instantaneous value is

$$i_P = {}^{v_P}\!/_{R_P} = \sqrt{2} \times {}^{V}\!/_{R_P} \sin \omega t = \sqrt{2} \times I_P \sin \omega t$$

where $I_P = V/R_P$ is the rms value of current in pressure coil.

Assuming that the pressure-coil resistance is sufficiently high to prevent branching out of any portion of the supply current towards the pressure coil, the current coil current can be written as

$$i_C = \sqrt{2} \times I \sin(\omega t - \varphi)$$

Thus, instantaneous torque from (7.9) can be written as

$$T_i = \sqrt{2} \times I_P \sin \omega t \times \sqrt{2} \times I \sin(\omega t - \varphi) \frac{dM}{d\theta}$$

$$= 2 I_p I \sin \omega t \sin(\omega t - \varphi) \frac{dM}{d\theta}$$

$$= I_P I \{\cos \varphi - \cos(2\omega t - \varphi)\} \frac{dM}{d\theta} \qquad (7.10)$$

Presence of the term containing $2w\ t$, indicates the instantaneous torque as shown in (7.10) varies at twice the frequency of voltage and current.

Average deflecting torque over a complete cycle is

$$T_d = \frac{1}{T}\int_0^T T_i\, d\omega t = \frac{1}{2\pi}\int_0^{2\pi} I_P I \{\cos \varphi - \cos(2\omega t - \varphi)\} \frac{dM}{d\theta}\, d\omega t$$

$$= \frac{I_P I}{2\pi} \left[\omega t \cos \varphi\right]_0^{2\pi} \frac{dM}{d\theta}$$

$$= I_P I \cos \varphi \frac{dM}{d\theta} \qquad (7.11)$$

$$= \frac{V}{R_P} I \cos \varphi \frac{dM}{d\theta}$$

$$= \frac{VI \cos \varphi}{R_P} \frac{dM}{d\theta} \qquad (7.12)$$

With a spring constant K, the controlling torque provided by the spring for a final steady-state deflection of θ is given by

$$T_C = K\theta$$

Under steady-state condition, the average deflecting torque will be balanced by the controlling torque provided by the spring. Thus, at balanced condition $T_C = T_d$

$$T_C = T_d$$

$$K\theta = \frac{VI \cos \varphi}{R_P} \frac{dM}{d\theta}$$

$$\theta = \frac{VI \cos \varphi}{KR_P} \frac{dM}{d\theta}$$

$$\theta = \left(K_1 \frac{dM}{d\theta}\right) P \qquad (7.13)$$

where, P is the power to be measured and $K_1 = 1/KR_P$ is a constant.

Steady-state deflection θ is thus found to be an indication of the power P to be measured.

7.4.3 Shape of scale in Electrodynamometer-type Wattmeter

Steady-state deflection θ can be made proportional to the power P to be measured, i.e., the deflection will vary linearly with variation in power if the rate of change of mutual inductance is constant over the range of deflection. In other words, the scale of measurement will be uniform if the mutual inductance between the fixed and moving coils varies linearly with angle of deflection. Such a variation in mutual inductance can be achieved by careful design of the instrument. Figure 7.12 shows the expected nature of variation of mutual inductance between fixed and moving coils with respect to angle of deflection.

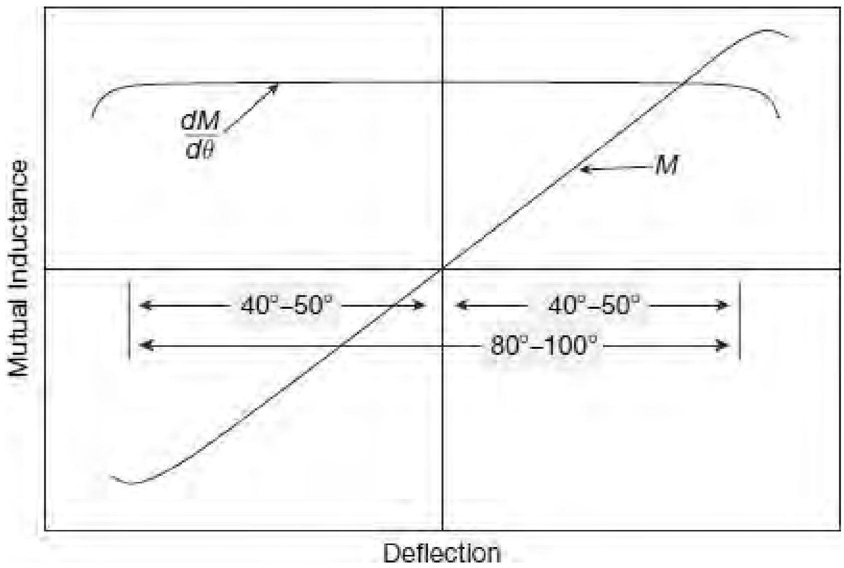


Figure 7.12 *Variation of mutual inductance with deflection*

By a suitable design, the mutual inductance between fixed and moving coils can be made to vary linearly with deflection angle over a range of 40° to 50° on either side of zero mutual inductance position, as shown in Figure 7.12. If the position of zero mutual inductance can be kept at the mid-scale, then the scale can be graduated to be uniform over 80° to 100°, which covers almost entire range of the scale.

7.4.4 Errors in Electrodynamometer-type Wattmeter

1. Error due to Pressure-Coil Inductance

It was assumed during the discussions so far that the pressure coil circuit is purely resistive. In reality, however, the pressure coil will have certain inductance along with resistance. This will introduce errors in measurement unless necessary compensations are taken care of. To have an estimate of such error, let us consider the following:

V = voltage applied to the pressure coil circuit (rms)

I = current in the current coil circuit (rms)

I_P = current in the voltage (pressure) coil circuit (rms)

r_P = resistance of pressure coil only

L = inductance of pressure coil

R_V = external resistance connected with pressure coil

R_P = resistance of pressure coil circuit (PC resistance + R_V)

Z_P = impedance of pressure coil circuit

M = mutual inductance between current coil and pressure coil

ω = angular frequency of supply in radian per second

φ = phase-angle lag of current I with respect to voltage V

Due to inherent inductance of the pressure coil circuit, the current and voltage in the pressure coil will no longer be in phase, rather the current through the pressure coil will lag the voltage across it by a certain angle given by

$$\alpha = \tan^{-1}\left(\frac{\omega L}{R_P}\right) = \tan^{-1}\left(\frac{\omega L}{r_P + R_V}\right)$$

As can be seen from Figure 7.13, current through the pressure coil lags voltage across it by a phase-angle which is less than that between the current coil current and the pressure coil voltage.

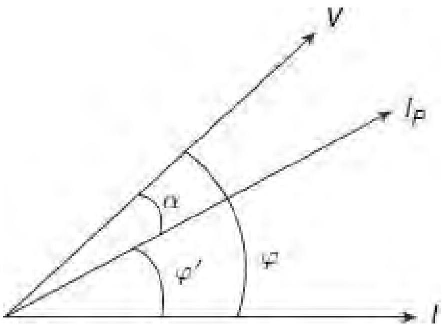


Figure 7.13 *Wattmeter phasor diagram with pressure coil inductance*

In such a case, phase-angle difference between the *with pressure coil inductance*pressure coil current and current coil current is

$$\varphi' = \varphi - \alpha$$

Following from (7.11), the wattmeter deflection will be

$$\theta' = \frac{I_P I}{K} \cos\varphi' \cdot \frac{dM}{d\theta}$$

$$\theta' = \frac{V}{Z_P K} I \cos(\varphi - \alpha) \cdot \frac{dM}{d\theta}$$

Relating to $R_P = Z_P \cos\alpha$ in the pressure coil circuit, the wattmeter deflection can be re-written as *VI dM*

$$\theta' = \frac{VI}{R_P K} \cos\alpha \cdot \cos(\varphi - \alpha) \cdot \frac{dM}{d\theta} \tag{7.14}$$

In the absence of inductance, $Z_P = R_P$ and $\alpha = 0$; wattmeter in that case will read true power, given by,

$$\theta = \frac{VI}{R_P K} \cos\varphi \frac{dM}{d\theta} \tag{7.15}$$

Taking the ratio of true power indication to actual wattmeter reading, we get

$$\frac{\text{True power indication}}{\text{Actual wattmeter reading}} = \frac{\theta}{\theta'} = \frac{\frac{VI}{R_p K}\cos\varphi \frac{dM}{d\theta}}{\frac{VI}{R_p K}\cos\alpha\cdot\cos(\varphi-\alpha)\cdot\frac{dM}{d\theta}} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)}$$

Thus, the correction factor can be identified as

$$CF = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)}$$

True power indication can thus be obtained from the actual wattmeter reading using the correction factor CF as

True power indication = CF X Actual wattmeter reading

Thus, for lagging power factor loads,

$$\text{True power indication} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi-\alpha)} \times \text{ Actual wattmeter readings}$$

The above relations, along with Figure 7.13 indicate that under lagging power factor loads, unless special precautions are taken, actual wattmeter reading will tend to display higher values as compared to true power consumed.

For leading power factor loads, however, the wattmeter phasor diagram will be as shown in Figure 7.14.

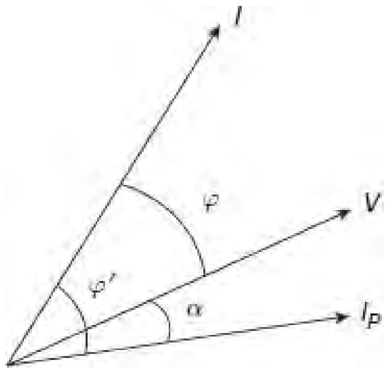


Figure 7.14 *Wattmeter phasor diagram with pressure coil inductance during leading load*

For leading power factor loads,

$$\text{True power indication} = \frac{\cos\varphi}{\cos\alpha\cdot\cos(\varphi+\alpha)} \times \text{ Actual wattmeter readings}$$

The above relations, along with Figure 7.13 indicate that under leading power factor loads, unless special precautions are taken, actual wattmeter reading will tend to display higher values as compared to true power consumed.

2. Compensation for Pressure Coil Inductance

A wattmeter can be compensated for pressure coil inductance by connecting a preset value of capacitance across a certain portion of the external resistance connected in series with the pressure coil, as shown in Figure 7.15.

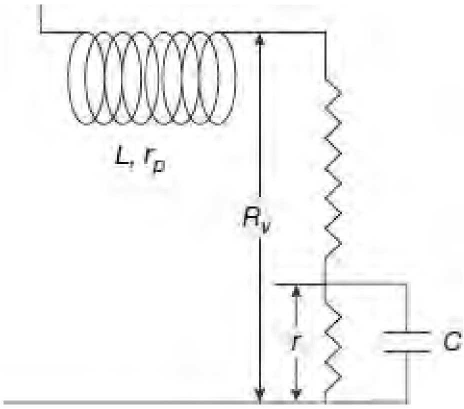


Figure 7.15 *Compensation for pressure coil inductance*

The total impedance of the circuit in such a case can be written as

$$Z_P = (r_p + R_V - r) + j\omega L + \frac{r - j\omega C r^2}{1 + \omega^2 C^2 r^2}$$

To make the entire circuit behave as purely resistive, if we can design the circuit parameters in such a case that for power frequencies

$$w^2 C^2 r^2 << 1$$

Then we can re-write the total impedance of the pressure coil as

$$Z_P = (r_p + R_V - r) + jwL + r - jwCr^2 = r_P + R_V + jw(L - Cr^2)$$

If by proper design, we can make $L = Cr^2$

Then, impedance = $r_p + R_V = R_P$

Thus error introduced due pressure coil inductance can be substantially eliminated.

3. Error due to Pressure Coil Capacitance

The voltage, or pressure coil circuit may have inherent capacitance in addition to inductance. This capacitance effect is mainly due to inter-turn capacitance of the winding and external series resistance. The effect of stray capacitance of the pressure coil is opposite to that due to inductance. Therefore, the wattmeter reads low on lagging power factors and high on leading power factors of the load. Actual reading of the wattmeter, thus, once again needs to be corrected by the corresponding correction factors to obtain the true reading. The effect of capacitance (as well as inductance) varies with variable frequency of the supply.

4. Error due to Connection

There are two alternate methods of connection of wattmeter to the circuit for measurement of power. These are shown in Figure 7.16. In either of these connection modes, errors are introduced in measurement due power losses in pressure coil and current coil.

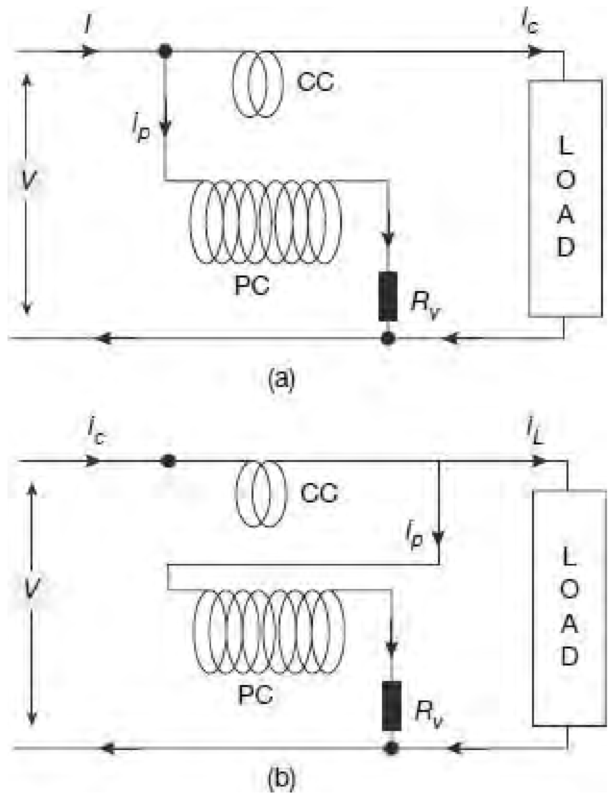


Figure 7.16 *Wattmeter connections*

In the connection of Figure 7.16(a), the pressure coil is connected across the supply, thus pressure coil measures the voltage across the load, plus the voltage drop across the current coil. Wattmeter reading in this case will thus include power loss in current coil as well, along with power consumed by the load.

Wattmeter reading = Power consumed by load + Power loss in CC

In the connection of Figure 7.16(b), the current coil is connected to the supply side; therefore, it carries load current plus the pressure coil current. Hence, wattmeter reading in this case includes, along with power consumed by the load, power loss in the pressure coil as well.

Wattmeter reading = Power consumed by load + Power loss in PC

In the case when load current is small, power loss in the current coil is small and hence the connection of Figure 7.16(a) will introduce comparatively less error in measurement.

On the other hand, when load current is large, current branching through the pressure coil is relatively small and error in measurement will be less if connection of Figure 7.16(b) is used.

Errors due to branching out of current through the pressure coil can be minimised by the use of compensating coil as schematically shown in Figure 7.17.

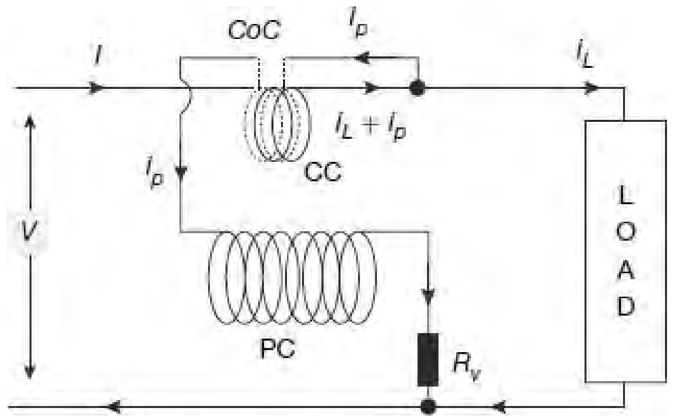


Figure 7.17 *Schematic connection diagram of compensated wattmeter*

In the compensated connection, the current coil consists of two windings, each winding having the same number of turns. The two windings are made as far as possible identical and coincident. One of the two windings (CC) is made of heavy wire that carries the load current plus the current for the pressure coil. The other winding (compensating coil–CoC) which is connected in series with the pressure coil, uses thin wire and carries only the current to the pressure coil. This current in the compensating coil is, however, in a direction opposite to the current in main current coil, creating a flux that opposes the main flux. The resultant magnetic field is thus due to the current coil only, effects of pressure coil current on the current coil flux mutually nullifying each other. Thus, error due to pressure coil current flowing in the current coil in cancelled out and the wattmeter indicates correct power.

5. Eddy-current Errors

Unless adequate precautions are adopted, Eddy-currents may be induced in metallic parts of the instrument and even within the thickness of the conductors by alternating magnetic field of the current coil. These Eddy-currents produce spurious magnetic fields of their own and distort the magnitude and phase of the main current coil magnetic field, thereby introducing error in measurement of power.

Error caused by Eddy-currents is not easy to estimate, and may become objectionable if metal parts are not carefully avoided from near the current coil. In fact, solid metal in coil supports and structural part should be kept to a minimum as far as practicable. Any metal that is used is kept away and is selected to have high resistivity so as to reduce Eddy-currents induced in it. Stranded conductors are recommended for the current coil to restrict generation of Eddy-current within the thickness of the conductor.

6. Stray Magnetic Field Errors

The operating field in electrodynamometer-type instruments being weak, special care must be taken to protect these instruments from external magnetic fields. Hence, these instruments should be shielded against effects of stray magnetic fields. Laminated iron shields are used in portable laboratory instruments, while steel casings are provided as shields in switchboard mounted wattmeter. Precision wattmeters, however, are not provided with metals shields, for that will introduce errors due to Eddy-current, and also some dc error due to permanent magnetization of the metal shield under influence of external magnetic field. Such wattmeters are manufactured to have *astatic* system as shown in Figure

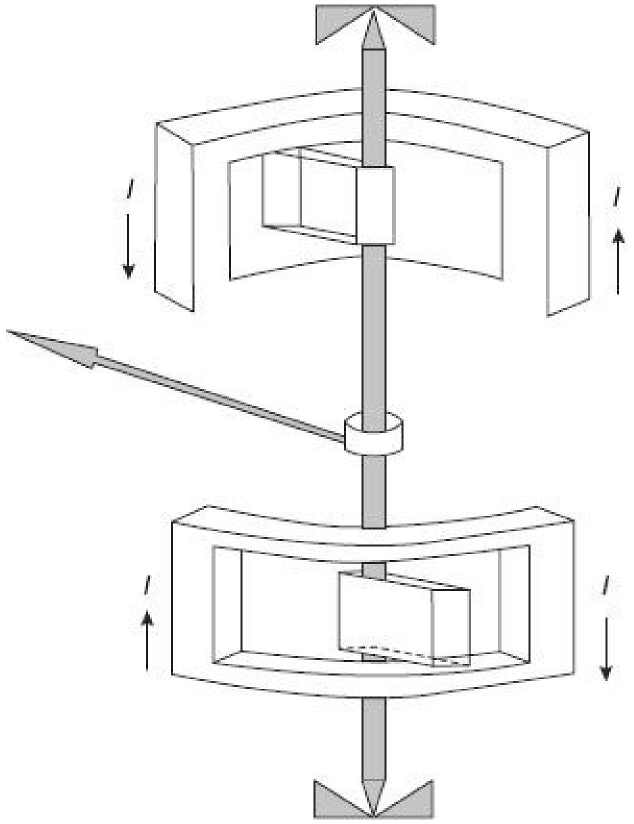


Figure 7.18 *Astatic systems for electrodynamometer wattmeter*

Astatic electrodynamometer instruments are constructed with two similar sets of fixed and moving coils mounted on the same shaft. The pair of fixed coils is so connected that their magnetic fields are in opposition. Similarly, the pair of moving coils is also connected to produce magnetic fields in opposite directions. This makes the deflecting torque acting on the two moving coils to be in the same direction. Deflection of the pointer is thus due to additive action of the two moving coils. However, since the two fields in the two pairs of fixed and moving coils are in opposition, any external uniform field will affect the two sets of pairs differently. The external field will reduce the field in one coil and will enhance the field in the other coil by identical amount. Therefore, the deflecting torque produced by one coil is increased and that by the other coil is reduced by an] equal amount. This makes the net torque on account of the external magnetic field to zero.

7. Error Caused by Vibration of the Moving System

The instantaneous torque on the moving system varies cyclically at twice the frequency of the voltage and current (7.10). If any part of the moving system, such as the spring or the pointer has natural frequency close to that of torque pulsation, then accidental resonance may take place. In such a case, the moving system may vibrate with considerable amplitude. These vibrations may pose problems while noting the pointer position on the scale. These errors due to vibrations may be avoided by designing the moving elements to have natural frequencies much further away from twice the frequency of the supply voltage.

8. Temperature Errors

Temperature changes may affect accuracy of wattmeter by altering the coil resistances. Temperature my change due to change in room temperature or even due to heating effects in conductors with flow of current. Change in temperature also affects the spring stiffness,

9 Cathode Ray Oscilloscope

9.1 INTRODUCTION

The Cathode Ray Oscilloscope (CRO) is a very useful and versatile laboratory instrument used for display, measurement and analysis of waveform and other phenomena in electrical and electronic circuits. CROs are, in fact, very fast *X*-*Y* plotters, displaying an input signal versus another signal or versus time. The 'stylus' of this 'plotter' is a luminous spot which moves over the display area in response to an input voltage. The luminous spot is produced by a beam of electrons striking a fluorescent screen. The extremely low inertia effects associated with a beam of electrons enables such a beam to be used following the changes in instantaneous values of rapidly varying voltages.

The normal form of a CRO uses a horizontal input voltage which is an internally generated ramp voltage called 'time base'. The horizontal voltage moves the luminous spot periodically in a horizontal direction from left to right over the display area or screen. The vertical input to the CRO is the voltage under investigation. The vertical input voltage moves the luminous spot up and down in accordance with the instantaneous value of the voltage. The luminous spot thus traces the waveform of the input voltage with respect to time. When the input voltage repeats itself at a fast rate, the display on the screen appears stationary on the screen. The CRO thus provides a means of visualising time-varying voltages. As such, the CRO has become a universal tool in all kinds of electrical and electronic investigation.

9.2 BLOCK DIAGRAM OF A CATHODE RAY TUBE (CRT)

The main part of the CRO is Cathode Ray Tube (CRT). It generates the electron beam, accelerates the beam to a high velocity, deflects the beam to create the image and contains a phosphor screen where the electron beam eventually becomes visible. The phosphor screen is coated with 'aquadag' to collect the secondary emitted electrons. For accomplishing these tasks, various electrical signals and voltages are required, which are provided by the power supply circuit of the oscilloscope. Low voltage supply is required for the heater of the electron gun for generation of electron beam and high voltage, of the order of few thousand volts, is required for cathode ray tube to accelerate the beam. Normal voltage supply, say a few hundred volts, is required for other control circuits of the oscilloscope.

Horizontal and vertical deflecting plates are fitted between the electron gun and screen to deflect the beam according to the input signal. The electron beam strikes the screen and creates a visible spot. This spot is deflected on the screen in the horizontal direction (*X*-

axis) with constant time dependent rate. This is accomplished by a time base circuit provided in the oscilloscope. The signal to be viewed is supplied to the vertical deflection plates through the vertical amplifier, which raises the potential of the input signal to a level that will provide usable deflection of the electron beam. Now electron beam deflects in two directions, horizontal on *X*-axis and vertical on *Y*-axis. A triggering circuit is provided for synchronising two types of deflections so that horizontal deflection starts at the same point of the input vertical signal each time it sweeps. A basic block diagram of a general-purpose oscilloscope is shown in Figure 9.1(a) and a schematic of internal parts of a CRT is shown in Figure 9.1(b).

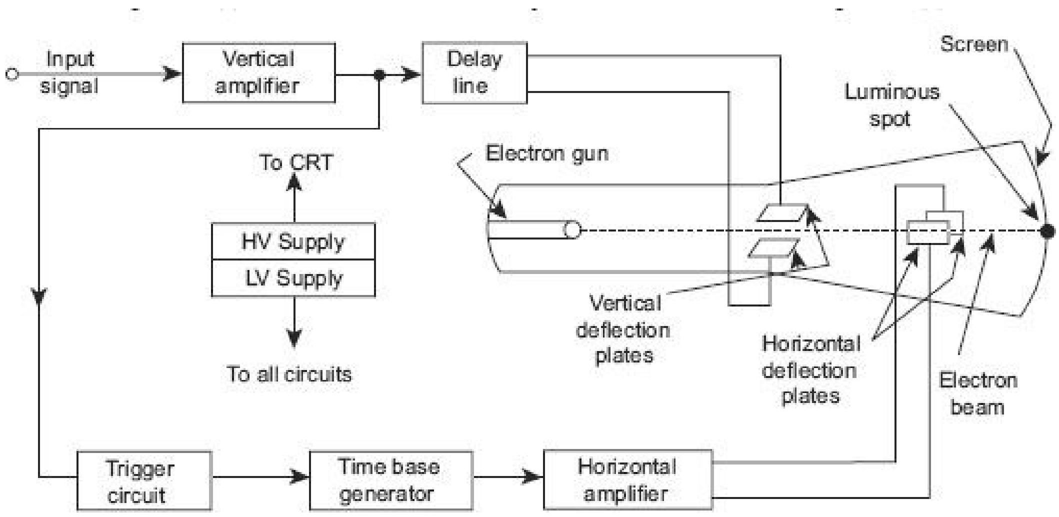


Figure 9.1 (a) *Block diagram of a general-purpose CRO*

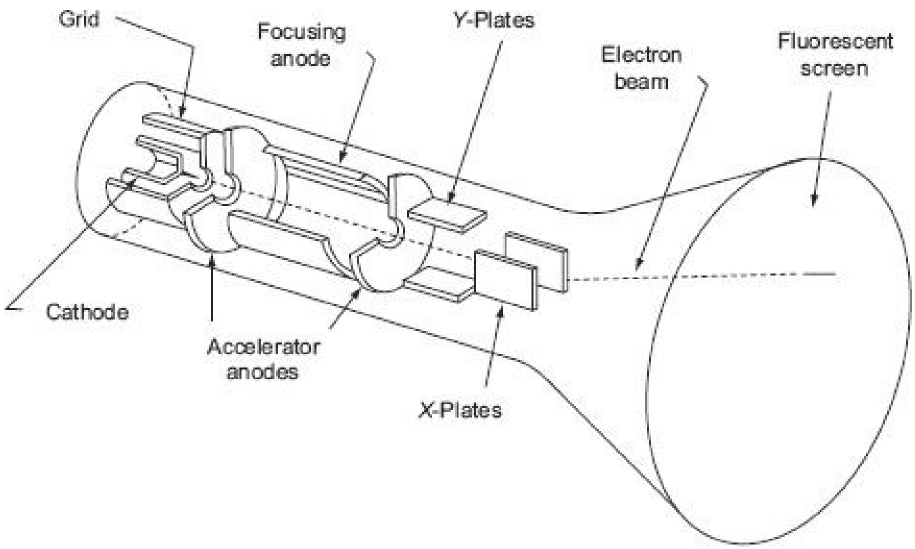


Figure 9.1 (b) *Cathode Ray Tube(CRT)*

9.3 ELECTROSTATIC DEFLECTION

Figure 9.2 shows a general arrangement for electrostatic deflection. There are two parallel plates with a potential applied between. These plates produce a uniform electrostatic filed in the *Y* direction. Thus any electron entering the field will experience a force in the *Y* direction and will be accelerate in that direction. There is no force either in *X* direction or *Z* direction and hence there will be no acceleration of electrons in these directions.

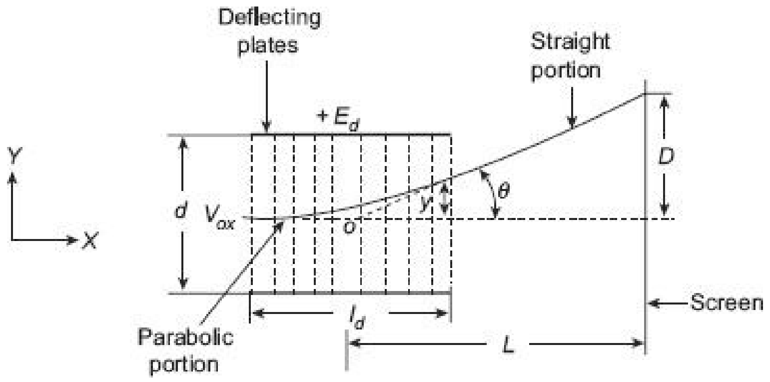


Figure 9.2 *Electrostatic deflection*

Let, E_a = voltage of pre-accelerating anode; (volt)

e = charge of an electron; (Coulomb)
m = mass of electron; (kg)
θ = deflection angle of the electron beam
v_{ox} = velocity of electron when entering the field of deflecting plates; (m/s)
E_d = potential difference between deflecting plates; (volt)
d = distance between deflecting plates; (m)
l_d = length of deflecting plates; (m)
L = distance between screen and the centre of the deflecting plates; (m)
y = displacement of the electron beam from the horizontal axis at time t

and D = deflection of the electron beam on the screen in Y direction; (m)

The loss of potential energy *(PE*) when the electron moves from cathode to accelerating anode;

$$PE = eE_a \tag{9.1}$$

The gain in kinetic energy *(KE*) by an electron

$$KE = \frac{1}{2} m v_{ox}^2 \tag{9.2}$$

Equating the two energies, we have $\boxed{v_{ox} = \sqrt{2eE_a/m}}$ (9.3)

This is the velocity of the electron in the *X* direction when it enters the deflecting plates. The velocity in the *X* direction remains same throughout the passage of electrons through the deflecting plates as there is no force acting in the direction.

The electric field intensity in the *Y* direction $\varepsilon_y = \frac{E_d}{d}$ (9.4)

Force acting on an electron in *Y* direction $= F_y = e\varepsilon_y = e\frac{E_d}{d}$ (9.5)

Suppose a_y is the acceleration of the electron in the *Y* direction, therefore,

$$F_y = ma_y \tag{9.6}$$

or, $$a_y = e\varepsilon_y / m \tag{9.7}$$

As there is no initial velocity in the Y direction [Eq. (9.8)], the displacement y at any instant t in the Y direction is

$$y = \frac{1}{2} a_y t^2 = \frac{1}{2} \frac{e\varepsilon_y}{m} t^2 \tag{9.8}$$

As the velocity in the X direction is constant, the displacement in X direction is given by

$$x = V_{ox} t \tag{9.9}$$

$$\therefore \qquad t = x / v_{ox} \tag{9.10}$$

Substituting the above value of t in Eq. (9.8), we have

$$y = \frac{1}{2} \frac{e\varepsilon_y}{mv_{ox}^2} x^2 \tag{9.11}$$

This is the equation of a parabola.

The slope at any point (x, y) is $\frac{dy}{dx} = \frac{e\varepsilon_y}{mv_{ox}^2} x$ (9.12)

Putting $x = l_d$ in Eq. (9.12), we get the value of tan θ.

$$\text{or} \qquad \tan\theta = \frac{e\varepsilon_y}{mv_{ox}^2} l_d = \frac{eE_d l_d}{mdv_{ox}^2} \tag{9.13}$$

After leaving the deflection plates, the electrons travel in a straight line. The straight line of travel of electron is tangent to the parabola at $x = l_d$ and this tangent intersects the X axis at point O'. The location of this point is given by

$$x = \frac{y}{\tan\theta} = \frac{e\varepsilon_y l_d^2}{2mv_{ox}^2} \Big/ \frac{e\varepsilon_y}{mv_{ox}^2} l_d = \frac{l_d}{2} \tag{9.14}$$

The apparent origin is thus the centre of the deflecting plates, the deflection D on the screen is given by

$$D = L\tan\theta = \frac{LeE_d l_d}{mdv_{ox}^2} \tag{9.15}$$

Substituting the value $v_{ox}^2 = \frac{2eE_a}{m}$ in Eq. (9.15), we get,

$$\boxed{D = \frac{LeE_d l_d}{md} \cdot \frac{m}{2eE_a} = \frac{LE_d l_d}{2dE_a}} \tag{9.16}$$

From Eq. (9.16) we conclude the following:

For a given accelerating voltage E_a, and for particular dimensions of CRT, the deflection of the electron beam is directly proportional to the deflecting voltage. This means that the CRT may be used as a linear indicating device.

The discussions above assume that E_d is a fixed dc voltage. The deflection voltage is usually a time varying quantity and the image on the screen thus follows the variation of the deflections voltage in a linear manner.

The deflection is independent of the (e/m) ratio. In a cathode ray tube, in addition to the electrons many types of negative ions such as oxygen, carbon, chlorine etc are present. With electrostatic deflection system, because deflection is independent of e/m, the ions travel with the electrons and are not concentrated at one point. Hence cathode ray tube with electrostatic deflection system does not produce an ion burn.

The *deflection sensitivity* of a CRT is defined as the deflection of the screen per unit deflection voltage.

Therefore, deflection sensitivity $\boxed{S = \frac{D}{E_d} = \frac{Ll_d}{2dE_a}}$ m/V (9.17)

The *deflection factor* of a CRT is defined as the reciprocal of sensitivity

Therefore, deflection factor $\boxed{G = \frac{1}{S} = \frac{2dE_a}{Ll_d}}$ V/m (9.18)

It is clear from Eq. (9.17), that the sensitivity can be increased by decreasing the value of accelerating voltage E_a. but this has a disadvantage as the luminosity of the spot is decreased with decrease in E_a. On the other hand a high value of E_a, produced a highly accelerated beam and thus produces a bright spot. However, a high accelerating voltage (E_a) requires a high deflection potential (E_d) for a given deflection. Also, highly accelerated beam is more difficult to deflect and is sometimes called *hard beam.*

Example 9.1

An electrically deflected CRT has a final anode voltage of 2000 V and parallel deflecting plates 1.5 cm long and 5 mm apart. If the screen is 50 cm from the centre of deflecting plates, find (a) beam speed, (b) the deflection sensitivity of the tube, and (c) the deflection factor of the tube.

Solution Velocity of the beam

$$v_{ox} = \sqrt{2eE_a/m} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 2000}{9.1 \times 10^{-31}}} = 26.5 \times 10^6 \text{ m/s}$$

Deflection sensitivity,

$$S = \frac{Ll_d}{2dE_a} = \frac{0.5 \times 1.5 \times 10^{-2}}{2 \times 5 \times 10^{-3} \times 2000} = 0.375 \text{ mm/V}$$

Deflection factor, $G = \frac{1}{S} = \frac{1}{0.375} = 2.66$ V/mm

Example 9.2

Calculate the maximum velocity of the beam of electrons in a CRT having a anode voltage of 800 V. Assume that the electrons to leave the anode with zero velocity. Charge of electron = 1.6×10^{-19} C and mass of electron = 9.1×10^{-31} kg.

Solution Velocity of electron is

$$v_{ox} = \sqrt{2eE_a/m} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 800}{9.1 \times 10^{-31}}} = 16.8 \times 10^6 \text{ m/s}$$

Example 9.3 *A CRT has an anode voltage of 2000 V and 2 cm long and 5 mm apart parallel deflecting plates. The screen is 30 cm from the centre of the plates. Find the input voltage required to deflect the beam through 3 cm. The input voltage is applied to the deflecting plates through amplifiers having an overall gain of 100.*

Solution

$$\text{Deflection } D = \frac{LeE_d l_d}{md} \cdot \frac{m}{2eE_a} = \frac{LE_d l_d}{2dE_a}$$

$$\therefore \text{ voltage applied to the deflecting plates } E_d = \frac{2dE_a D}{Ll_d}$$

$$= \frac{2 \times 5 \times 10^{-3} \times 2000 \times 3 \times 10^{-2}}{0.3 \times 2 \times 10^{-2}} = 100\text{ V}$$

$$\therefore \text{ input voltage required for a deflection of 3 cm} = \frac{E_d}{\text{gain}} = \frac{100}{100} = 1\text{ V}.$$

9.4 TIME BASE GENERATOR

Generally, oscilloscopes are used to display a waveform that varies as a function of time. For the waveform to be accurately reproduced, the beam must have a constant horizontal velocity. Since the beam velocity is a function of the deflecting voltage, the deflecting voltage must increase linearly with time. A voltage with this characteristic is called a *ramp voltage*. If the voltage decreases rapidly to zero with the waveform repeatedly reproduced, as shown in Figure 9.3, the pattern is generally called a sawtooth waveform.

During the sweep time, T_s, the beam moves from left to right across the CRT screen. The beam is deflected to the right by the increasing amplitude of the ramp voltage and the fact that the positive voltage attracts the negative electrons. During the retrace time or flyback time, T_r, the beam returns quickly to the left side of the screen. This action would cause a retrace line to be printed on the CRT screen. To overcome this problem the control grid is generally 'gated off', which blanks out the beam during retrace time and prevents an undesirable retrace pattern from appearing on the screen.

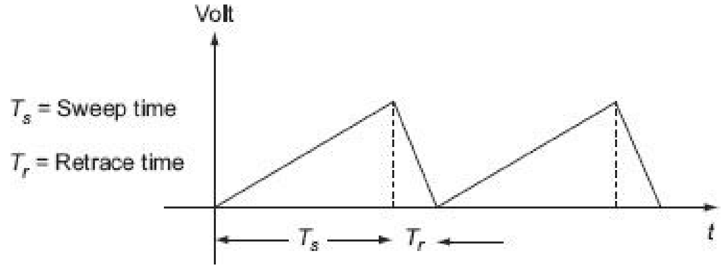


Figure 9.3 *Typical sawtooth waveform applied to the horizontal deflection plates*

In low-cost oscilloscopes the time base is said to be free running, although the time base oscillator may, in fact, be synchronised to the vertical amplifier signal. Unless the time base is so synchronous, the waveform marches across the screen and remains unstable.

Synchronisation means that the time base signal sweeps across the screen in a time that is equal to an integer number of vertical waveform periods. The vertical waveform will then appear locked on the CRT screen.

The vertical sector consists of a wideband preamplifier and power amplifier combination that drivers the CRT vertical deflection plates. The vertical amplifier has a high gain, so large signals must be passed through an attenuator or, in low cost oscilloscopes, a vertical gain controller.

9.5 VERTICAL INPUT AND SWEEP GENERATOR SIGNAL SYNCHRONISATION

Several waveforms that are needed to be observed with the help of CRO will be changing at a rate much faster than the human eye can sense, perhaps many million times per second. To observe such rapid changes, the beam must retrace the same pattern repeatedly. If the pattern is retraced in such a manner that the pattern always occupies the same location on the screen, it will appear as stationary. The beam will retrace the same pattern at a rapid rate. If the vertical input signal and the sweep generator signal are synchronised, which means that the frequency of vertical input signal must be equal to or an exact multiple of the sweep generator signal frequency, as shown in Figure 9.4. If the vertical input frequency is not exactly equal to or an exact multiple of the sawtooth frequency, the waveform will not be synchronised and the display moves across the screen. If the pattern moves towards the right, the frequency of the sawtooth waveform is too high. Movement of the pattern towards the left indicates that the frequency of the sawtooth is too low.

The vertical input signal and the sawtooth generator signal can be synchronized in two different ways:

1. Free running sweep
2. Triggered sweep

9.5.1 Free Running Sweep

In low-cost oscilloscopes, the time base is said to be free running. In these oscilloscopes, the sweep generator is continuously charging and discharging a capacitor. One ramp voltage is followed immediately by another; hence, the sawtooth pattern appears. A sweep generator working in this manner is said to be ‘free running’. In order to present a stationary display on the CRT screen, the sweep generator signal must be forced to run in synchronisation with the vertical input signal. In basic or low-cost oscilloscopes this is accomplished by carefully adjusting the sweep frequency to a value very close to the exact frequency of the vertical input signal or a submultiple of this frequency. When both signals are at same frequency, an internal synchronising pulse will lock the sweep generator into the vertical input signal. This method of synchronisation has some serious limitations when an attempt is made to observe low amplitude signals, because it is very difficult to observe that a very low amplitude signal is stationary or movable in the CRT screen. However, the most serious limitation is probably the inability of the instrument to maintain synchronisation when the amplitude or frequency of the vertical signal is not constant, such as variable frequency audio signal or voice.

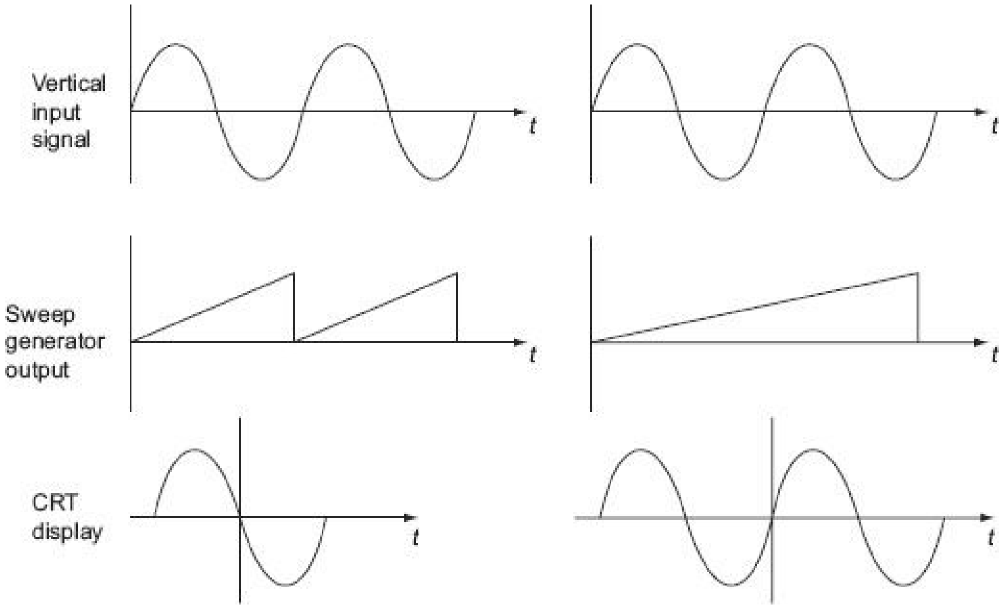


Figure 9.4 *Synchronised waveforms and CRT display*

9.5.2 Triggered Sweep

In free running sweep oscillators, it is not possible to observe the signals of variable frequency. The limitation is overcome by incorporating a trigger circuit into the oscilloscope as shown in Figure 9.5. The trigger circuit may receive an input from one of three sources depending on the setting of the trigger selecting switch. The input signal may come from an external source when the trigger selector switch is set to EXT, from a low amplitude ac voltage at line frequency when the switch is set to line, or from the vertical amplifier when the switch is set to INT. When set for Internal Triggering (INT), the trigger circuit receives its input from the vertical amplifier. When the vertical input signal that is being amplified by the vertical amplifier matches a certain level, the trigger circuit provides a pulse to the sweep generator, thereby ensuring that the sweep generator output is synchronised with the signal that triggers it.

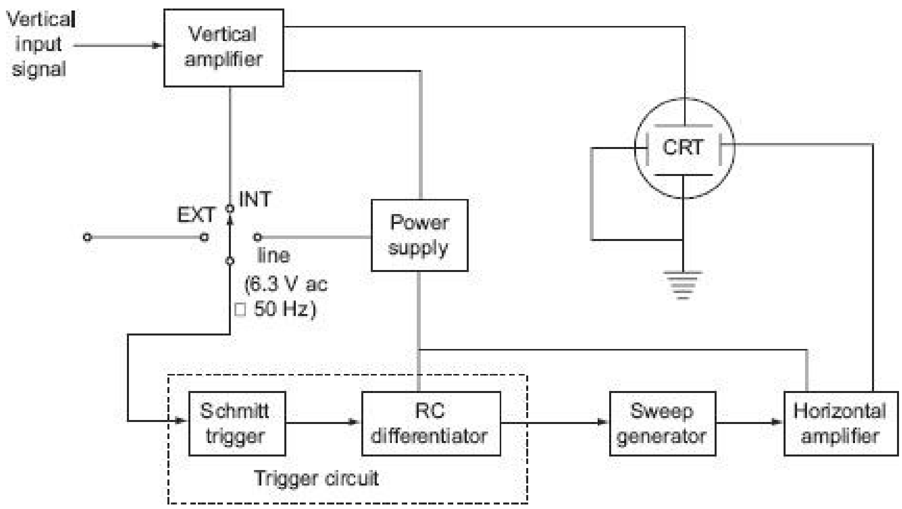


Figure 9.5 *Block diagram of an oscilloscope with triggered sweep.*

Schmitt trigger or a voltage level detector circuit is frequently used in the ‘trigger circuit’ block of Figure 9.5. Basically, the Schmitt trigger compares an input voltage, in this case from the vertical amplifier, with a pre-set voltage.

9.6 MEASUREMENT OF ELECTRICAL QUANTITIES WITH CRO

The CRO is a very versatile instrument in laboratory for measurement of voltage, current, frequency and phase angle of any electrical quantity. But before we go ahead with the discussion on measurement of electrical quantities with a CRO, we should understand some basic oscilloscope patterns.

Basic Oscilloscope Patterns

Assume that a sinusoidal voltage is applied to the horizontal deflecting plates without any voltage signal to the vertical deflecting plates, as shown in Figure 9.6. One horizontal line will appear on the screen of the CRO. This line would be in the central position on the screen vertically.

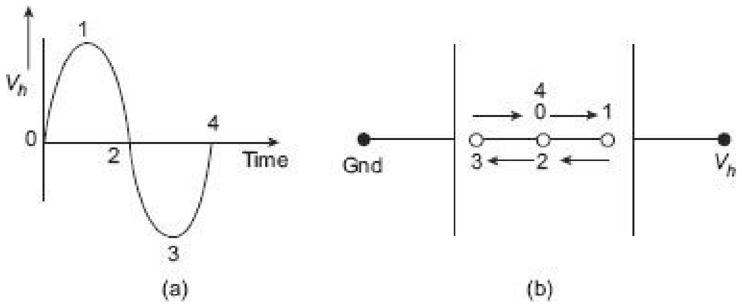


Figure 9.6 *Deflection for a sinusoidal voltage applied to the horizontal deflection plates*

If a sinusoidal voltage signal is applied to the vertical deflecting plates without applying any voltage signal to the horizontal deflecting plates then we get a vertical line on the screen of CRO, as shown in Figure 9.7. This line would be in the central position on the screen horizontally.

Now we would discuss what happens when both vertical and horizontal deflection plates are supplied with sinusoidal voltage signals simultaneously. Let us consider when two sinusoidal signals equal in magnitude and frequency and in phase with each other are applied to both of the horizontal and vertical deflection plates, as shown in Figure 9.8. Here we get a straight line inclined at 45° to the positive X-axis.

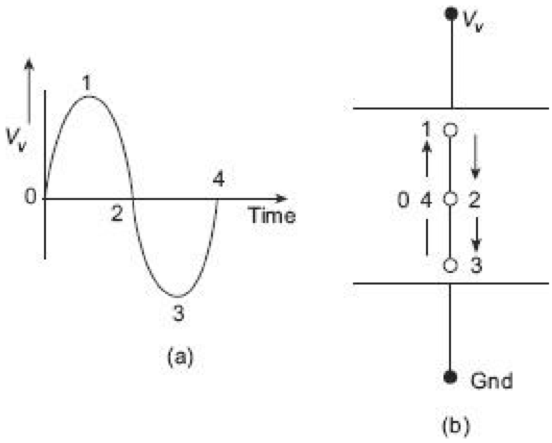


Figure 9.7 *Deflection for a sinusoidal voltage applied to the vertical deflecting plates*

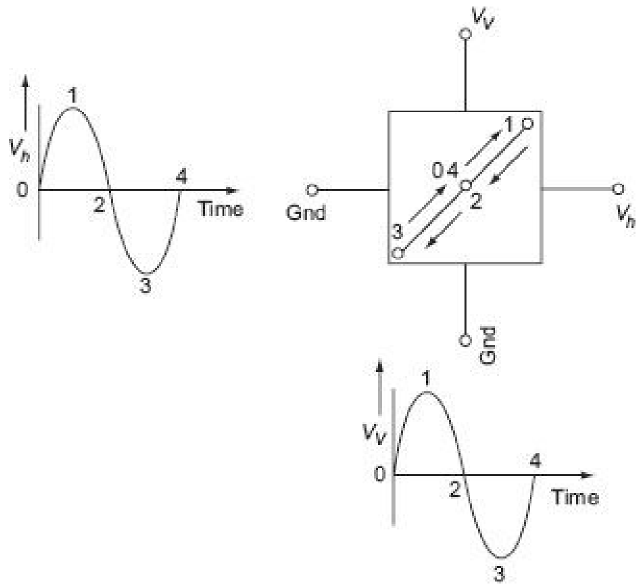


Figure 9.8 *Deflection for sinusoidal voltage signals in phase and equal in magnitude and frequency, applied to horizontal and vertical deflection plates*

Now let us consider a case when two sinusoidal voltage signals applied to the horizontal and vertical deflection plates are of equal magnitude and equal frequency but opposite in phase, as shown in Figure 9.9. We get a straight line inclined at 135° to the positive *X*-axis.

In the last case, if the two sinusoidal voltage signals, 90° out of phase and of equal magnitude and equal frequency, are applied to the horizontal and vertical deflection plates, a circle would appear on the screen as shown in Figure 9.10.

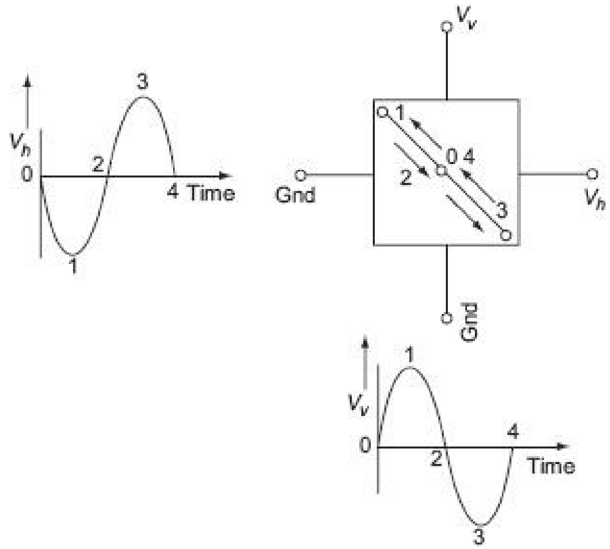


Figure 9.9 *Deflection for sinusoidal voltage signals equal in magnitude and frequency but opposite in phase, applied to horizontal and vertical deflection plates*

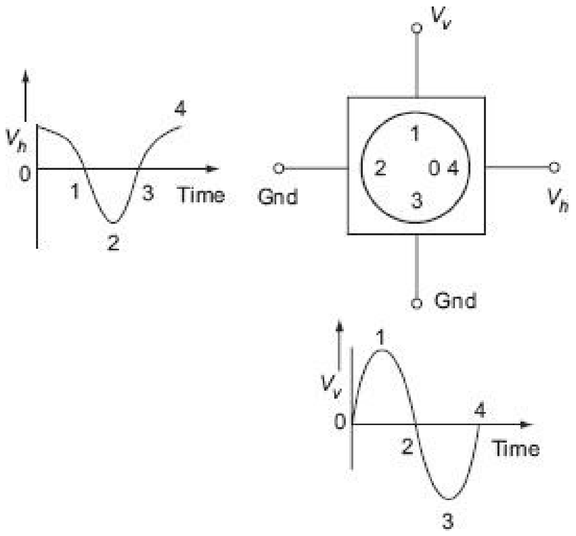


Figure 9.10 *Deflection for sinusoidal voltage signals equal in magnitude and frequency but 90° out of phase, applied to horizontal and vertical deflection plates*

9.7 MEASUREMENT OF VOLTAGE AND CURRENT

The expression for electrostatic deflection is $D = \frac{LI_d E_d}{2dE_a}$, where

L = distance between screen and the centre of the deflecting plates

I_d = length of deflecting plates

E_d = potential between deflecting plates

d = distance between deflecting plates

E_a = voltage of pre accelerating anode

So deflection is proportional to the deflecting-plate voltage. Thus, the cathode ray tube will measure voltage. It is used to calibrate the tube under the given operating conditions by observing the deflection produced by a known voltage. Direct voltage may be obtained from the static deflection of the spot, alternating voltage from the length of the line produced when the voltage is applied to *Y*-plates while no voltage is applied to X-plates. The length of the line corresponds to the peak to peak voltage. While dealing with sinusoidal voltages, the rms value is given by dividing the peak to peak voltage by $2\sqrt{2}$.

For measurement of current, the current under measurement is passed through a known non inductive resistance and the voltage drop across it is measured by CRO, as mentioned above. The current can be determined simply by dividing the voltage drop measured by the value of non inductive resistance. When the current to be measured is of very small magnitude, the voltage drop across noninductive resistance (small value) is usually amplified by a calibrated amplifier.

9.8 MEASUREMENT OF FREQUENCY

It is interesting to consider the characteristics of patterns that appear on the screen of a CRO when sinusoidal voltages are simultaneously applied to the horizontal and vertical plates. These patterns are called *Lissajous patterns.*

Lissajous patterns may be used for accurate measurement of frequency. The signal, whose frequency is to be measured, is applied to the *Y*-plates. An accurately calibrated standard variable frequency source is used to supply voltage to the *X*-plates, with the internal sweep generator switched off. The standard frequency is adjusted until the pattern appears as a circle or an ellipse, indicating that both signals are of the same frequency. Where it is not possible to adjust the standard signal frequency to the exact frequency of the unknown signal, the standard is adjusted to a multiple or submultiple of the frequency of the unknown source so that the pattern appears stationary.

Let us consider an example. Suppose sine waves are applied to *X* and *Y* plates as shown in Figure 9.11. Let the frequency of wave applied to *Y* plates is twice that of the voltage applied to the *X* plates. This means that the CRT spot travels two complete cycles in the vertical direction against one of the horizontal direction.

The two waves start at the same instant. A Lissajous pattern may be constructed in the usual way and a 8 shaped pattern with two loops is obtained. If the two waves do not start at the same instant we get different pattern for the same frequency ratio. The Lissajous pattern for the other frequency ratios can be similarly drawn. Some of these patterns are shown in Figure 9.12.

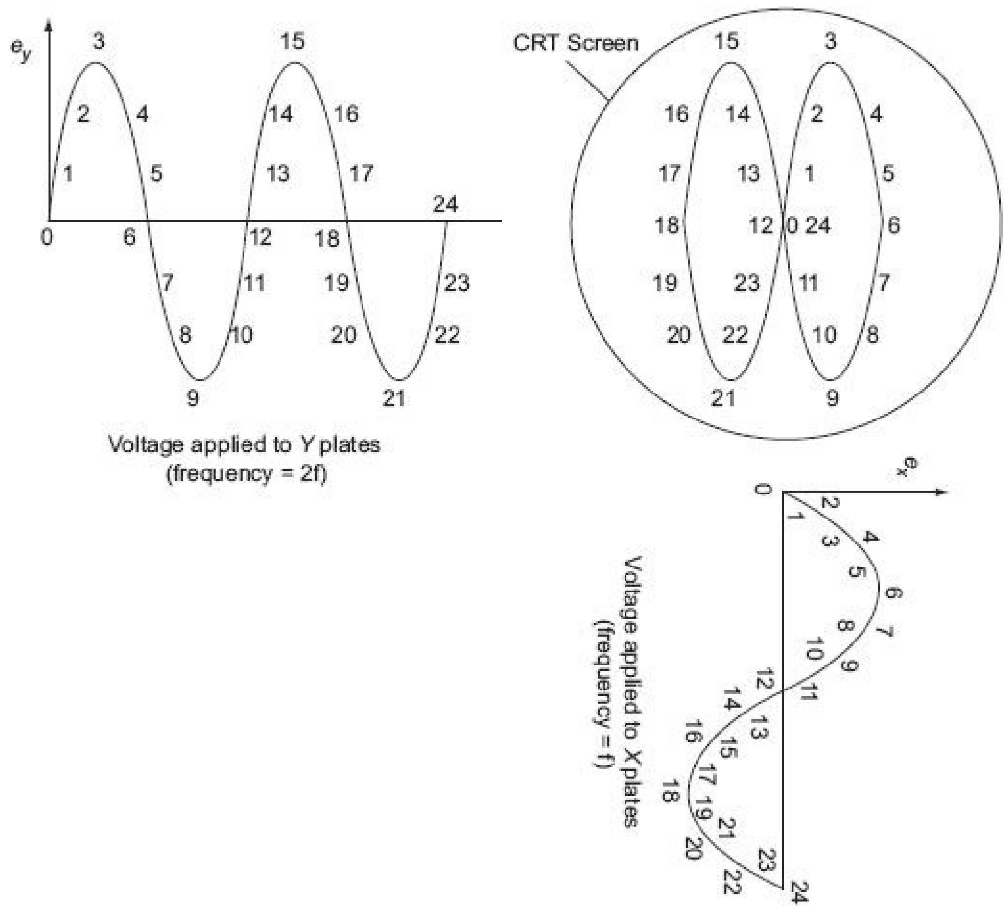


Figure 9.11 *Lissajous pattern with frequency ratio 2:1*

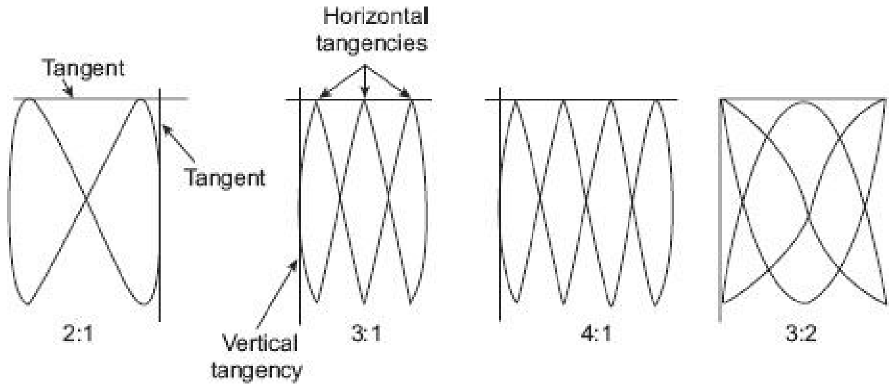


Figure 9.12 *Lissajous patterns with different frequency ratio*

It can be shown that for all the above cases, the ratios of the two frequencies is

$$\frac{f_y}{f_x} = \frac{\text{Number of times tangent touches top or bottom}}{\text{Number of times tangent touches either side}}$$
$$= \frac{\text{Number of horizontal tangencies}}{\text{Number of vertical tangencies}}$$

where f_y = Frequency of signal applied to Y plates

f_x = Frequency of signal applied to X plates

The above rule, however, does not hold for the Lissajous patterns with free ends as shown in Figure 9.13. The simple rule mentioned above needs the following modifications:

Two lines are drawn, one horizontal and the other vertical so that they do not pass through any intersections of different parts of the Lissajous curve. The number of intersections of the horizontal and the vertical lines with the Lissajous curve are individually counted. The frequency ratio is given by

$$\frac{f_y}{f_x} = \frac{\text{Number of intersections of the horizontal line with the curve}}{\text{Number of intersection of the vertical line with the curve}}$$

The applications of these rules to Figure 9.13(a) gives a frequency ratio $\frac{f_y}{f_x} = \frac{5}{2}$.

The modified rule is applicable in all cases whether the Lissajous pattern is open or closed.

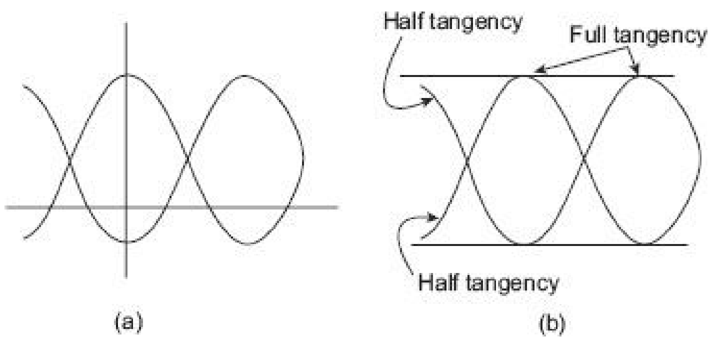


Figure 9.13 *Lissajous pattern with half tangencies*

The ratio of frequencies when open ended Lissajous patterns are obtained can also be found by treating the open ends as half tangencies as shown in Figure 9.13(b).

$$\therefore \quad \frac{f_y}{f_x} = \frac{\text{Number of horizontal tangencies}}{\text{Number of vertical tangencies}} = \frac{2 + 1/2}{1} = \frac{5}{2}$$

There are some restrictions on the frequencies which can be applied to the deflection plates. One obviously, is that the CRO must have the bandwidth required for these frequencies. The other restriction is that the ratio of the two frequencies should not be such as to make the pattern too complicated otherwise determination of frequency would become difficult. As a rule, ratios as high as 10:1 and as low as 10:9 can be determined comfortably.

9.9 MEASUREMENT OF PHASE DIFFERENCE

When two sinusoidal voltages of equal frequency which are in phase with each other are applied to the horizontal and vertical deflecting plates, the pattern appearing on the screen is a straight line as is clear from Figure 9.14.

Thus when two equal voltages of equal frequency but with 90° phase displacement are applied to a CRO, the trace on the screen is a circle. This is shown in Figure 9.15.

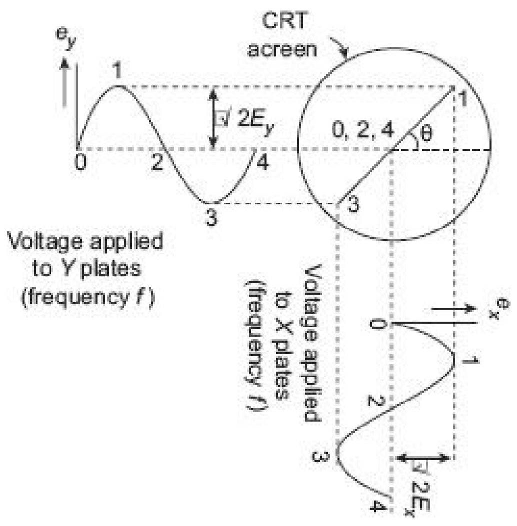


Figure 9.14 *Lissajous pattern with equal frequency voltages and zero phase shift*

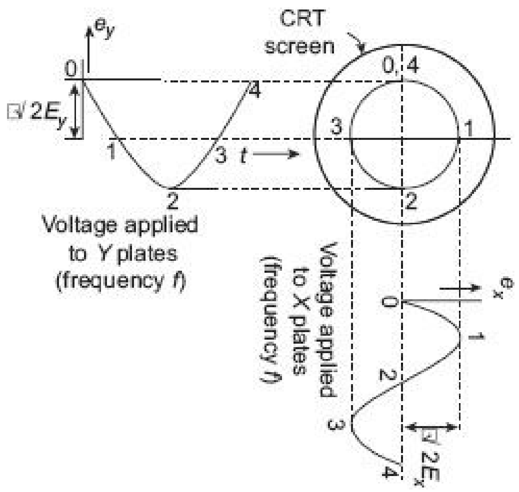


Figure 9.15 *Lissajous pattern with equal voltages and a phase shift of 90°*

When two equal voltages of equal frequency but with a phase shift Φ (not equal to 0 or 90°) are applied to a CRO, we obtain an ellipse as shown in Figure 9.16. An ellipse is also obtained when unequal voltages of same frequency are applied to the CRO.

A number of conclusions can be drawn from the above discussions. When two sinusoidal voltages of same frequency are applied, a straight line results when the two voltages are equal and are either in phase with each other or 180° out of phase with each other. The angle formed with the horizontal is 45° when the magnitudes of voltages are equal. An increase in the vertical deflecting voltage causes the line to have an angle greater than 45° with the horizontal.

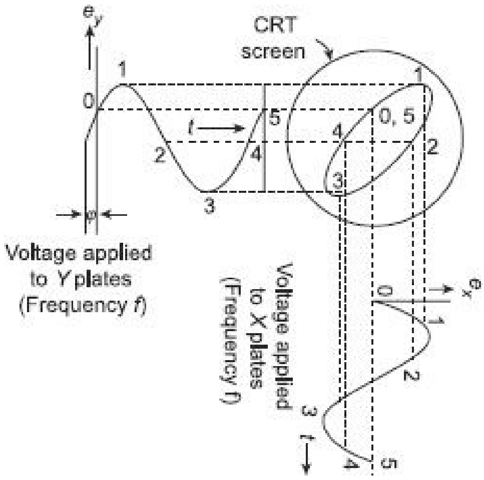


Figure 9.16 *Lissajous pattern with two equal voltages of same frequency and phase shift of Φ*

Two sinusoidal waveforms of the same frequency produce a Lissajous pattern which may be a straight line, a circle or an ellipse depending upon the phase and the magnitude of the voltages.

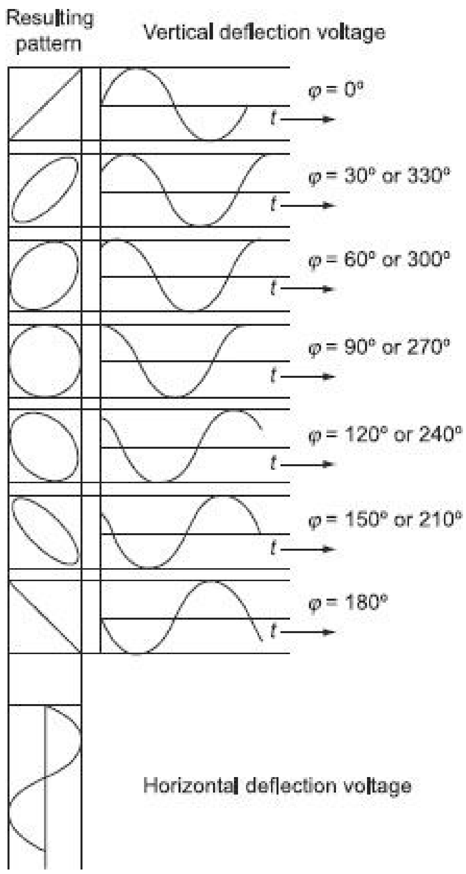


Figure 9.17 *Lissajous pattern with different phase shift*

A circle can be formed only when the magnitude of the two signals are equal and the phase difference between them is either 90° or 270°. However, if the two voltages are out of phase an ellipse is formed.

It is clear from Figure 9.17 that for equal voltages of same frequency, progressive variation of phase voltage causes the pattern to vary from a straight diagonal line to ellipse of different eccentricities and then to a circle, after that through another series of ellipses and finally a diagonal straight line again.

Regardless of the amplitudes of the applied voltages the ellipse provides a simple means of finding phase difference between two voltages. Referring to Figure 9.18, the sine of the phase angle between the voltages is given by

$$\sin \varphi = \frac{Y_1}{Y_2} = \frac{X_1}{X_2}$$

For convenience, the gains of the vertical and horizontal amplifiers are adjusted so the ellipse fits exactly into a square marked by the lines of the graticule.

If the major axis of the ellipse lies in the first and third quadrants (i.e., positive slope) as in Figure 9.18 (a), the phase angle is either between 0° to 90° or between 270° to 360°. When the major axis of ellipse lies in second and fourth quadrants, i.e., when its slope is negative as in Figure 9.18 (b), the phase angle is either between 90° and 180° or between 180° and 270°.

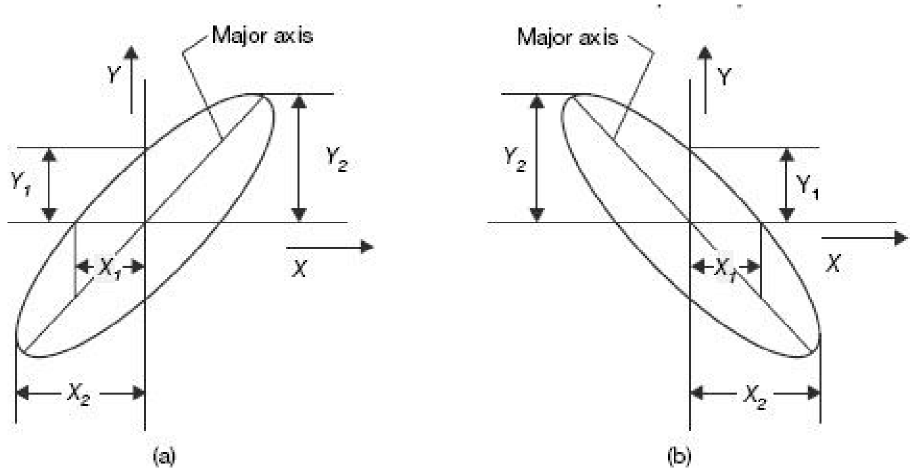


Figure 9.18 *Determination of angle of phase shift*

9.10 SAMPLING OSCILLOSCOPE

This oscilloscope is specially used for observing very high repetitive electrical signals by sampling the input waveform and reconstructing its shape from the sample. Such high frequency signals cannot be viewed by a conventional oscilloscope because its frequency range is limited by the gain bandwidth product of its vertical amplifier. The sampling frequency may be as low as $1/100^{th}$ of the input signal frequency, i.e., an ordinary oscilloscope having a bandwidth of 10 MHz can be used for observing input signal of frequency as high as 1000 MHz. As many as 1000 samples are used to reconstruct the original waveform.

A block diagram of a sampling oscilloscope is given in Figure 9.19. The input waveform, which must be repetitive, as applied to the sampling gate. Sampling pulses momentarily bias the diodes of the balanced sampling gate in the forward direction, thereby briefly connecting the gate input capacitance to the test point. These capacitors are slightly charged toward the voltage level of the input circuit. The capacitor voltage is amplified by a vertical amplifier and applied to the vertical deflecting plates. The sampling must be synchronised with the input signal frequency. The signal is delayed in the vertical amplifier, allowing the horizontal sweep to be initiated by the input signal.

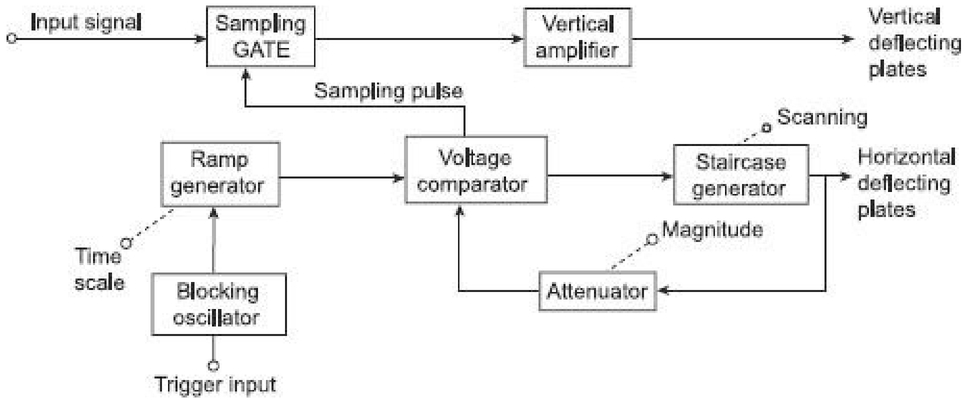


Figure 9.19 *Block diagram of sampling oscilloscope*

At the beginning of each sampling cycle, the trigger pulse activates an oscillator and a linear ramp voltage is generated. The ramp voltage is applied to a voltage comparator which compares the ramp voltage to a staircase generator output voltage. When the two voltages are equal in amplitude, the staircase generator is allowed to advance one step and simultaneously a sampling pulse is applied to the sampling gate. At this moment, a sample of the input voltage is taken, amplified and applied to the vertical deflecting plates.

The resolution of the final image on the screen of the CRT is determined by the size of the steps of the staircase generator. The smaller the size of these steps, the larger the number of samples and the higher the resolution of the image.

The sampling oscilloscope can be employed beyond 50 MHz into the UHF range around 500 MHz and beyond up to 10 GHz. However, sampling techniques cannot be used for the display of transients waveforms as they are not repetitive signals.

9.11 STORAGE OSCILLOSCOPE

There are two types of storage oscilloscopes, namely,

1. Analog storage oscilloscope
2. Digital storage oscilloscop

9.11.1 Analog Storage Oscilloscope

Storage targets can be distinguished from standard phosphor targets by their ability to retain a waveform pattern for a long time (10 to 15 hours after the pattern is produced on the screen). In a conventional CRT, the persistence of the phosphor varies from a few milliseconds to several seconds as a result of which, where persistence of the screen is smaller than the rate at which the signal sweeps across the screen, and the start of the display would fade before the end is written.

An analog storage oscilloscope uses the phenomenon of secondary electron emission to build up and store electrostatic charges on the surface of an insulated target. Such oscilloscopes are widely used (i) for real-time observation of events that occur only once, and (ii) for displaying the waveform of a very low frequency (VLF) signal.

The construction of a CRT using variable persistence storage technique, called the half-tone or mesh storage CRT is shown in Figure 9.20. With the variable persistence the slow swept trace can be stored on display continuously by adjusting the persistence of the CRT screen to match the sweep time.

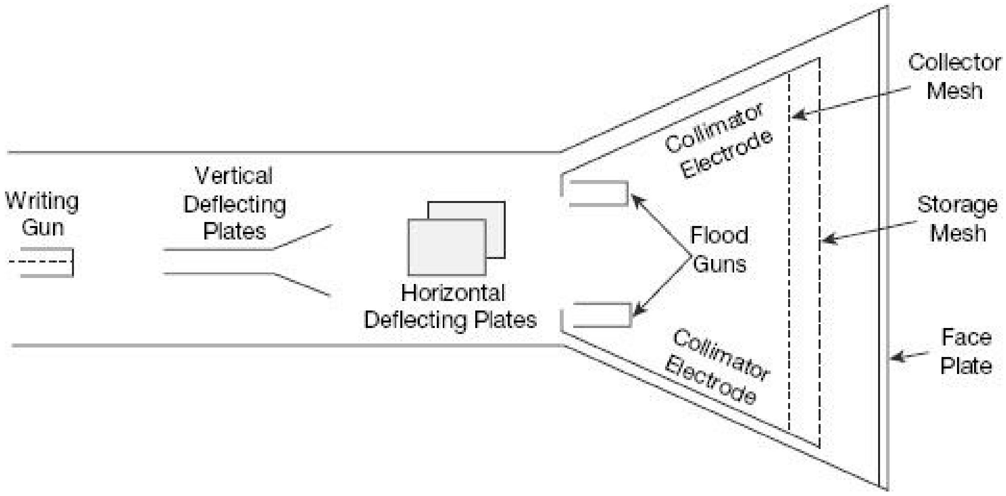


Figure 9.20 *Analog storage oscilloscope*

A mesh storage CRT, illustrated in Figure 9.20, contains a storage mesh, flood guns and a collimator, in addition to all the elements of a standard CRT. The storage mesh that is the storage target behind the phosphor screen is a conductive mesh covered with dielectric material consisting of a thin layer of material such as magnesium fluoride. The writing gun is a high-energy electron gun similar to the conventional gun, giving a narrow focused beam which can be deflected and used to write the information to be stored. The writing gun etches a positively charged pattern on the storage mesh or target by knocking off secondary emission electrons. This positively charged pattern remains exactly in the position on the storage target where it is deposited. This is due to the excellent insulating property of the magnesium fluoride coating on the storage target. The electron beam, which is deflected in the conventional manner, both in horizontal and vertical direction, traces out the wave pattern on the storage mesh. In order to make the pattern visible, even after several hours, special electron guns, known as the flood guns are switched on.

The flood guns are of simple construction and are placed inside the CRT in a position between the direction plates and the storage target and they emit low-velocity electrons covering a large area towards the screen. The electron paths are adjusted by the collimator electrodes consisting of a conductive coating on the inside surface of the CRT. The collimator electrodes are biased so as to distribute the flood gun electrons evenly over the target surface and causes the electrons to be perpendicular to the storage mesh. Most of the flood electrons are stopped and collected by the collector mesh and, therefore, never reach the phosphor screen. Only electrons near the stored positive charge are pulled to the storage target with sufficient force to hit the phosphor screen. The CRT display, therefore, will be an exact replica of the pattern which was initially stored on the target and the display will remain visible as long as the flood gun operates. For erasing of the pattern on the storage target, a negative charge is applied to neutralise the stored positive charge.

For achieving variable persistence, the erase voltage is applied in the form of pulses instead of a steady dc voltage; by varying the width of these pulses the rate of erase is controlled.

9.11.2 Digital Storage Oscilloscope

There are a number of distinct disadvantages of the analog storage oscilloscope. These disadvantages are listed below:

1. There is a finite amount of time that the storage tube can preserve a stored waveform. Eventually, the waveform will be lost. The power to the storage tube must be present as long as the image is to be stored.
2. The trace of a storage tube is, generally, not as fine as a normal cathode ray tube. Thus, the stored trace is not as crisp as a conventional oscilloscope trace.
3. The writing rate of the storage tube is less than a conventional cathode ray tube, which limits the speed of the storage oscilloscope.
4. The storage cathode ray tube is considerably more expensive than a conventional tube and requires additional power supplies.
5. Only one image can be stored. If two traces are to be compared, they must be superimposed on the same screen and displayed together.

A superior method if trace storage is the digital storage oscilloscope (DSO). In this technique, the waveform to be stored is digitised, stored in a digital memory and retrieved for display on the storage oscilloscope. The stored waveform is continually displayed by repeatedly scanning the stored waveform and, therefore, a conventional CRT can be employed for the display and thus some of the cost of the additional circuitry for digitizing and storing the input waveform is offset. The stored display can be displayed indefinitely as long as the power is applied to the memory, which can be supplied with a small battery. The digitised waveform can be further analysed by either the oscilloscope or by loading the content of the memory into a computer. Some of the digital storage oscilloscope use 12-bit converter, giving 0.025% resolution and 0.1% accuracy on voltage and time readings, which are better than the 2.5% of analog storage oscilloscopes. Split screen capabilities (simultaneously displaying live analog traces and replayed stored ones) enable easy comparison of the two signals. Pre-trigger capability is also an important advantage. The display of stored data is possible in both amplitude versus time, and *X*-*Y* modes. In addition to the fast memory readout employed for CRT display, a slow readout is possible for developing hard copy with external plotters.

The only drawback of digital storage oscilloscopes is limited bandwidth by the speed of their analog-to-digital converters (ADCs). However, 20 MHz digitising rates available on some oscilloscopes yield a bandwidth of 5 MHz, which is adequate for most of the applications.

Figure 9.21 gives the block diagram of a digital storage oscilloscope (DSO). It uses both of digital-to-Analog and Analog-to-Digital (DACs and ADCs) for digitising, storing and displaying analog waveforms. The overall operation is controlled and synchronised by the control circuits. Which usually have microprocessor executing a control program stored in Read-Only Memory (ROM). The data acquisition portion of the system contains a sample-and-hold (S/H) and a analog-to-digital converter that repetitively samples and digitized the input signal at a rate determined by the sample clock, and transmits the digitised data to memory for storage. The control circuit makes sure that successive data points are stored in successive memory locations by continually updating the memory's *address counter.*

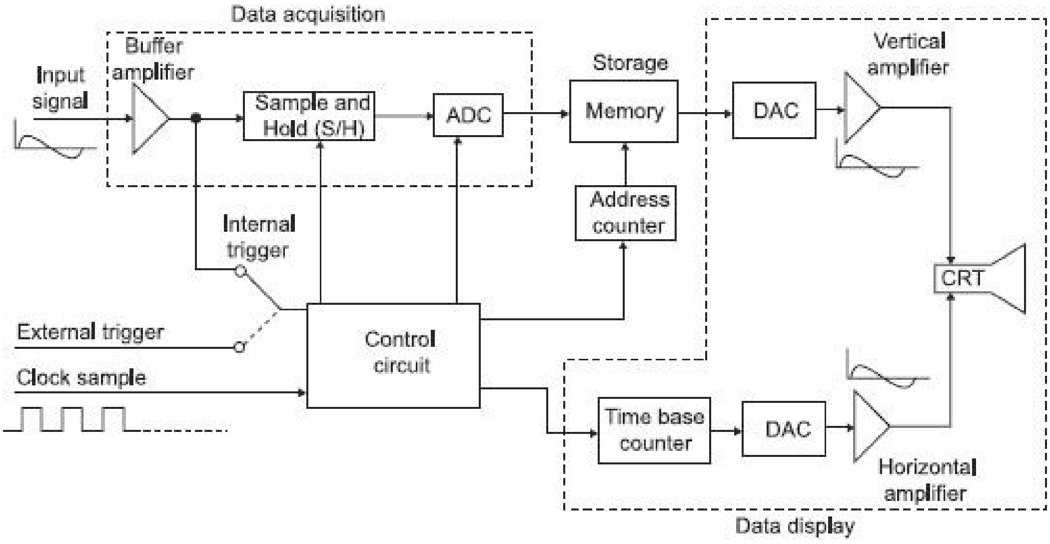


Figure 9.21 *Block diagram of Digital Storage Oscilloscope (DSO)*

When memory is full, the next data point from the ADC is stored in the first memory location writing over the old data, and so on for successive data points. This data acquisition and the storage process continue until the control circuit receives a trigger signal from either the input waveform (internal trigger) or an external trigger source. When the triggering occurs, the system stops acquiring data further and enters the display mode of operation, in which all or part of the memory data is repetitively displayed on the Cathode Ray Tube (CRT).

In display operation two DACs are employed for providing the vertical and horizontal deflecting voltages for the cathode ray tube. Data from memory produce the vertical deflection of the electron beam, while the time base counter provides the horizontal deflection in the form of a staircase sweep signal. The control circuits synchronize the display operation by incrementing the memory address counter and the time base counter at the same time so that each horizontal step of the electron beam is accompanied by a new data value from the memory to the vertical DAC. The counters are continuously recycled so that the stored data points are repetitively re-plotted on the screen of the CRT. The screen display consists of discrete dots representing the various data points but the number of dots is usually so large (typically 1000 or more) that they tend to blend together and appear to be a continuous waveform.

The display operation is transmitted when the operator presses a front panel button that commends the digital storage oscilloscope to begin a new data acquisition cycle.

9.12 MULTI-INPUT OSCILLOSCOPES

Modem oscilloscopes have the multi-input facility. They display the multi input simultaneously. Two inputs is most generally used, although four and eight inputs are available for special applications. There are two primary types: single beam and dual beam. A single beam can be converted into several traces. A dual beam, on the other hand may also subsequently be converted into a further number of traces. Two input oscilloscopes are described in this chapter, although the principles are applicable to any number of inputs.

9.12.1 Dual Trace Oscilloscopes

The block diagram of a dual trace oscilloscope is shown in Figure 9.22. There are two separate vertical input channels, *A* and B, and these use separate attenuator and preamplifier stages. Therefore the amplitude of each input, as viewed on the oscilloscope, can be individually controlled. After preamplification, the two channels meet at the electronic switch. This has the ability to pass one channel at a time into the vertical amplifier, via the delay line.

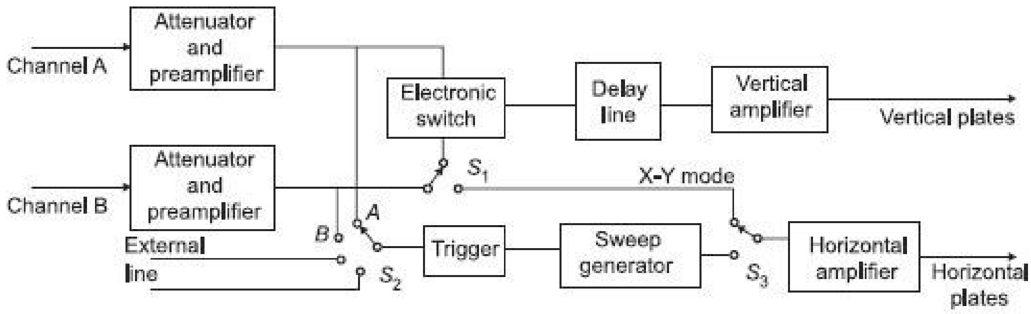


Figure 9.22 *Block diagram of a dual trace oscilloscope*

There are two common operating modes for the electronic switch, called *alternate* and *chop*, and these are selected from the instrument's front panel. The alternate mode is illustrated in Figure 9.23. In this figure, the electronic switch alternates between channels *A* and B, letting each through for one cycle of the horizontal sweep. The display is blanked during the flyback and hold-off periods, as in the conventional oscilloscope. Provided the sweep speed is much greater than the decay time of the CRT phosphor, the screen will show a stable of both the waveform at channels *A* and *B*. The alternate mode cannot be used for displaying very low-frequency signals.

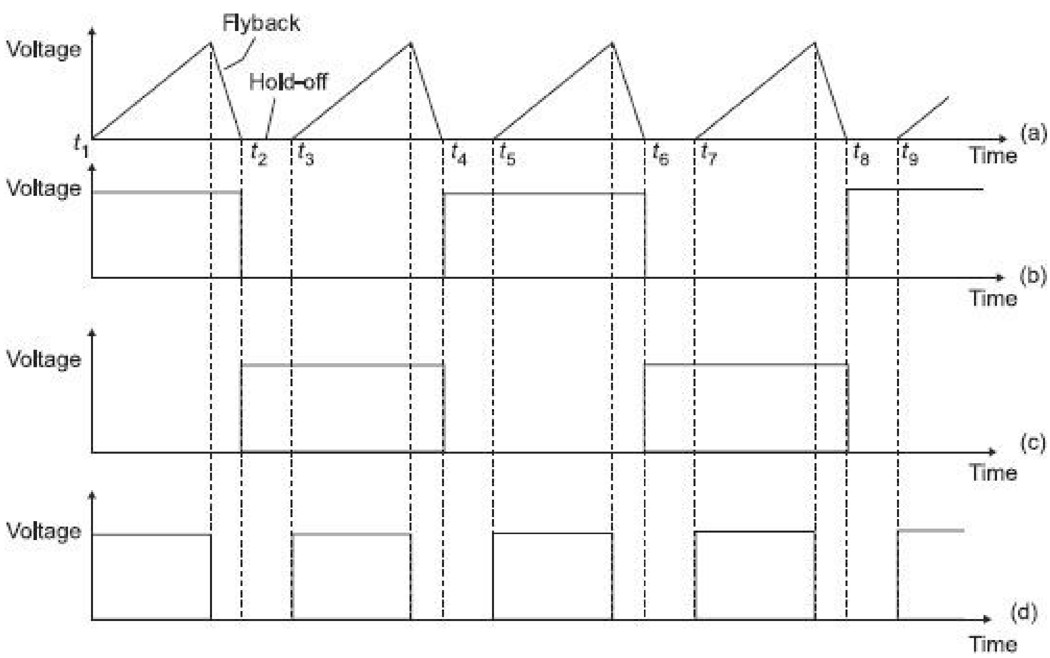


Figure 9.23 *Waveforms for a dual channel oscilloscope operating in alternating mode: (a) Horizontal Sweep voltage (b) Voltage to channel A (c) Voltage to channel B, (d) Grid control voltage*

The chopped operating mode of the electronic switch is shown in Figure 9.24. In this mode the electronics switch free runs at a high frequency of the order of 100 kHz to 500 kHz. The result is that small segments from channels *A* and *B* are connected alternately to the vertical amplifier, and displaying on the screen. Provided the chopping rate is much faster than the horizontal sweep rate, the display will show a continuous line for each channel. If the sweep rate approaches the chopping rate then the individual segments will be visible, and the alternate mode should now be used.

The time base circuit shown in Figure 9.22 is similar to that of a single input oscilloscope. Switch S_2 allow the circuit to be triggered on either the *A* or *B* channel waveforms, or on line frequency, or on an external signal. The horizontal amplifier can be fed from the sweep generator, or the *B* channel via switch S_1. This is the *X-Y* mode and the oscilloscope operates from channel *A* as the vertical signal and channel *B* as the horizontal signal, giving very accurate *X-Y* measurements. Several operating modes can be selected from the front panel for display, such as channel *A* only, channel *B* only, channels *A* and *B* as two traces, and signals $A + B$, $A - B$, $B - A$ or $-(A + B)$ as a single trace.

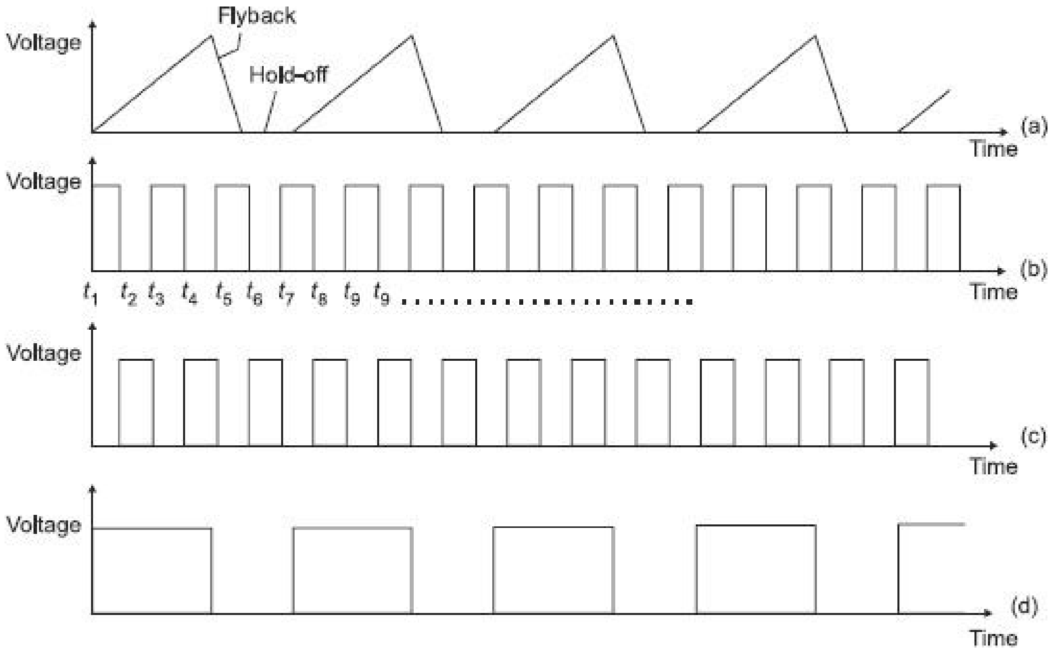


Figure 9.24 *Waveforms for a dual channel oscilloscope operating in chopped mode: (a) Horizontal sweep voltage (b) Voltage to channel A (c) Voltage to channel B, (d) Grid control voltage*

9.12.2 Dual Beam Oscilloscopes

The dual trace oscilloscope cannot capture two fast transient events, as it cannot switch quickly enough between traces. The dual beam oscilloscope has two separate electron beams, and therefore two completely separate vertical channels, as in Figure 9.23. The two channels may have a common time base system, as in Figure 9.22, or they may have independent time base circuits, as in Figure 9.25. An independent time base allows different sweeps rates for the two channels but increases the size and weight of the oscilloscope.

Two methods are used for generating the two electron beams within the CRT. The first method used a double gun tube. This allows the brightness and focus of each beam to be controlled separately but it is bulkier than a split beam tube.

In the second method, known as split beam, a single electron gun is used. A horizontal splitter plate is placed between the last anode and the *Y* deflection plates. This plate is held at the same potential as the anode, and it goes along the length of the tube, between the two vertical deflection plates. It therefore isolates the two channels. The split beam arrangement has half the brightness of a single beam, which has disadvantages at high frequency operation. An alternative method of splitting the beam, which improves its brightness, is to have two apertures in the last anode, instead of one, so that two beams emerge from it.

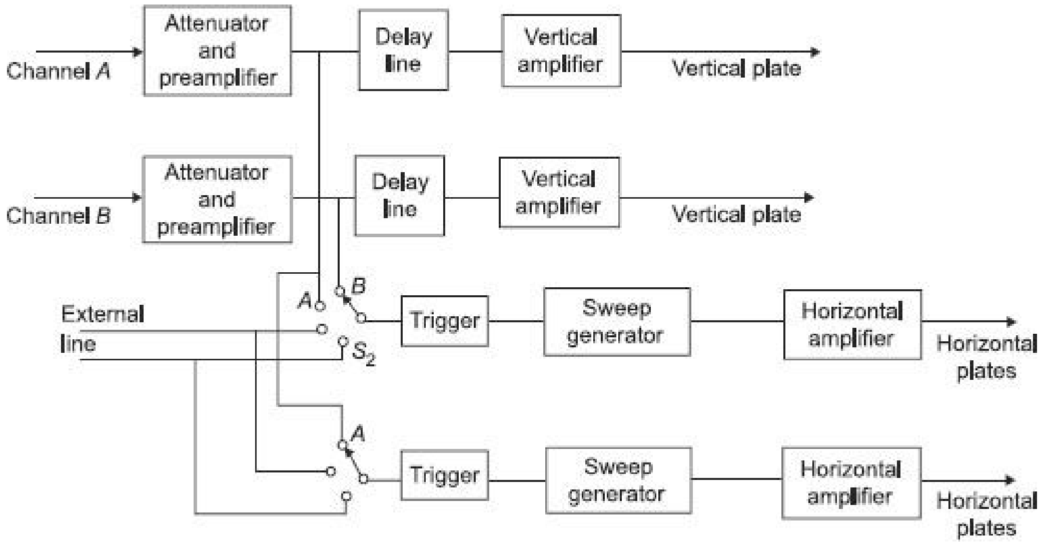


Figure 9.25 *Block diagram of a dual beam oscilloscope with independent time base*

The disadvantage of the split beam construction is that the two displays may have noticeably different brightness, if operated at widely spaced sweep speeds. The brightness and focus controls also affect the two traces at the same time.

9.13 FREQUENCY LIMITATION OF CRO

Deflection of electron beam on the screen in ***Y*** direction is given by $D = \frac{Ll_d E_d}{2dE_a}$, where E_d is the potential between deflecting plates. In this derivation, the plate voltage is assumed constant during the motion of the electrons through the deflecting field. If the voltage applied to the vertical deflecting plates change during the transit time of the electrons through the horizontal plates, the deflection sensitivity gets decreased.

Transit time $t_1 = \frac{l}{V_{ox}}$,

where l = length of deflecting plates

V_{ox} = velocity of electron while entering the field of deflecting plates.

The transit time impose a limitation of the upper frequency limit. An upper frequency is defined as that frequency at which the transit time is equal to one quarter (¼) of the period of the voltage applied to vertical plates.

$\therefore$ upper limiting frequency $= f_c = \frac{1}{4t_1} = \frac{V_{ox}}{4l}$

The frequency range of the oscilloscope can be increased by sub-dividing the deflecting plates in a number of sections in the path of electron beam. The voltage being measured is applied to the vertical plates through an iterative network, whose propagation time corresponds to the velocity of electron; thereby the voltage applied to the vertical plates is made to synchronise with the velocity of the beam. The use of this technique allows the CRO to be used up to frequencies of 500 MHz and above.

EXERCISE

Objective-type Questions

1. The time base signal in a CRO is
   (a) a sinusoidal signal
   (b) a sawtooth signal
   (c) a square wave signal
   (d) a triangular wave signal
2. In CRT aquadag carries
   (a) secondary emission electrons
   (b) sweep voltage
   (c) aqueous solution of graphite
   (d) none of these
3. In a CRO, the sawtooth voltage is applied at the
   (a) cathode
   (b) accelerating anode
   (c) vertical deflecting plates
   (d) horizontal deflecting plates
4. The purpose of the synchronising control in a CRO is to
   (a) adjust the amplitude of display
   (b) control the intensity of the spot
   (c) focus the spot on the screen
   (d) lock the display of signal
5. Retrace period for an ideal sawtooth waveform is
   (a) 0 second
   (b) equal to tracing period
   (c) infinite
   (d) none of these
6. In a CRT, the highest positive potential is given to
   (a) cathode
   (b) focusing electrodes
   (c) vertical deflecting plates
   (d) post-deflection acceleration anode

7. The X and Y inputs of a CRO are respectively $V \sin \omega t$ and $-V \sin \omega t$. The resulting Lissajous pattern will be

(a) a straight line

(b) a circle

(c) the shape of 8

(d) an ellipse

8. The patterns used to measure phase and frequency with a cathode ray oscilloscope are called

(a) Faraday's pattern

(b) Ohm's patterns

(c) Lissajous pattern

(d) Phillips pattern

9. The voltage $10 \cos \omega t$ and $V \cos (\omega t + \alpha)$ are applied to the X and Y plates of a CRO. The Lissajous figure observed on the screen is a straight line of 60° to the positive axis. Then

(a) $V = 10$, $\alpha = 60°$

(b) $V = 10$, $\alpha = 0°$

(c) $V = 10\sqrt{3}$, $\alpha = 60°$

(d) $V = 10\sqrt{3}$, $\alpha = 0°$

10. Sampling oscilloscopes are specially designed to measure

(a) very high frequency

(b) very low frequency

(c) microwave frequency

(d) none of these

11. Which of the following statements is not correct for a storage-type oscilloscope?

(a) Secondary emission electrons etch a positively charged pattern.

(b) The flood guns used for display, emit high velocity electrons.

(c) The flood guns are placed between the deflection plates and storage target.

(d) The storage target is a conductive mesh covered with magnesium fluoride.

12. In a digital oscilloscope, the A/D converters are usually

(a) ramp type

(b) flash type

(c) integrating type

(d) successive approximate type

13. A double beam oscilloscope has

(a) two screens

(b) two electron guns

(c) two different phosphor coatings

(d) one waveform divided into two parts

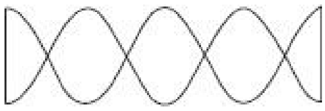
14. Two equal voltages of same frequency applied to the X and Y plates of a CRO, produce a circle on the screen. The phase difference between the two voltages is

(a) 150°

(b) 90°

(c) 60°

(d) 30°

15. The Lissajous pattern on a CRO screen is shown in the given figure:  The frequency ratio of the vertical signal to the horizontal one is

(a) 3 : 2

(b) 2 : 3

(c) 5 : 1

(d) 1 : 5

Answers

1. (b)	2. (c)	3. (d)	4. (d)	5. (a)	6. (d)	7. (a)
8. (c)	9. (a)	10. (a)	11. (b)	12. (c)	13. (b)	14. (b)
15. (c)						

Short-answer Questions

1. What is meant by the *deflection factor* and *deflection sensitivity* of a CRO? What is aquadag?
2. Discuss the advantages and disadvantages of analog and digital type of oscilloscope.
3. Explain the functioning of the time base generator in a CRO with proper diagram.
4. Describe the phenomenon of synchronisation of vertical input signal to its sweep generator.
5. Discuss the *triggered sweep* in a CRO.
6. Why is a CRO considered one of the most important tools in the field of modern electronics? What is the heart of a CRO?
7. How is the frequency of an ac signal measured with the help of CRO?
8. How is the phase difference between two signals measured with the help of the CRO?
9. "The focusing system of a CRO named as an electrostatic lens." Explain.
10. What are the differences between dual trace and dual beam oscilloscopes?

Long-answer Questions

1. (a) Draw the block diagram of a CRO and explain the different components.

   (b) The deflection sensitivity of an oscilloscope is 35 V/cm. If the distance from the deflection plates to the CRT screen is 16 cm, the length of the deflection plates is 2.5 cm and the distance between the deflection plates is 1.2 cm, what is the acceleration anode voltage?

2. (a) Derive an expression for the vertical deflection on the screen of a cathode ray tube in terms of length of plates, separation distance, accelerating voltage and distance of screen from the origin.

   (b) In a CRT, the distance between the deflecting plates is 1.0 cm, the length of the deflecting plates is 4.5 cm and the distance of the screen from the centre of the deflecting plates is 33 cm. If the accelerating voltage supply is 300 volt, calculate deflecting sensitivity of the tube.

3. What are Lissajous patterns? From the Lissajous patterns, how can the frequency and the phase difference be measured?
4. What are the advantages of dual trace over double beam for multitrace oscilloscopes? Explain the working of a dual trace CRO with the help of the proper block diagram.
5. Explain the working principle of a sampling oscilloscope with the help of proper block diagram. What precaution should be taken when using the sampling oscilloscope?
6. Draw the block diagram of a storage-type oscilloscope and explain the working of each block. How does the digital oscilloscope differ from the conventional analog storage oscilloscope?
7. Write short notes on the following.

   (a) Vertical amplifier

(b) Electromagnetic focusing

(c) Delay line

(d) Frequency and phase measurement by CRO

(e) High frequency oscilloscope

(f) Free running sweep

(g) Oscilloscope limitations

10 Electronic Instruments

10.1 INTRODUCTION

Nowadays everyone is familiar with analog signals. They are used for the movement of an electromagnetic meter to measure voltage, current, resistance, power, etc. Although the bridges and multipliers use electrical components for these measurements, the instruments described use no amplifiers to increase the sensitivity of the measurements. The heart of these instruments was the d'Arsonval meter, which typically cannot be constructed with a full-scale sensitivity of less than about 50 μA. Any measurement system using the d'Arsonval meter, without amplifiers, must obtain at least 50 μA from the circuit under test for a full-scale deflection. For the measurement of currents of less than 50 μA full scale, an amplifier must be employed. The resistance of a sensitive meter, such as a 50 μA meter for a volt-ohm- milliammeter, is of the order of a few hundred ohms and represents a small but finite amount of power. As an example, 50 μA through a 200 Ω meter represents ½ microwatt. This represents the power required for a meter for full-scale deflection and does not represent the power dissipated in the series resistor, and thus the total power required by the example meter would be greater than ½ μW and would depend on the voltage range. This does not sound like much power, but many electronics circuits cannot tolerate this much power being drained from them. So the electronics instruments are required for measuring very small current and voltage.

Electronic instruments, mainly electronic voltmeters, used either transistors or vacuum tubes. The later one is called the Vacuum Tube Voltmeter (VTVM) and the former one is called the Transistorised Voltmeter (TVM). In almost every field of electronics, VTVMs have been replaced by TVMs because of their numerous advantages. In TVM, due to the absence of a heating element, warm-up time is not required. It is portable due to the light weight of the transistor. VTVMs cannot measure current due to the very high resistance whereas due to the low resistance of the TVM, it can measure the current directly from the circuit. VTVMs also cannot measure high-frequency signals. The only disadvantage of TVM over VTVM is that the TVM has very low input impedance. But using FET (Field Effect Transistor) in the input stage of the voltmeter overcomes this low-impedance problem, because an FET offers input impedance almost equal to a vacuum tube.

10.2 MERITS AND DEMERITS OF DIGITAL INSTRUMENTS OVER ANALOG ONES

Although electronics are usually more costly than electrical instruments but are becoming more and more popular because of their various advantages over conventional ones, some of the main advantages are discussed below:

1.Detection of Low Level Signals As indicated above analog instruments use PMMC movement for indication. This movement cannot be constructed with a full scale sensitivity of less than 50 mA. Any measurement using a PMMC movement must draw a current of 50 mA from the measured quantity for its operation for full scale deflection if conventional voltmeters are used. This would produce great loading effects especially in electronic and communication circuits. Electronic voltmeter avoids the loading errors by supplying the power required for measurement by using external circuits like amplifiers. The amplifiers not only supply power for the operation but make it possible for low level signals, which produce a current less than 50 mA for full scale deflection, to be detected which otherwise cannot be detected in the absence of amplifiers. Let us examine the loading effect. A typical meter has a resistance of 200 W and its operating current at full scale is 50 mA. This means the power consumed is only $(50\ ¥10^{-6})^2$ ¥ 200 = 0.5 mW. This is extremely a low power for the power circuits but not for many electronic and communication circuits. If this power is taken from the measurand, the signal gets greatly distorted in case power level the circuit is very small and to offset this power is supplied from outside through use of amplifiers.

Let us have a look at the voltage. A meter with 200 W internal resistance and full scale current of 50 mA will have full scale voltage range of 50 ¥ 10^{-6} ¥ 200 = 10 mV. Now in case of lower ranges of voltages have to be measured, the use of a voltage amplifier becomes absolutely necessary. This is only possible through use of electronic voltmeters which allow the use of an amplifier. Therefore, it is possible to measure currents below 50 mA, voltages below 10 mV and keep drawn the power drainage below 0.5 mW by using electronic voltmeters through use of amplifiers which is otherwise not possible with conventional types of meter using PMMC movement. For the case of ac measurements, the use of an amplifier for detection of low level signals is even more necessary for sensitive measurements.

2.High Input Impedance A conventional PMMC voltmeter is a rugged and an accurate instrument, but it suffers from certain disadvantages. The principle problem is that it lacks both high sensitivity and high input resistance. It has a sensitivity of 20 kW/V with a 0 – 0.5 V range and has an input resistance of only 10 kW at its 0.5 V range with the result it has a full scale current of 50 mA which loads the measurand considerably. In electronics and communication circuits even this low value of current may not be tolerable on account of the fact that these circuits have very low operating currents. The electronic voltmeter (EVM), on the other hand, can have input resistances from 10 MW to 100 MW with the input resistance remaining constant over all ranges instead of being different at different ranges, the EVM gives for less loading effects.

3.Low Power Consumption Electronic voltmeters utilize the amplifying properties of vacuum tubes and transistors and therefore the power required for operating the instrument can be supplied from an auxiliary source. Thus, while the circuit whose voltage is being measured controls the sensing element of the voltmeter, the power drawn from the circuit under measurement is very small or even negligible. This can be interpreted as the voltmeter circuit has very high input impedance. This feature of electronic voltmeter is indispensable for voltage measurement in many high impedance circuits such as encountered in communicating equipments.

4.High Frequency Range The most important feature of electronic voltmeters is that their response can be made practically independent of frequency within extremely wide limits. Some electronic voltmeters permit the measurement of voltage from direct current to frequency of the order of hundreds of MHz. the high frequency range may also be attributed to low input capacitance of most electronics devices. The capacitance may be of the order of a few pF.

5.Better Resolution Resolution (smallest reading perceivable) of analog instruments is limited by space on the scale markings and also by ability of the human operator to read such small deviations in scale markings. Whereas in a digital instrument, the measured value is displayed directly on a LED or LCD panel whose resolution is solely determined by resolution of the analog to digital converter (ADC). Use of 12 bit (or higher) ADC can make a digital instrument to read as small as 0.001 V in 0 – 5 V range.

6.Storage Facility Digital instruments have an additional optional advantage that their readings can be stored for future reference. Since the value displayed is obtained through an ADC, the digital data can be easily stored in a microprocessor or PC memory. Such storage facility can only be made available in analog instruments by the use of chart recorders where the pointer has a ink source that keeps on marking the values on a roll of moving paper.

7.Accuracy Since there are very few moving parts (or even no moving parts) in the digital instruments, in general they are usually more accurate than the analog instruments. Even the human error involved in reading these instruments is very less, which adds to the accuracy of digital instruments. However, overall accuracy of a digital instrument will largely depend on accuracies of the large number of individual electronic components used for building the instrument.

In addition, digital instruments are more user friendly as they are easy to read, takes up smaller space, suitable for mass production, and also sometimes less costly.

Disadvantages of Digital Instruments

1. Effects on noise in more predominant on digital instruments than analog instruments. Analog instruments, due to inertia of its moving parts, normally remain insensitive to fast varying noise, while digital instruments continue to show erratic variations in presence of noise.
2. Analog instruments have higher overload capacity than digital instruments. The sensitive electronic components used in digital instruments are more prone to damage in case of even momentary overloads.
3. Digital instruments can sometimes loose its reliability and tend to indicate erratic values due to faulty electronic circuit components or damaged display.
4. Digital instruments and their internal electronic components are very much sensitive to external atmospheric conditions. In case of high humidity and corrosive atmosphere the internal parts may get damaged and indicate the faulty values.

PERFORMANCE CHARACTERISTICS OF DIGITAL

10.3 METERS

The performance characteristics of digital instruments are resolution, accuracy, linear errors, monotonicity, settling time and temperature sensitivity.

1. Resolution

It is the reciprocal of the number of discrete steps in the Digital to Analog (D/A) converter input. Resolution defines the smallest increment in voltage that can be discerned. Evidently resolution depends on the number of bits, i.e., the smallest increment in output voltage is determined by the Least Significant Bit (LSB). Percentage resolution is $[1/(2^n - 1)] \times 100$, where N is the number of bits.

2. Accuracy

It is a measure of the difference between actual output and expected output. It is expressed as a percentage of the maximum output voltage. If the maximum output voltage (or full-scale deflection) is 5 volt and accuracy is ±0.1%, then the maximum error is (0.1/100) × 5 = 0.005 volt or 5 mV. Ideally the accuracy should be better than ±½ of LSB. In a 8-bit converter, LSB is 1/255 or 0.39% of full scale. The accuracy should be better than 0.2%.

3. Linear Error

Linearity means that equal increments in digital input of digital instruments should result in equal increment in analog output voltage. If the values of resistances are very accurate and the other components are also ideal, there would be perfectly linear relation between output and input and output–input graph would be a straight line. Because of the fact that resistances used in the circuit have some tolerance, a perfectly linear relation between input and output is not obtained. A special case of linear error is offset error which is the output voltage when digital input is 0000.

4. Monotonicity

A Digital to Analog (D/A) converter is monotonic if it does not take any reverse step when it is sequenced over the entire range of input bits.

5. Settling Time

When the digital input signal changes, it is desirable that analog output signal should immediately show the new output value. However, in actual practice, the D/A converter takes some time to settle at the new position of the output voltage. Settling time is defined as the time taken by the D/A converter to settle within ±½ LSB of its final value when a change in input digital signal occurs. The finite time taken to settle down to new value is due to the transients and oscillations in the output voltage. Figure

6. Temperature Sensitivity

The reference voltage supplied to the resistors of a D/A converter are all temperature sensitive. Therefore, the analog output voltage depends, at least to some extent, on the temperature. The temperature sensitivity of the offset voltage and the bias current of OP-AMP also affect the output voltage. The range of temperature sensitivity for a D/A converter is from about ±50 to ±1.5 ppm/°C.

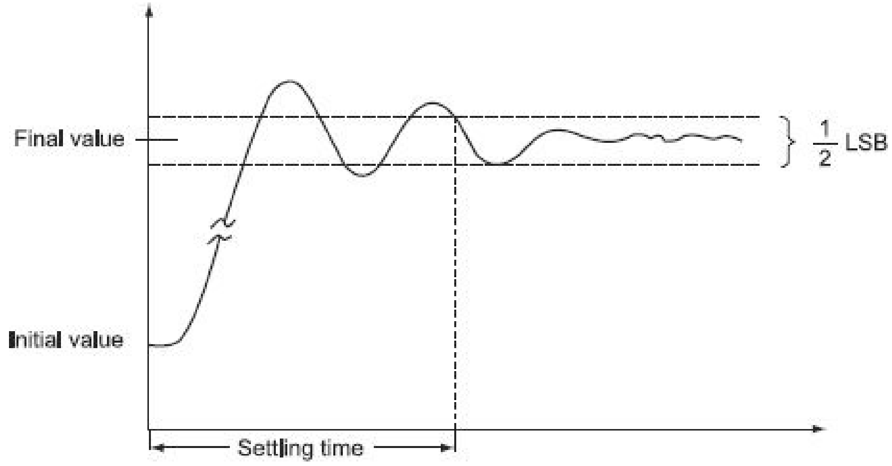


Figure 10.1 *Settling time of a digital instrument*

10.4 DIGITAL MULTIMETER

A digital multimeter is an electronic instrument which can measure very precisely the dc and ac voltage, current (dc and ac), and resistance. All quantities other than dc voltage is first converted into an equivalent dc voltage by some device and then measured with the help of digital voltmeter.

The block diagram of a digital multimeter is shown in Figure 10.2. The procedures of measurement of different quantities are described below.

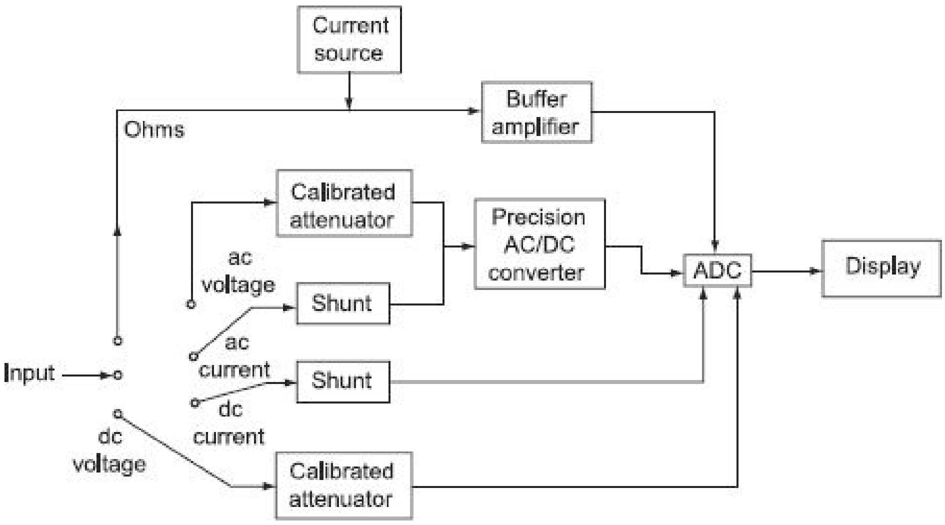


Figure 10.2 *Block diagram of a digital multimeter*

For measurement of ac voltage, the input voltage, is fed through a calibrated, compensated attenuator, to a precision full-wave rectifier circuit followed by a ripple reduction filter. The resulting dc is fed to an Analog Digital Converter (ADC) and the subsequent display system. Many manufacturers provide the same attenuator for both ac and dc measurements.

For current measurement, the drop across an internal calibrated shunt is measured directly by the ADC in the 'dc current mode', and after ac to dc conversion in the 'ac current mode'. This drop is often in the range of 200 mV (corresponding to full scale).

Due to the lack of precision in the ac–dc conversions, the accuracy in the ac range is generally of the order of 0.2 to 0.5%. In addition, the measurement range is often limited to about 50 Hz at the lower frequency end due to the ripple in the rectified signal becoming a non-negligible percentage of the display and hence results in fluctuation of the displayed number. At the higher frequency end, deterioration of the performance of the ADC converter limits the accuracy. In ac measurement the reading is often average or rms values of the unknown current. Sometimes for measurement of current, a current-tovoltage converter may also be used, asblock diagram in Figure 10.3.

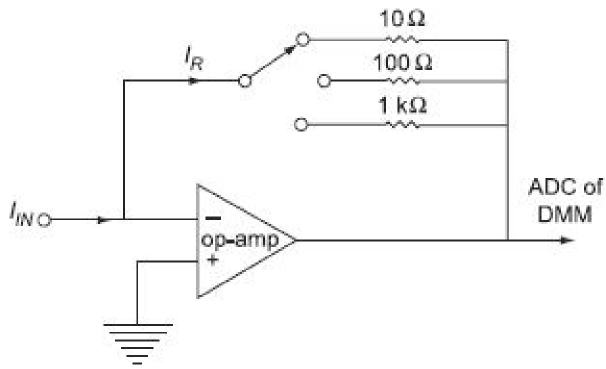


Figure 10.3 *Block diagram of a current-to-voltage converter*

The current under measurement is applied to the summing junction at the input of the op-amp. The current in the feedback resistor I_R is equal to the input current I_{IN} because of very high input impedance of the op-amp. The current I_R causes a voltage drop across one of the resistors, which is proportional to the input current I_{IN}. Different resistors are employed for different ranges.

For resistance measurement the digital multimeter operates by measuring the voltage across the externally connected resistance, resulting from a current forced through it from a calibrated internal current source. The accuracy of the resistance measurement is of the order of 0.1 to 0.5% depending on the accuracy and stability of the internal current sources. The accuracy may be proper in the highest range which is often about 10 to 20 MΩ. In the lowest range, the full scale may be nearly equal to>200 Ω with a resolution of about 0.01 Ω for a 4½ digit digital multimeter. In this range of resistance measurement, the effect of the load resistance will have to be carefully considered.

Table 10.1 *Comparison between analog and digital multimeter*

Analog Multimeter	*Digital Multimeter*
No external power supply required.	An external power supply is required.
Visual indication of change in reading is better observable.	Less observable.
Less effect of electronic noise.	More affected by electronic noise.
Less isolation problems.	More isolation problems.
It has less accuracy.	Highly accurate instrument.
Interface of the output with external equipment is not possible.	Possible to connect an external instrument with the output reading.
Simple in construction.	Very complicated in construction.
Big in size.	Small in size.

Low cost.	More costlier instrument.
The output is ambiguous in many times.	Unambiguous reading due to digital indication.

10.5 DIGITAL FREQUENCY METER

A frequency counter is a digital instrument that can measure and display the frequency of any periodic waveform. It operates on the principle of gating the unknown input signal into the counter for a predetermined time. For example, if the unknown input signal were gated into the counter for exactly 1 second, the number of counts allowed into the counter would be precisely the frequency of the input signal. The term *gated* comes from the fact that an AND or an OR gate is employed for allowing the unknown input signal into the counter to be accumulated.

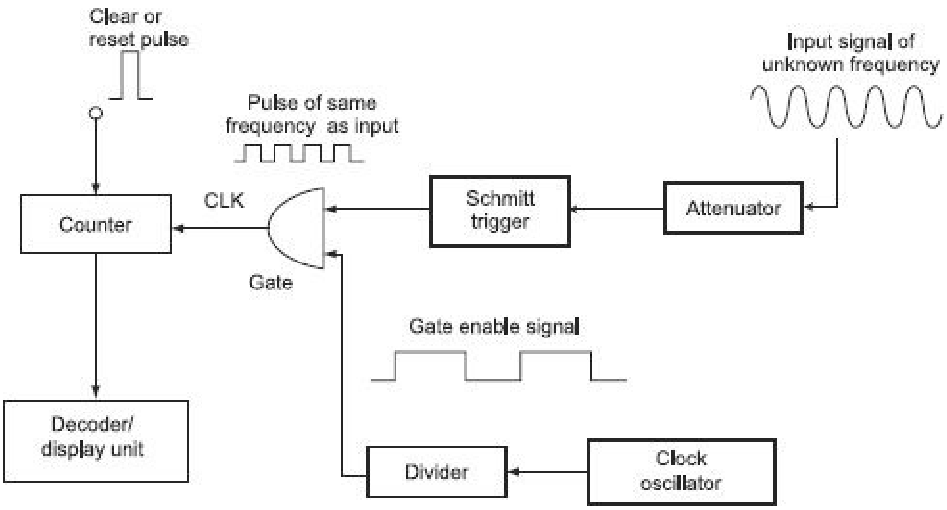


Figure 10.4 *Block diagram of frequency counter*

One of the most straightforward methods of constructing a frequency counter is shown in Figure 10.4 in simplified form. It consist of a counter with its associated display/decoder circuitry, clock oscillator, a divider and an AND gate. The counter is usually made up of cascaded Binary Coded Decimal (BCD) counters and the display/decoder unit converts the BCD outputs into a decimal display for easy monitoring. A GATE ENABLE signal of known time period is generated with a clock oscillator and a divider circuit and is applied to one leg of an AND gate. The unknown signal is applied to the other leg of the AND gate and acts as the clock for the counter. The counter advances one count for each transition of the unknown signal, and at the end of the known time interval, the contents of the counter will be equal to the number of periods of the unknown input signal that have occurred during time interval, t. In other words, the counter contents will be proportional to the frequency of the unknown input signal. For instance if the gate signal is of a time of exactly 1 second and the unknown input signal is a 600-Hz square wave, at the end of 1 second the counter will counts up to 600, which is exactly the frequency of the unknown input signal.

The waveform in Figure 10.5 shows that a clear pulse is applied to the counter at t_0 to set the counter at zero. Prior to t_1, the GATE ENABLE signal is LOW, and so the output of the AND gate will be LOW and the counter will not be counting. The GATE ENABLE goes HIGH from t_1 tot_2 and during this time interval t (= $t_2 - t_1$), the unknown input signal pulses will pass through the AND gate and will be counted by the counter. After t_2, the AND gate output will be again LOW and the counter will stop counting. Thus, the counter will have counted the number of pulses that occurred during the time interval, t of the GATE ENABLE SIGNAL, and the resulting contents of the counter are a direct measure of the frequency of the input signal.

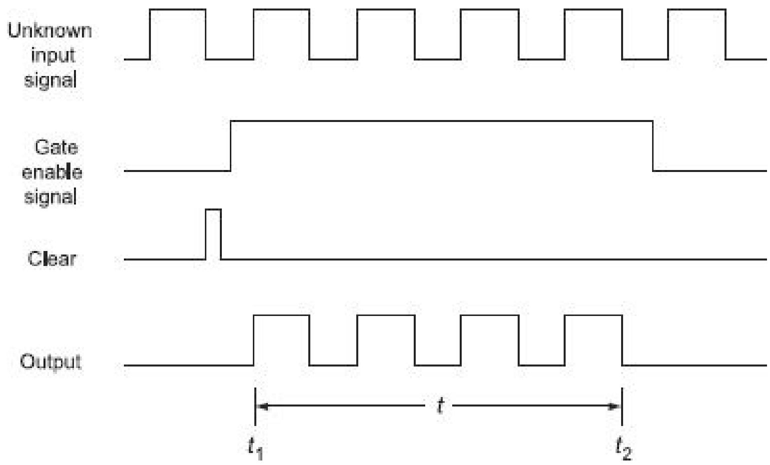


Figure 10.5 *Different waveforms in a frequency counter*

The accuracy of the measurement depends almost entirely on the time interval of the GATE ENABLE signal, which needs to be controlled very accurately. A commonly used method for obtaining very accurate GATE ENABLE signal is shown in Figure 10.6. A crystal controlled oscillator is employed for generating a very accurate 100 kHz waveform, which is shaped into the square pulses and fed to a series of decade counters that are being used to successively divide this 100 kHz frequency by 10. The frequencies at the outputs of each decade counter are as accurate as the crystal frequency.

The switch is used to select one of the decade counter output frequencies to be supplied to a single Flip-flop to be divided by 2. For instance in switch position 1, the 1 Hz pulses are supplied to flip-flop Q, which acts as a toggle flip-flop so that its output will be a square wave with a period if $T = 2$ s and a Tpulse duration,$t = \frac{T}{2} = 1\,s$ 1s . In position 2, the pulse duration would be 0.1s, and so on in other positions of the select switch.

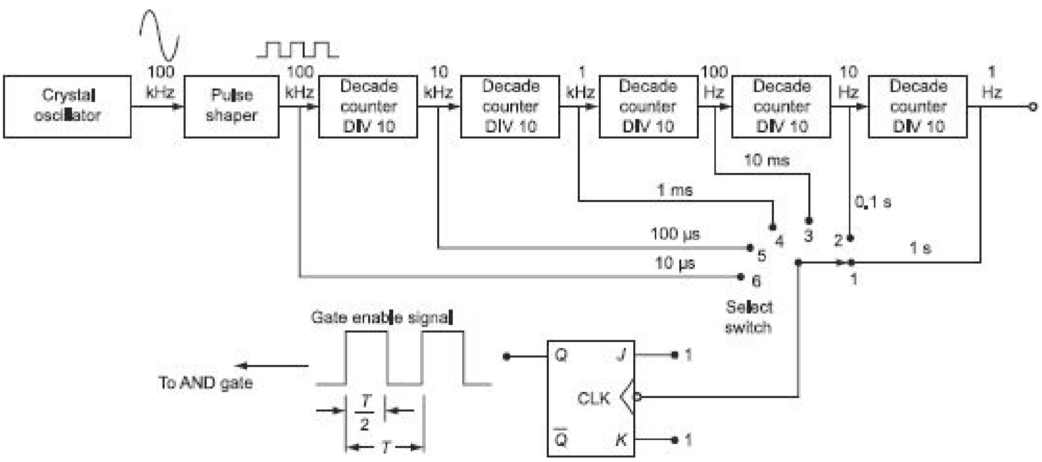


Figure 10.6 *Method of obtaining very accurate GATE ENABLE signal*

DIGITAL VOLTMETERS (DVMS)

10.6

The Digital Voltmeter (DVM) displays measurement of ac or dc voltages as discrete numbers instead of a pointer deflection on a continuous scale as in analog instruments. It is a versatile and accurate instrument that is employed in many laboratory measurement applications. Because of development and perfection of IC modules, their size, power consumptions and cost of the digital voltmeters has been drastically reduced and, therefore, DVMs are widely used for all measurement purposes.

The block diagram of a simple digital voltmeter is shown in Figure 10.7. The unknown signal is fed to the pulse generator which generates a pulse whose width is directly proportional to the input unknown voltage.

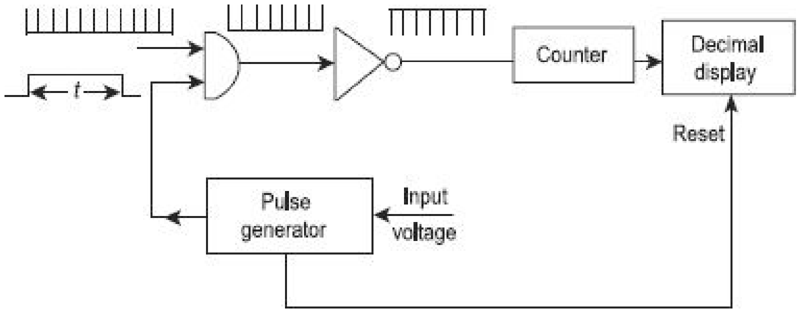


Figure 10.7 *Block diagram of DVM*

The output of the pulse generator is applied to one leg of an AND gate. The input signal to the other leg of the AND gate is a train of pulses. The output of the AND gate is, thus, a positive trigger train of duration *t* second and the inverter converts it into a negative trigger train. The counter counts the number of triggers in *t* seconds which is proportional to the voltage under measurement. Thus, the counter can be calibrated to indicate voltage in volts directly.

Thus, the DVMs described above is an Analog to Digital Converter (ADC) which converts an analog signal into a train of pulses, the number of which is proportional to the input voltage. So a digital voltmeter can be made by using any one of the analog to digital conversion methods and can be represented by a block diagram shown in Figure 10.8. So the DVMs can be classified on the basis of ADCs used.

Figure 10.8 *Representation of DVM using blocks*

The input range of the DVM may vary from ±1.00000 V to ±1000.00 V and its limiting accuracy is as high as ±0.005 percent of the reading. Its resolution may be 1 part in 10^6, giving 1 μV reading of the 1 V input range. It has high input resistance of the order of 10 MΩ and input capacitance of the order of 40 pF.

Digital voltmeters employing different analog to digital conversion methods are described below:

10.6.1 Ramp-Type DVM

The operation of a ramp-type DVM is based on the measurement of the time that a linear ramp voltage takes to change from the level of the input voltage to zero voltage or vice-versa. This time interval is measured with an electronic time interval counter and the count is displayed as a number of digits on the electronic indicating tubes of the voltmeter output readouts.

The operating principle and block diagram of a ramp-type DVM are given in Figures 10.9(a) and 10.9(b) respectively.

At the start of measurement, a ramp voltage is initiated, this voltage can be positive going or negative going. In Figure 10.9(a), negative going voltage ramp is illustrated.

The ramp voltage is continuously compared with the voltage under measurement (unknown voltage). At the instant the value of ramp voltage becomes equal to the voltage under measurement, a coincidence circuit, called the *input comparator*, generates a pulse which opens a gate, as shown in Figure 10.9(b). The ramp voltage continues to fall till it reaches zero value (or ground value). At this instant another comparator, called the *ground comparator*, generates a stop pulse. The output pulse from this ground comparator closes the gate. The time duration of the gate opening is proportional to the value of the dc input voltage.

The time elapsed between opening and closing the gate is t, as illustrated in Figure 10.9(a). During this time interval pulses from a clock pulse oscillator pass through the gate and are counted and displayed. The decimal number indicated by the readout is a measure of the value of the input voltage.

The sample rate multivibrator determines the rate at which the measurement cycles are initiated. The sample rate circuit provides an initiating pulse for the ramp generator to start its next ramp voltage. At the same time a reset pulse is generated, which resets the counter to zero state.

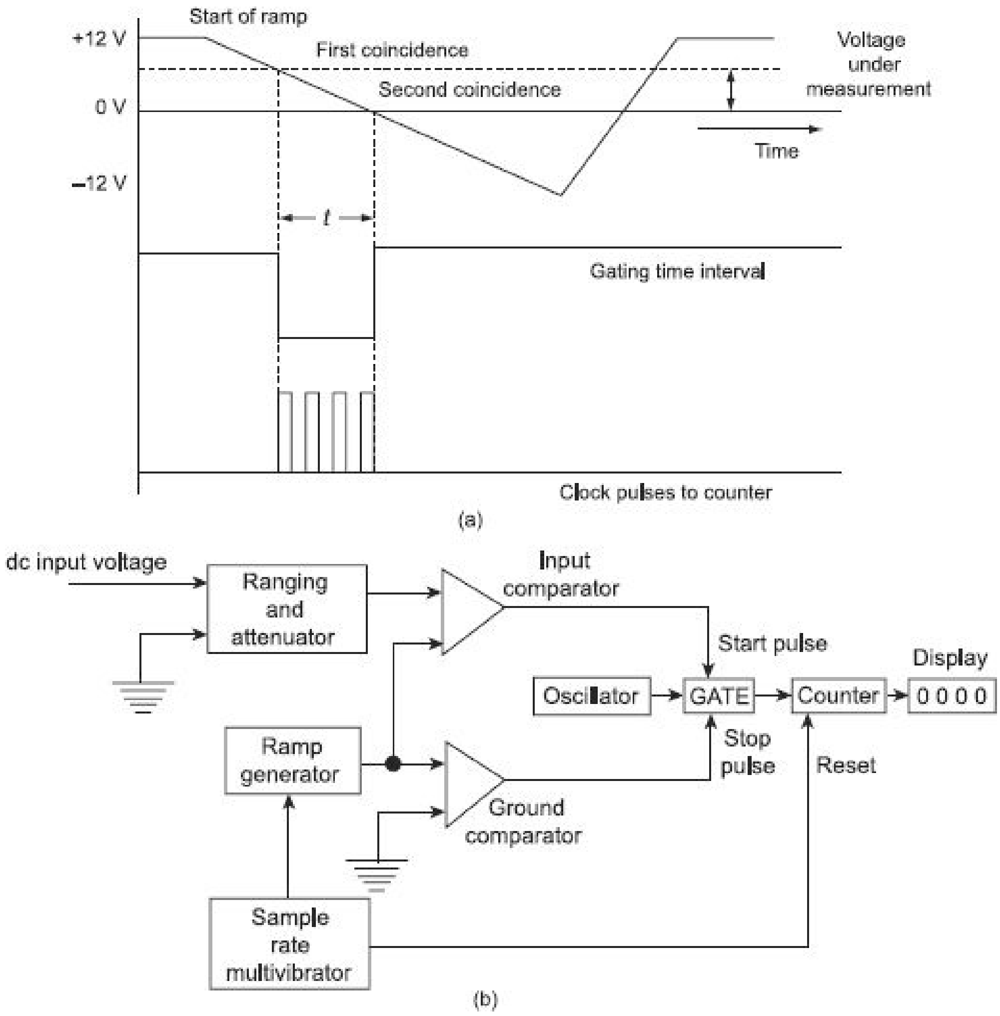


Figure 10.9 *(a) Voltage-to-time conversion (b) Block diagram of a ramp-type DVM*

10.6.2 Dual-Slope Integrating-Type DVM

The block diagram of a dual-slope integrating-type DVM is given in Figure 10.10(a). The dual slope ADC consists of five blocks namely an OP-AMP employed as an integrator, a level comparator, a basic clock for generating time pulses, a set of decimal counter and a block of logic circuitry.

For a fixed time interval (usually, the full count range of the counter), the analog input voltage to be measured, is applied through the switch *S* to the integrator which raises the voltage in the comparator to some negative value, as illustrated in Figure 10.10(b), obviously at the end of fixed time interval the voltage from the integrator will be greater in magnitude for larger input voltage, i.e., rate of increase of voltage or slope is proportional to the analog input voltage. At the end of the fixed count interval, the count is set at zero and the switch *S* is shifted to the reference voltage V_{REF} of opposite polarity. The integrator output or input to the capacitor then starts increasing at a fixed rate, as illustrated in

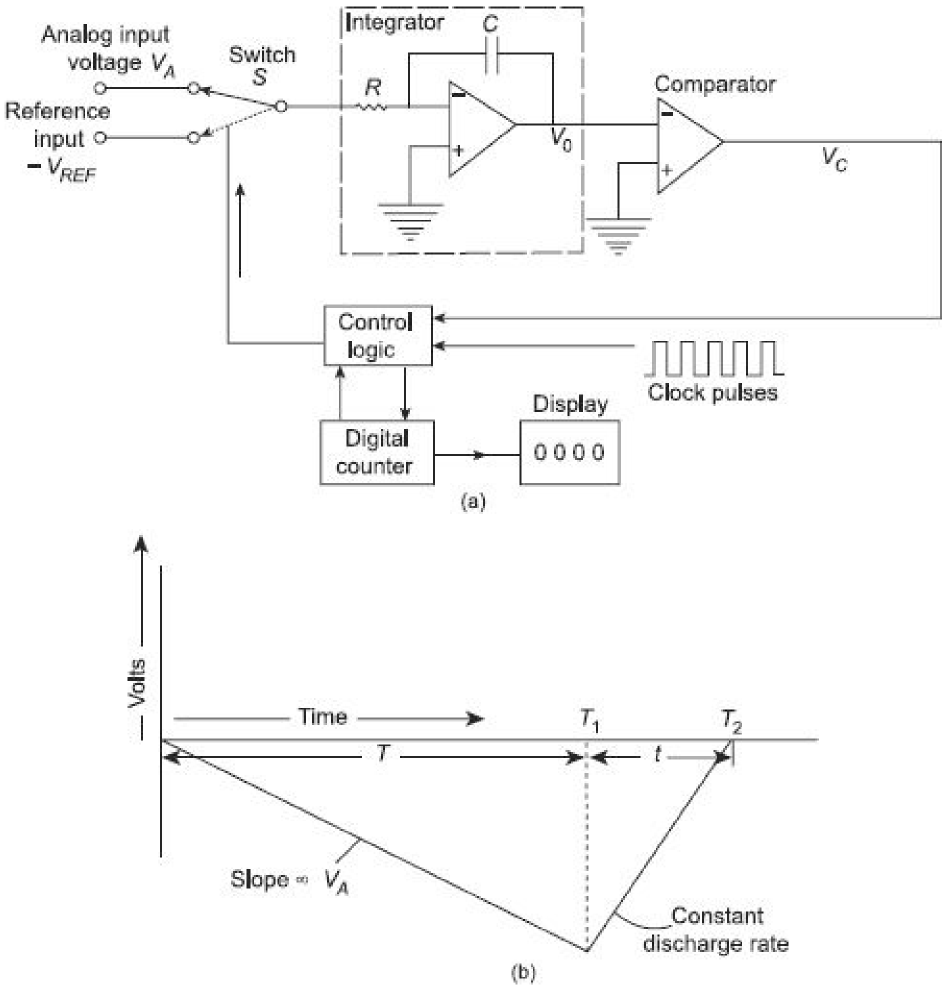


Figure 10.10 *(a) Block diagram of a Dual-Slope Integrating-type DVM*
(b) Principle of operation of dual-slope-type DVM

Figure 10.10(b). The counter advances during this time. The integrator output voltage increases at a fixed rate until it rises above the comparator reference voltage, at which the control logic receives a signal (the comparator output) to stop the count. The pulse counted by the counter thus has a direct relation with the input voltage V_A.

During charging, the output voltage V_0 is given as

$$V_0 = \frac{-1}{RC}\int_0^t V_A\,dt = \frac{-V_A t}{RC}$$

at $t = T_1$ (charging time) the value of the integrator output voltage $V_0 = \frac{-V_A T_1}{RC}$ (10.1)

Now the capacitor C has already a voltage of $\frac{-V_A T_1}{RC}$ (initial voltage),

During discharging, the output voltage is given as

$$V_0 = \text{Initial voltage of the capacitor} + \frac{-1}{RC}\int_0^t (-V_{REF})\,dt = \frac{-V_A T_1}{RC} + \frac{V_{REF}t}{RC}$$

At $t = T_2$ (the time measured by the counter), the output voltage of the integrator becomes zero

So, $$0 = \frac{-V_A T_1}{RC} + \frac{V_{REF} T_2}{RC}$$

or, $$V_A = V_{REF}\frac{T_2}{T_1} \quad (10.2)$$

Thus, the count shown by the counter (T_2) is proportional to the input voltage to be measured, V_A. The display unit displays the measured voltage.

The averaging characteristics and cancellation of errors that usually limit the performance of a ramp-type DVM are the main advantages of dual slope integrating type DVM. The integration characteristics provide the average value of the input signal during the period of first integration. Consequently, disturbances, such as spurious noise pulses, are minimised. Long-term drifts in the time constant as may result from temperature variations or aging, do not affect conversion accuracy. Also, long-term alternations in clock frequency have no effect.

10.6.3 Integrating-Type DVM (Voltage to Frequency Conversion)

Such a digital voltmeter makes use of an integration technique which employs a voltage to frequency (V/f) conversion. This voltmeter indicates the true average value of the unknown voltage over a fixed measuring period.

An analog voltage can be converted into digital form by producing pulses whose frequency is proportional to the analog input voltage. These pulses are counted by a counter for a fixed duration and the reading of the counter will be proportional to the frequency of the pulses, and hence, to the analog voltage.

A block diagram of a voltage to frequency ADC is shown in Figure 10.11. The analog input voltage V_A is applied to an integrator which in turn produces a ramp signal whose slope is proportional to the input voltage. When the output voltage V_0 attains a certain value (a preset threshold level), a trigger pulse is produced and also a current pulse is generated which is used to discharge the integrator capacitor C. Now a new ramp is initiated. The time between successive threshold level crossings is inversely proportional to the slope of the ramp. Since the ramp slope is proportional to the input analog voltage V_A, the frequency of the output pulses from the comparator is, therefore, directly proportional to the input analog voltage. This output frequency may be measured with the help of a digital frequency counter.

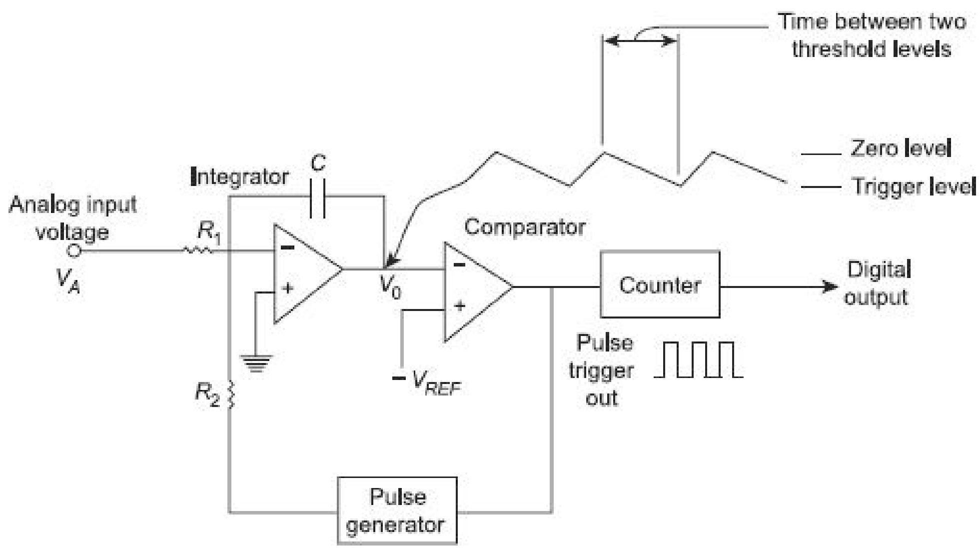


Figure 10.11 *Block diagram of an integrating-type DVM*

The above method provides measurement of the true average of the input signal over the ramp duration, and so provides high discrimination against noise present at the input.

However, the digitising rates are slow because of high integration durations. The accuracy of this method is comparable with the ramp type ADC, and is limited by the stability of the integrator time constant, and the stability and accuracy of the comparator.

This DVM has the drawback that it needs excellent characteristics in the linearity of the ramp. The ac noise and supply noise are averaged out.

10.6.4 Applications of DVMs

DVMs are often used in ‘data processing systems’ or ‘data logging systems’. In such systems, a number of analog input signals are scanned sequentially by an electronic system and then each signal is converted to an equivalent digital value by the A/D converter in the DVM.

The digital value is then transmitted to a pointer along with the information about the input line from which the signal has been derived. The whole data is then printed out.

In this way, a large number of input signals can be automatically scanned or processed and their values either printed or logged.

10.7 SIGNAL GENERATORS

A signal generator is numerously known as test signal generator, tone generator, arbitrary waveform generator, frequency generator, digital pattern generator, function generator, etc. It is an electronic device which produces repeating or non-repeating electronic signals (either analog or in digital patterns). These signals are utilised in testing, designing, troubleshooting and repairing electronic devices; apart from their artistic uses as well. Signal generators also modulate sinusoidal output signal with other signals. This feature is the main distinguisher between the signal generator and oscillator. When an unmodulated sinusoidal output is generated by the signal generators then they are said to be producing CW (continuous height wave) signal. When they produce modulated output signals then they can be in the form of square waves, externally applied sine waves, pulses, triangular waves, or more complex signals, as well as internally generated sine waves. Although Amplitude Modulation (AM) or Frequency Modulation (FM) can be used, yet amplitude modulation is generally employed. In Figures 10.12(a) and 10.12(b), the principles of amplitude modulation and frequency modulation are shown respectively.

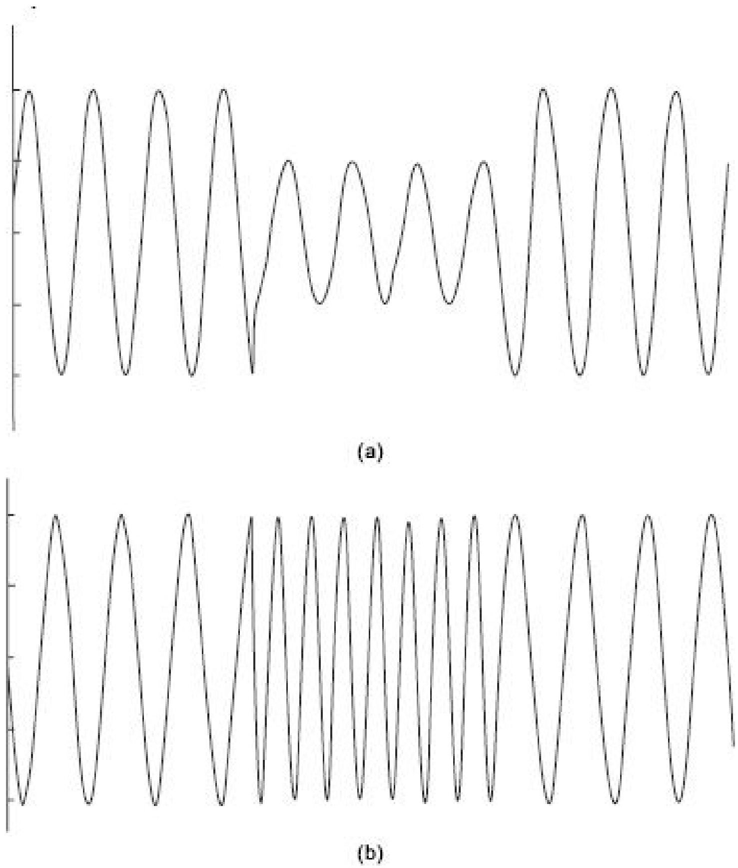


Figure 10.12 *(a) Amplitude modulation (b) Frequency modulation*

For providing appropriate signals for calibration and testing, signal generators are mainly employed.They are also used for troubleshooting of the amplifier circuits used in electronic and communications circuit amplifiers. Signal generators also measure the features of antennas and transmission lines.

Figure 10.13 illustrates the block diagram of a signal generator. A Radio Frequency (RF) oscillatoris applied for producing a carrier waveform whose frequency can be attuned typically from about 100 kHz to 40 MHz. With the help of vernier dial setting and range selector switch, carrier wave frequencies can be varied and displayed. Frequency dividers are employed to determine the range. An oscillators frequency stability is kept very high at all frequency ranges.

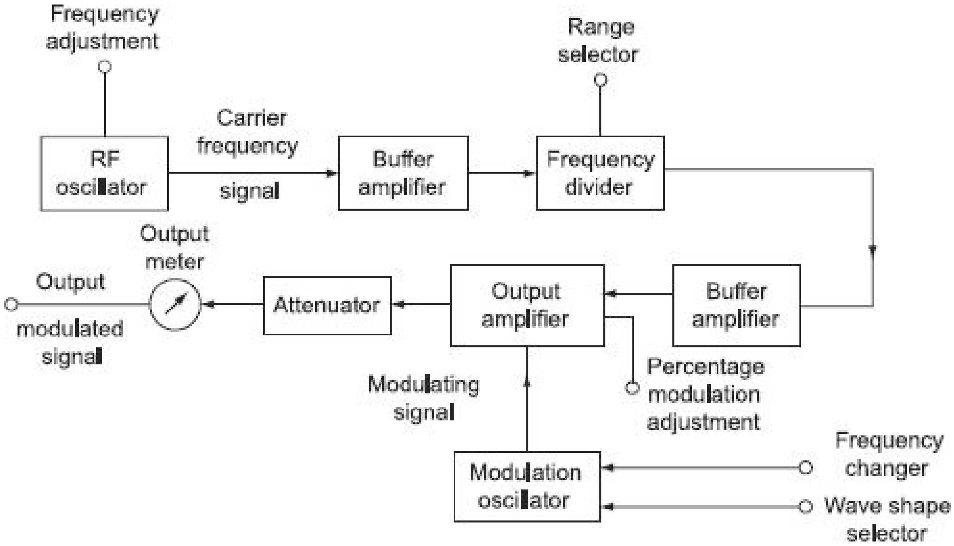


Figure 10.13 *Block diagram of an AM signal generator*

The following measures are taken in order to attain stable frequency output.

1. Regulated power supply is employed to reduce the change in supply voltage as it changes frequency of output voltage.
2. To separate the oscillator circuit from output circuit, buffer amplifiers are used. This is done so that any change in the circuit linked to the output does not affect the amplitude and frequency of the oscillator.
3. Temperature compensating devices are used to stable the temperature which causes change in oscillator frequency.
4. In place of an *LC* oscillator, a quartz crystal oscillator is employed to achieve high *Q*-factor, for instance, 20000.
5. When an audio frequency modulating signal is generated in another very stable oscillator, it is called the *modulation oscillator*. For changing the amplitude and the frequency of the produced signal, provision is made in the modulation oscillator.
6. Provision is also employed to get several types of waveforms such as the pulses, square, triangular oscillator. The modulation frequency and radio frequency signals are applied to a broadband amplifier, called the *output amplifier*. Modulation percentage is indicated and adjusted by the meter.
7. A control device can adjust modulation level up to 95%. The amplifier output is then sent to an attenuator and at last the signal reaches the output of the signal generator. An output meter reads or displays the final output signal.
8. An important specification of the signal generator performance is depending by the accuracy to which the frequency of the RF oscillator is tuned. Most laboratory-type signal generators are generally calibrated to be within 0.5–1.0% of the dial setting. For most measurements, this accuracy is sufficient. If greater accuracy is required then a crystal oscillator (frequency
9. Depending upon the different purposes and applications, various types of signal generators are available, however no device is suitable for all possible applications. Though traditional signal generators have embedded hardware units, with the advancement of multimedia-computers, audio software, tone generators signal generators have become more user-friendly and versatile.

EXERCISE

Objective-type Questions

1. Which one of the following statements is correct?
   An electronic voltmeter is more reliable as compared to multimeter for measuring voltages across low impedance because

   (a) its sensitivity is high

   (b) it offers high input impedance

   (c) it does not alter the measured voltage

   (d) its sensitivity and input impedance are high and do not alter the measured value

2. VTVM can be used to measure

   (a) dc voltage

   (b) ac voltage of high frequency

(c) dc voltage and ac voltage up to the order of 5 MHz frequency

(d) ac voltage of low frequency

3. Transistor voltmeter

(a cannot measure ac voltage

(b) cannot measure high frequency voltage

(c) cannot be designed to measure resistance as well as voltage

(d) can measure ac voltage

4. An electronic voltmeter provides more accurate readings in high-resistance circuits as compared to a nonelectronic voltmeter because of its

(a) low meter resistance

(b) high Ω/V ratings

(c) high V/Ω ratings

(d) high resolution

5. The input impedance of a TVM as compared to that of a VTVM is

(a) low

(b) high

(c) same

(d) not comparable

6. The power consumption of a dc voltmeter using a direct coupled amplifier when measuring a voltage of 0.5 volt and having an input impedance of 10 MΩ is

(a) a few nanowatts

(b) a few milliwatts

(c) a few microwatts

(d) a few watts

7. FETs are used in the amplifier to get

(a) high output impedance

(b) high input impedance

(c) low output impedance

(d) low input impedance

8. A direct voltage is applied to a peak diode voltmeter whose scale is calibrated to read rms voltage of a sine wave. If the meter reading is 36 V_{rms}, the value of the applied direct voltage is

(a) 51 volts

(b) 25 volts

(c) 36 volts

(d) 71 volts

9. A multimeter is used for the measurement of the following:

1. Both ac and dc voltage
2. Both ac and dc current
3. Resistance
4. Frequency
5. Power

Select the correct answer using the codes given:

(a) 1, 2 and 4

(b) 1, 2 and 5

(c) 1, 3 and 5

(d) 1, 2 and 3

10. Modern electronic multimeters measure resistance by

(a) using an electronic bridge compensator for nulling

(b) forcing a constant current and measuring the voltage across the unknown resistor

(c) using a bridge circuit

(d) applying a constant voltage and measuring the current through the unknown resistor

11. An n-bit A/D converter is required to convert an analog input in the range of 0–5 volt to an accuracy of 10 mV. The value of n should be

(a) 8

(b) 9

(c) 10

(d) 16

12. Which of the following is not true for a digital frequency counter?

(a) less costly

(b) high accuracy

(c) accepts inputs in the form of train of pulses

(d) wide range of frequency measurement

13. The resolution of a 12-bit analog to digital converter in per cent is

(a) 0.04882

(b) 0.09760

(c) 0.01220

(d) 0.02441

14. A DVM measures

(a) peak value

(b) rms value

(c) average value

(d) peak to peak value

15. Which of the following measurements can be made with the help of a frequency counter?

1. Fundamental frequency of input signal
2. Frequency components of the input signal at least up to third harmonics
3. Time interval between two pulses
4. Pulse width

Select the correct answer using the codes given below:

(a) 2 and 4

(b) 1 and 2

(c) 1, 2 and 3

(d) 1, 3 and 4

16. Accuracy of DVM is specified as

(a) percentage of the full scale reading

(b) percentage of the actual reading

(c) number of least significant digit

(d) all of these

Answers

1. (d)	2. (c)	3. (d)	4. (b)	5. (a)	6. (b)	7. (b)
8. (a)	9. (d)	10. (b)	11. (b)	12. (a)	13. (d)	14. (c)
15. (d)	16. (a)					

Short-answer Questions

1. What are the merits of the digital system over analog system?
2. Briefly describe the performance characteristics of digital measurement.
3. Write down the comparison between analog and digital multimeters.
4. What is the basic principle of digital frequency meter?
5. How many types of digital voltmeters are there? Explain any one of them briefly.
6. What are the applications of the digital voltmeter?
7. How does an electronic voltmeter (EVM) measure ac signal?

Long-answer Questions

1. (a) What are the advantages of digital systems over analog?

   (b) Explain the following terms as applied to digital measurement.

   (i) Resolution

   (ii) Sensitivity of digital meter

   (iii) Accuracy specification of digital meters

2. (a) Explain the operating principle of a DVM using a suitable block diagram.

   (b) With a neat sketch, describe the operating principle of dual slop integrating type of DVM.

3. Explain with the help of a functional block diagram, the principle of operation of a digital frequency meter.
4. With the help of a functional block diagram, describe the principle of operation of a digital multimeter.
5. What are the advantages of a digital voltmeter over analog type? What are its types? With a block diagram, explain the working of an integrating type. Compare its performance with other types.

11 Sensors and Transducers

11.1 INTRODUCTION

The physical quantity under measurement makes its first contact in the time of measurement with a sensor. A *sensor* is a device that measures a physical quantity and converts it into a signal which can be read by an observer or by an instrument. For example, a mercury thermometer converts the measured temperature into expansion and contraction of a liquid which can be read on a calibrated glass tube. A thermocouple converts temperature to an output voltage which can be read by a voltmeter. For accuracy, all sensors need to be calibrated against known standards.

In everyday life, sensors are used everywhere such as touch sensitive mobile phones, laptop's touch pad, touch controller light, etc. People use so many applications of sensors in their everyday lifestyle that even they are not aware about it. Examples of such applications are in the field of medicine, machines, cars, aerospace, robotics and manufacturing plants. The sensitivity of the sensors is the change of sensor's output when the measured quantity changes. For example, the output increases 1 volt when the temperature in the thermocouple junction increases 1°C. The sensitivity of the thermocouple element is 1 volt/°C. To measure very small charges, the sensors should have very high sensitivity.

A *transducer* is a device, usually electrical, electronic, electro-mechanical, electromagnetic, photonic, or photovoltaic that converts one type of energy or physical attribute to another (generally electrical or mechanical) for various measurement purposes including measurement or information transfer (for example, pressure sensors).

The term transducer is commonly used in two senses; the sensor, used to detect a parameter in one form and report it in another (usually an electrical or digital signal), and the audio loudspeaker, which converts electrical voltage variations representing music or speech to mechanical cone vibration and hence vibrates air molecules creating sound.

11.2 ELECTRICAL TRANSDUCERS

Electrical transducers are defined as the transducers which convert one form of energy to electrical energy for measurement purposes. The quantities which cannot be measured directly, such as pressure, displacement, temperature, humidity, fluid flow, etc., are required to be sensed and changed into electrical signal first for easy measurement.

The advantages of electrical transducers are the following:

- Power requirement is very low for controlling the electrical or electronic system.

- An amplifier may be used for amplifying the electrical signal according to the requirement.
- Friction effect is minimised.
- Mass-inertia effect are also minimised, because in case of electrical or electronics signals the inertia effect is due to the mass of the electrons, which can be negligible.
- The output can be indicated and recorded remotely from the sensing element.

Basic Requirements of a Transducer

The main objective of a transducer is to react only for the measurement under specified limits for which it is designed. It is, therefore, necessary to know the relationship between the input and output quantities and it should be fixed. A transducer should have the following basic requirements:

1. Linearity

Its input vs output characteristics should be linear and it should produce these characteristics in balanced way.

2. Ruggedness

A transducer should be capable of withstanding overload and some safety arrangements must be provided with it for overload protection.

3. Repeatability

The device should reproduce the same output signal when the same input signal is applied again and again under unchanged environmental conditions, e.g., temperature, pressure, humidity, etc.

4. High Reliability and Stability

The transducer should give minimum error in measurement for temperature variations, vibrations and other various changes in surroundings.

5. High Output Signal Quality

The quality of output signal should be good, i.e., the ratio of the signal to the noise should be high and the amplitude of the output signal should be enough.

6. No Hysteresis

It should not give any hysteresis during measurement while input signal is varied from its low value to high value and vice versa.

7. Residual Reformation

There should not be any deformation on removal of input signal after long period of use.

11.3 LINEAR VARIABLE DIFFERENTIAL TRANSFORMER (LVDT)

The most widely used inductive transducer to translate the linear motion into electrical signals is the Linear Variable Differential Transformer (LVDT). The basic construction of very effective as a short circuit for the voltage induced in the flowing liquid metal. When metallic pipes are used as with dc excitation no special electrodes are necessary. The output voltage is tapped off the metal pipe itself at the points of maximum potential difference.

11.6 TEMPERATURE TRANSDUCERS

Application of heat or its withdrawal from a body produces various primary effects on this body such as

- Change in its physical or chemical state such as phase transition
- Change in its physical dimensions
- Variations in its electrical properties
- Generation of an emf at the junction of two dissimilar metals
- Change in the intensity of the emitted radiation

Any of these effects can be employed to measure the temperature of a body, though the first one is generally used for standardisation of temperature sensors rather than for direct measurement of temperature.

11.6.1 Resistance Thermometers

Resistance temperature detectors, or resistance thermometers, employ a sensitive element of extremely pure platinum, copper or nickel wire that provides a definite resistance value at each temperature within its range. The relationship between temperature and resistance of conductors in the temperature range near 0°C can be calculated from the equation

$$R_t = R_{ref}(1 + \alpha \Delta t) \quad (11.12)$$

where R_t = resistance of the conductor at temperature t (°C)

R_{ref} = resistance at the reference temperature, usually 0°C

α = temperature coefficient of resistance

Δt = difference between operating and reference temperature

Almost all metallic conductors have a positive temperature coefficient of resistance so that their resistance increases with an increase in temperature. Some materials, such as carbon and germanium, have a negative temperature coefficient of resistance that signifies that the resistance decreases with an increase in temperature. A high value of α is desirable in a temperature sensing element so that a substantial change in resistance occurs for a relatively small change in temperature. This change in resistance *(ΔR)* can be measured with a Wheatstone bridge, which may be calibrated to indicate the temperature that caused the resistance change rather than the resistance change itself.

Figure 11.7 indicates the variation of resistance with temperature for several commonly used materials. The graph shows that the resistance of platinum and copper increases almost linearly with increasing temperature, while the characteristic for nickel is decidedly nonlinear.

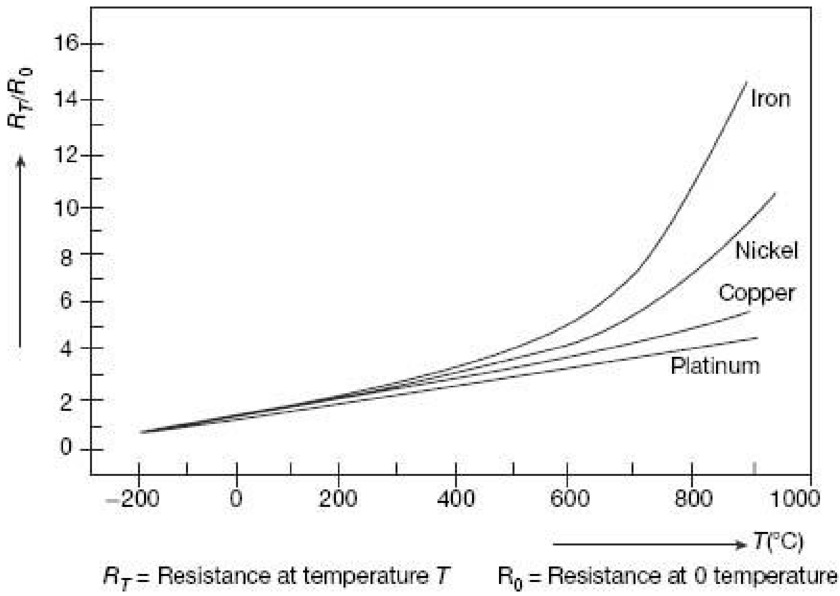


Figure 11.7 *Relative resistance (R_T/R_0) versus temperature for some pure metals*

The sensing element of a resistance thermometer is selected according to the intended application. Table 11.1 summarises the characteristics of the three most commonly used resistance materials. Platinum wire is used for most laboratory work and for industrial measurements of high accuracy. Nickel wire and copper wire are less expensive and easier to manufacture than platinum wire elements, and they are often used in low-range industrial applications.

Table 11.1 *Resistance thermometer elements*

Type	*Temperature range*	*Accuracy*	*Advantages*	*Disadvantages*
Platinum	−3000°F to +15000°F	±0.1°F	Low cost, high stability, wide operating range	Relatively slow response time (15s), not as linear as copper thermometers.
Copper	−3250°F to +2500°F	±0.50°F	High linearity, high accuracy in ambient temperature range, high stability	Limited temperature range (to 2500°F)
Nickel	−320°F to + 1500°F	±0.50°F	Longer life, high sensitivity, high temperature coefficient	More nonlinear than copper, limited temperature range (to 1500°F)

Resistance thermometers are generally of the probe type for immersion in the medium whose temperature is to be measured or controlled. A typical sensing element for a probe-type thermometer is constructed by coating a small platinum or silver tube with ceramic material, winding the resistance wire over the coated tube, and coating the finished winding again with ceramic. This small assembly is then fired at high temperature to assure annealing of the winding and then it is placed at the tip of the probe. The probe is protected by a sheath to produce the complete sensing elements as shown in Figure 11.8.

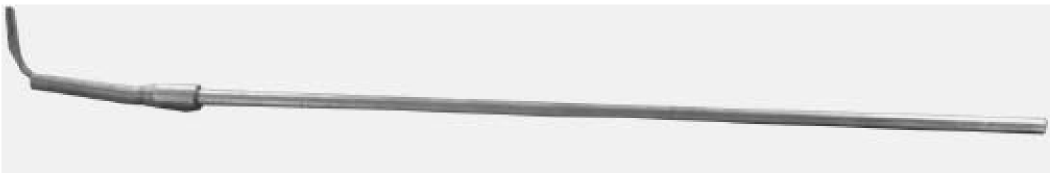


Figure 11.8 *Resistance thermometer in protecting cover*

Practically, all resistance thermometers for industrial applications are mounted in a tube or well to provide protection against mechanical damage and to guard against contamination and eventual failure. Protecting tubes are used at atmospheric pressure;

when they are equipped inside a pipe thread bushing, they may be exposed to low or medium pressures. Metal tubes offer adequate protection to the sensing element at temperatures up to 100°F, although they may become slightly porous at temperatures above 1500°F and then fail to protect against contamination.

Protecting covers are designed for use in liquid or gases at high pressure such as in pipe lines, steam power plants, pressure tanks, pumping stations, etc. The use of a protecting cover becomes imperative at pressures above three times of atmospheric pressure. Protective wells are drilled from solid bar stock, usually carbon steel or stainless steel, and the sensing element is mounted inside. A waterproof junction box with provision for conduit coupling is attached to the top of the tube or well.

A typical bridge circuit with resistance thermometer R_t in the unknown position is shown in Figure 11.9. The function switch connects three different resistors in the circuit. R_{Ref} is a fixed resistor whose resistance is equal to that of the thermometer element at the reference temperature (say, 0°C). With the function switch in the 'REFERENCE' position, the zero adjust resistor is varied until the bridge indicator reads zero. R_{fs} is another fixed resistor whose resistance equals that of the thermometer element for full-scale reading of the current indicator. With the function switch in the 'FULL SCALE' position, the full scale adjust resistor is varied until the indicator reads the full scale. The function switch is then set to the 'MEASUREMENT' position, connecting the resistance thermometer R_t in the circuit. When the resistance temperature characteristic of the thermometer element is linear, the galvanometer indication can be interpolated linearly between the set of values of reference temperature and full scale temperature.

The Wheatstone bridge has certain disadvantages when it is used to measure the resistance variations of the resistance thermometer. These are the effects of contact resistances of connections to the bridge terminals, heating of the elements by the unbalance current and heating of the wires connecting the thermometer to the bridge. Slight modifications of the Wheatstone bridge, such as the double slide wire bridge eliminates most of these problems. Despite these measurement difficulties, the resistance thermometer method is so accurate that it is one of the standard methods of temperature measurement within the range –150° to 650°C.

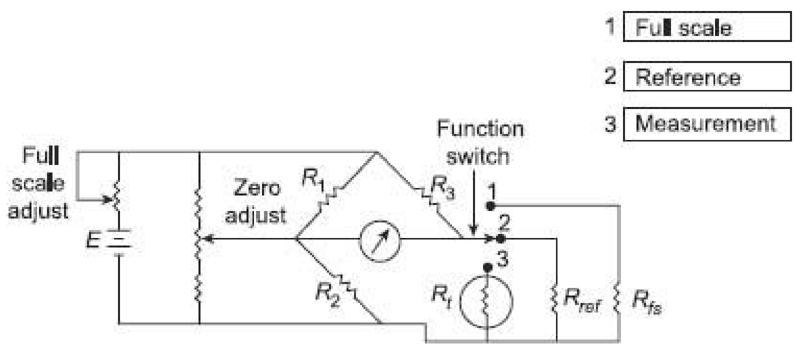


Figure 11.9 *Wheatstone bridge circuit with a resistance thermometer as one of the bridge elements*

11.6.2 Thermocouple

Thomas Seebeck discovered in 1821 that when two dissimilar metals were in contact, a voltage was generated where the voltage was a function of temperature. The device, consisting of two dissimilar metals joined together, is called a *thermocouple* and the voltage is called the *Seebeck voltage*, according to the name of the discoverer.

As an example, joining copper and constantan produces a voltage on the order of a few tens of millivolts (Figure 11.10) with the positive potential at the copper side. An increase in temperature causes an increase in voltage.

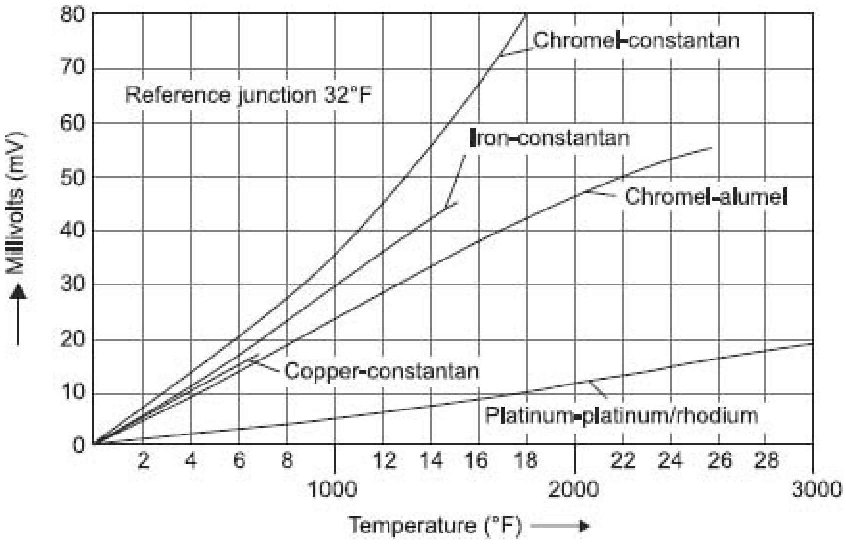


Figure 11.10 *Thermocouple output voltage as a function of temperature for various thermocouple materials*

There are several methods of joining the two dissimilar metals. One is to weld the wires together. This produces a brittle joint, and if not protected from stresses, this type of thermocouple can fracture and break apart. During the welding process, gases from the welding can diffuse into the metal and cause a change in the characteristic of the thermocouple. Another method of joining the two dissimilar metals is to solder the wires together. This has the disadvantage of introducing a third dissimilar metal. Fortunately, if both sides of the thermocouple are at the same temperature, the Seebeck voltage due to thermocouple action between the two metals of the thermocouple and the solder will have equal and opposite voltages and the effect will cancel. A more significant disadvantage is that in many cases the temperatures to be measured are higher than the melting point of the solder and the thermocouple will come apart.

It would appear to be a simple matter to measure the Seebeck voltage and create an electronic thermometer. To do this, wires could be connected as shown in Figure 11.11 to make the measurement. This connection of the wires causes a problem of measurement, as shown in Figure 11.12.

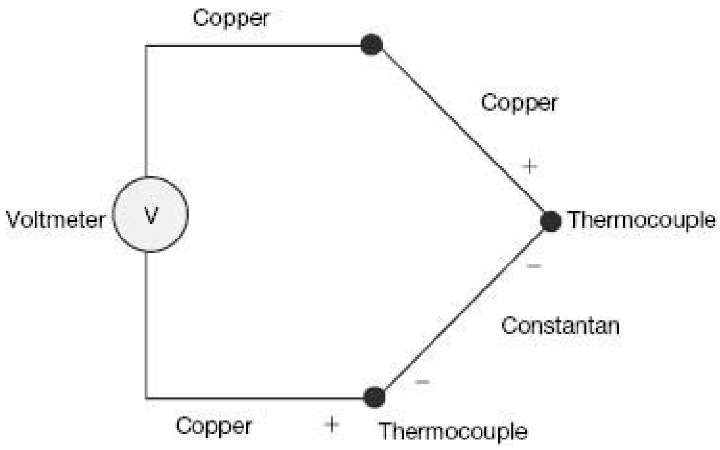


Figure 11.11 *Effects of additional parasitic thermocouple*

Assume that the meter uses copper wires as shown. In this case, where the two copper wires come in contact there is no problem, but where the copper comes in contact with another metal, such as the constantan thermocouple wire, the two dissimilar metals create another thermocouple, which generates its own Seeback voltage. For this example, copper interconnecting wires were used and the thermocouple was copper and constantan. The composition of the wires is immaterial, as any combination will produce these parasitic thermocouples with the problems of additional Seeback voltages. It is an inescapable fact that there will be at least two thermocouple junctions in the system. To contend with this, it is necessary that the temperature of one of the junctions be known and constant. Therefore, there is a fixed offset voltage in the measuring system. In early times, it is mandatory to place this junction in a mixture of ice and water, thus stabilising the temperature to 0°C as shown in Figure 11.13. In modern-age electronic reference junction is used and it is called the reference or cold junction because this junction was traditionally placed in the ice bath.

Figure 11.12 *Connection of wires*

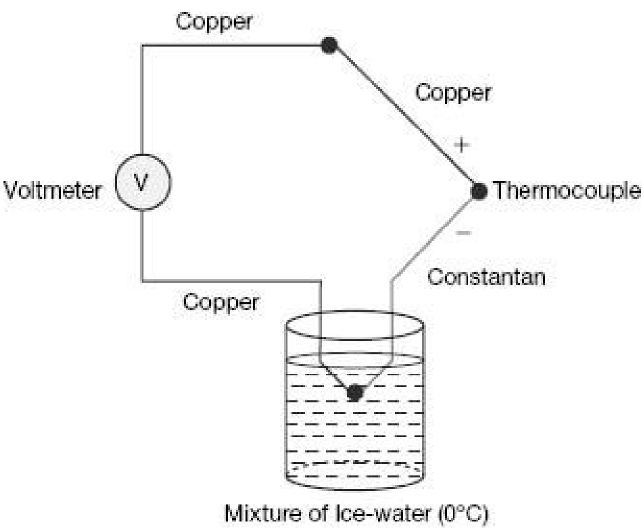


Figure 11.13 *Application of a reference junction*

The classic method of measuring thermocouple voltages was the use of a potentiometer. This was a mechanical device and is no longer used. Completely electronic devices are used to measure thermocouple voltages and to convert from the Seeback voltage to temperature, and to compensate for the reference junction.

11.6.3 Errors occurring During the Measurement using Thermocouple

1. Open Junction

There are many sources of an open junction. Usually, the error introduced by an open junction is of such an extreme magnitude that an open junction is easily spotted. By simply measuring the resistance of the thermocouple, the open junction is easily identified.

2. Insulation Degradation

The thermocouple is often used at very high temperatures. In some cases, the insulation can break down and causes a significant leakage resistance which will cause an error in the measurement of the Seeback voltage. In addition, chemicals in the insulation can defuse into the thermocouple wire and cause decalibration.

3. Thermal Conduction

The thermocouple wire will shunt heat energy away from the source to be measured. For small temperature to be measured, small diameter thermocouple wire could be used. However, the small diameter wire is more susceptible to the effects. If a reasonable compromise between the degrading effects of small thermocouple wire and the loss of thermal energy and the resultant temperature error cannot be found, thermocouple extension wire can be used. This allows the thermocouple to be made of small diameter wire, while the extension wire covers majority of the connecting distance.

4. Galvanic Action

Chemicals coming in contact with the thermocouple wire can cause a galvanic action. This resultant voltage can be as much as 100 times the Seebeck voltage, causing extreme errors.

5. Decalibration

This error is a potentially serious fault, as it can cause slight error that may escape detection. Decalibration is due to altering the characteristics of the thermocouple wire, thus changing the Seeback voltage. This can be caused due to subjecting the wire to excessively high temperatures, diffusion of particles from the atmosphere into the wire, or by cold working the wire.

11.6.4 Thermistors

Thermistors (construction of thermal resistor) are semiconductors which behave as resistors with a high negative temperature coefficient of resistance.

Manganese, nickel or cobalt oxides are milled, mixed in proper proportion with binders, passes into desired shapes and then sintered to form thermistors in the form of beads, rods or discs. Sometimes, a glass envelope is provided to protect a thermistor form contaminations.

The resistance R_T of a thermistor at temperature T (Kelvin) can be written as

$$R_T = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right] \tag{11.13}$$

where R_T and R_0 are the resistances in ohms of the thermistor at absolute temperatures T

and T_0.β is a thermistor constant ranging from 3500 K to 5000 K. The reference temperature T_0 is usually taken as 298 K or 25°C.

Now the temperature coefficient of the resistance

$$\alpha = \frac{1}{R_T}\frac{dR_T}{dT} = -\frac{\beta}{T^2} \text{ [from equation (11.13)]}$$

At T = 298 K, the value of α is

$$\alpha = -\frac{4000}{298^2} = -0.045\ \Omega/°\text{C}\ \frac{\beta}{T^2} \text{ [assume } \beta = 4000 \text{ K]} \quad (11.14)$$

This is evidently a rather high temperature coefficient because for a platinum resistance thermometer the corresponding figure is 0.0035/°C.

The plot of resistivity (ρ) versus temperature (Figure 11.14) will also demonstrate this comparison.

Equation (11.13) can be rearranged to the form

$$\frac{1}{T} = \left(\frac{1}{T_0} - \frac{1}{\beta}\ln R_0\right) + \frac{1}{\beta}\ln R_T \quad (11.15)$$

$$\frac{1}{T} = A + B\ln R_T \quad (11.16)$$

where A and B are constants. Eq. (11.16) may alternatively be used to find temperatures by evaluating A and B from two pairs of known values of R_T and T.

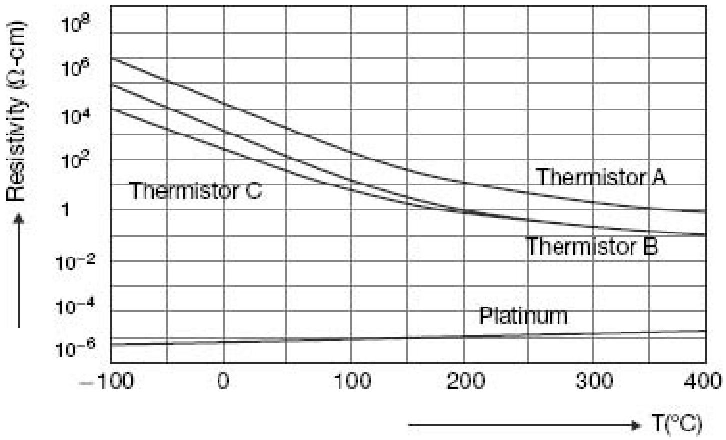


Figure 11.14 *Comparison of resistivity of platinum with that of thermistors*

Thermistors are very popular as temperature transducers because (i) they are compact, rugged, inexpensive, (ii) their calibration is stable, (iii) they have a small response time, (iv) they are amenable to remote measurements, and above all, (v) their accuracy is high.

Example 11.2

For a certain thermistor β = 3100 K and its resistance at 20°C is known to be 1050 Ω. The thermistor is used for temperature measurement and the resistance measured is 2300 Ω. Find the measured temperature if the temperature-resistance characteristics of the thermistor is given by

$$R = R_0 \exp\left[\beta\left(\frac{1}{T} - \frac{1}{T_0}\right)\right]$$